Engineering Tripos Part 1A - 1996

Paper 1 - MECHANICAL ENGINEERING - ANSWERS

2. (c) 493 °C, 91% (d) 37%, 164 kg/s

3. (a) $\eta \leq 1 - \frac{T_2}{T_1}$ (b) $\frac{1 - \frac{T_2}{T_1}}{1 - \frac{T_2}{T_3}} Q_1$

4. 1010 K, 288 m/s, 56.0 J/K

Section B

5. b) $\sqrt{gh}\ \underline{j}$

$\frac{\sqrt{gh}}{3}\left(-\sqrt{2}\sin\theta\underline{i} + 2\underline{j} - \sqrt{2}\cos\theta\underline{k}\right)$

6. a) 91 mm/s to the right

0.1625 rad/s anticlockwise

0.65 rad/s clockwise

46 mm/s

b) i) 1.46 Nm

ii) 3.42 Nm

7. a) $\dot{\theta} + \dot{\phi}$

b) $-R\dot{\theta}\underline{e}_1 + R\phi\left(\dot{\theta} + \dot{\phi}\right)\underline{e}_2$

$-R\phi\left(\dot{\theta} + \dot{\phi}\right)^2\underline{e}_1 R\left(\phi\ddot{\phi} + \dot{\phi}^2 - \dot{\theta}^2\right)\underline{e}_2$

c) $4mR^2\,\theta\,\dot{\theta}^2$

8. a) i) 150 cos α mm/s

ii) 300 + 150 sin α mm/s

b) 13.19 rad/s

Section C

9. a) 12 s 24.95 s

b) 38.9 s

c) 37.4 V 51.5°

10. 130.8 kN/m

375.8 Ns/m

98.1 m/s^2

75.7 m/s^2

11. b) $0.521\sqrt{k/J}$: $\theta_2 = 2.186\,\theta_1$

$1.108\sqrt{k/J}$: $\theta_2 = -0.686\,\theta_1$

c) $\sqrt{k/J}$, T/k

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Paper 2 Section A - STRUCTURES

Answers

Q1 (a) $\sqrt{3} = 1.73$

(b) 4.30 and 0.697

Q2 (a)

DG	DC	GF	GC	CF	CB
$-W\sqrt{2}$	W	-W	W	$-W\sqrt{2}$	2W
$-2WL/AE$	WL/AE	$-WL/AE$	WL/AE	$-2WL/AE$	$2WL/AE$

(b) $(3+2\sqrt{2})WL/AE$ down, WL/AE to the left

(c) L/50

Q3 (a) $-q_o L^2/16$

(b) $-q_o L^2/48$

(c) $-q_o L^2/160$

Q4 $M = Wa - Wx + W[x-a]$

$$v = -\frac{Wax^2}{2EI} + \frac{Wx^3}{6EI} - \frac{W}{6EI}[x-a]^3$$

$$v(L) = -\frac{WaL^2}{2EI} + \frac{WL^3}{6EI} - \frac{W}{6EI}[L-a]^3 = -\frac{Wa^2}{6EI}(3L-a)$$

Q5 173 MPa $10738 \times 10^4\ mm^4$ 96 MPa 13.6 kN

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PAPER 2, SECTION B (MATERIALS) : NUMERICAL SOLUTIONS

6. $\left(\frac{dF}{dr}\right)_{r=r_0} = S_0 = -\frac{2A}{r_0^3} + \frac{n(n+1)B}{r_0^{(n+2)}}$

$r_0 = 2.3$ Å

7. $\sigma_n = \frac{\sigma_0[\ln(1+\varepsilon_n)]^n}{(1+\varepsilon_n)}$

Annealed 70/30 brass (70% copper and 30% zinc)

$F < 31$ kN

8. $N_f \sim 10^5$ cycles

9. $n \sim 5$, power law creep

$Q = 188872$ J mol $^{-1}$

$A = 1.07 \times 10\text{-}35$ Pa-5 s-1

Time = 752 hours

10. $W^2 = K_1 t$ ($n = 2$ and $K_1 = 0.0266$)

$\Delta W = 4$ mg cm^{-2}

Nickel, tin, cadmium.

Part IA, 1996

Electrical Paper

Answers

Q.1 $R = 9M\Omega$; $\bar{Z}_{in} = \frac{10^6 - j100\pi f}{1+10^{-8}\pi^2 f^2}$; $\bar{V} = 387\angle -78.5°$

Q.2 $R_1 = 80k\Omega$, $R_2 = 10k\Omega$; GAIN = - 44.4, $R_{\text{out}} = 44.4k\Omega$; $C_1 = 195nF$

Q.3 $R_2 = 1M\Omega$; $R_{in} = R_1 = 10k\Omega$; $C_1 = 31.8pF$; GAIN = 39.7 d B

Q.4 6.34A, PF = 0.22 log ; 276W, 1464 Vars, 60.3 W (cable) ; 84.4 μ F, 2.13 W, 0.67 Vars

5. $Y = AC\overline{D} + \overline{A}BC$

To produce a hazard-free solution the additional term $BC\overline{D}$ may be ORed in to the expression.

