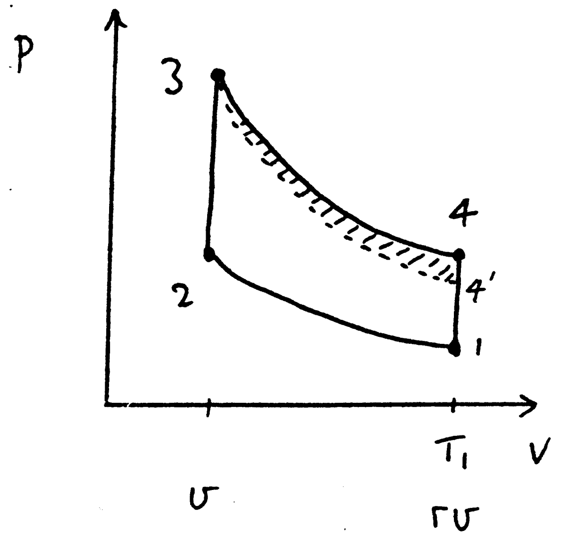
Part IA Paper 1 1996

1. Petrol is more volatile than diesel fuel. Its more "violent" combustion leads to mechanical problems if used in a compression ignition cycle. Petrol engines are thus limited to compression ratios which avoid compression ignition and use a spark to initiate combustion.



$1 \to 2$ Adiabatic + reversible $\Rightarrow$ isentropic

$\Rightarrow Tv^{\gamma-1} = \text{const}$

$\therefore T_2 = T_1 r^{\gamma-1}$

$2 \to 3 \quad T_3 = \beta T_2 = \beta T_1 r^{\gamma-1}$

$3 \to 4 \quad T_4 = \dfrac{T_3}{r^{\gamma-1}} = \beta T_1$

Heat transfer:

$Q_{12} = 0 \qquad Q_{23} = c_v(T_3 - T_2)$ (heating at const volume

$Q_{34} = 0 \qquad Q_{41} = c_v(T_1 - T_4)$

$$\therefore \quad \eta = \frac{Q_{NET}}{Q_{23}} = 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{(\beta-1)T_1}{(\beta-1)r^{\gamma-1}T_1}$$

$$= 1 - \frac{1}{r^{\gamma-1}}.$$

Heat transfer during $3 \to 4$ will be out of gas $\Rightarrow$ temperature at 4 reduced (to 4' say).