6. The unused states are 000, 011, 100, 110

$J_A = \overline{Q}_C\overline{M} + Q_C M$ $\qquad$ $K_A = Q_B\overline{M} + \overline{Q}_B M$

$J_B = 1$ $\qquad$ $K_B = 1$

$J_C = 1$

$K_C = Q_A\overline{Q}_B M + \overline{Q}_A\overline{M}$

7(b) $51_{10} = 00110011_2$
$126_{10} = 01111110_2$
$-126_{10} = 10000010_2$
$98_{10} = 01100010_2$

(c) With outputs Q and Q':

$Q = \overline{\overline{S}.Q'} = S + \overline{Q'}$

$Q' = \overline{\overline{R}.Q} = R + \overline{Q}$

8. At the end of the code segment, $40 contains the total number of 1 bits set in the ten data values, i.e. 0+1+1+2+1+2+2+3+1+2=15.

Q.9 W = $\frac{1}{2}$ Q V ; F = $\frac{1}{2}\frac{A\sigma}{d}V - \frac{1}{2}\frac{A\sigma^2}{\epsilon_o d}x$; $8.9 \times 10^{-8} Cm^{-2}$

Q.10 $C = 2\pi\epsilon_o / \left(\frac{1}{\epsilon_1}\ell n\left(\frac{r_3}{r_1}\right) + \frac{1}{\epsilon_2}\ell n\left(\frac{r_2 - t_2}{r_3}\right)\right)$; $4.6 \times 10^{-10} Fm^{-1}$; $5.5 \times 10^4 Vm^{-1}$

Q.11 0.95 amp ; 0.2 T ; less than 2T

Numerical Answers to 1996 Part 1A, "Mathematical Methods"

Question 1

equation 1: $\underline{\mathbf{r}} = (1\ \ 1\ \ 2)^t + \lambda(1\ -2\ \ 2)^t + \mu(3\ -2\ \ 1)^t$ plus various other combinations

equation 2: $\underline{\mathbf{r}} \bullet \underline{\mathbf{n}} = \underline{\mathbf{s}}$ i.e. $(1\ \ 1\ \ 2)^t \bullet (2\ \ 5\ \ 4)^t = 15$ (or $2x + 5y + 4z = 15$)

unit normal: $\pm\frac{1}{\sqrt{45}}(2\ \ 5\ \ 4)$

angle: $90 - cos^{-1}\frac{5}{\sqrt{45}}$. point: 1/3(2 5 4). distance: $\frac{15}{\sqrt{45}}$.

Question 2.

(i) $z = \pm i \sinh \pi$

(ii) limit = -2i

(iii) series = $\frac{1}{\sqrt{2}}\left[1 + 2x + x^2 + O\left(x^4\right)\right]$

Question 3.

A followed by **B**: $\frac{1}{2}\begin{bmatrix} \sqrt{2} & -1 & -1 \\ 0 & \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & 1 & 1 \end{bmatrix}$. det **BA** = 1. $(\mathbf{BA})^{-1} = (\mathbf{BA})^t$.

$x = 4\sqrt{2}$, $y = 2\sqrt{2} - 2$, $z = -2\sqrt{2} - 2$

B followed by **A**: $\frac{1}{2}\begin{bmatrix} \sqrt{2} & 0 & -\sqrt{2} \\ 0 & \sqrt{2} & -1 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$

Question 4.

General Solution: $y = e^x(A\sin x + B\cos x) + e^{-x}(\sin x + \cos x)$.

When $x = \pi$, $y = -e^{-\pi}$.

Question 5.

Let $m = n + 1/2$ (n odd) and $m = n/2$ (n even).

Then $a_m = \frac{2}{3} + \frac{1}{12}\cdot 4^m$ (odd terms).

and $a_m = \frac{2}{3} - \frac{1}{12}\cdot 4^m$ (even terms).

Question 6.

eigenvectors of $\mathbf{A}^{-1}$ same as for **A**; eigenvalues of $\mathbf{A}^{-1}$ are 1/a, 1/b, 1/c respectively

$\mathbf{A} = \begin{bmatrix} \frac{a+c}{2} & \frac{a-c}{2} & 0 \\ \frac{a-c}{2} & \frac{a+c}{2} & 0 \\ 0 & 0 & b \end{bmatrix}$; if a = 3, b = 2, c = 1, $\mathbf{A}^{-1} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & 0 \\ -\frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$.

7. (a) Step response: $y(t) = 1 - e^{-t/\alpha}$ for $t \geq 0$.

   (b) $g(t) = \frac{1}{\alpha} e^{-t/\alpha}$

   (d) $y(t) = \frac{e^{-t/\alpha} - e^{-t/\beta}}{\alpha - \beta}$.
   Tends to impulse response as $\beta \to 0$.

8. $f(t) = \sinh(1) + 2\sinh(1) \sum_{n=1}^{\infty} \frac{1}{1 + n^2\pi^2} \cos(n\pi t)$

9. $x(t) = \frac{1}{2} \sin 2t + \cos 2t - \frac{1}{2} h(t-3) + \frac{1}{2} \cos\left(2(t-3)\right) h\left(t-3\right)$ for $t \geq 0$.

10. $\alpha = -1,\ \beta = -2$.
    $f(x,y) = \frac{1}{\sqrt{2}} \arctan\left(\frac{x}{\sqrt{2}}\right) - \frac{x}{y} + y +$ constant.
    Steepest ascent direction: $(-1, 3)\,\frac{1}{\sqrt{10}}$.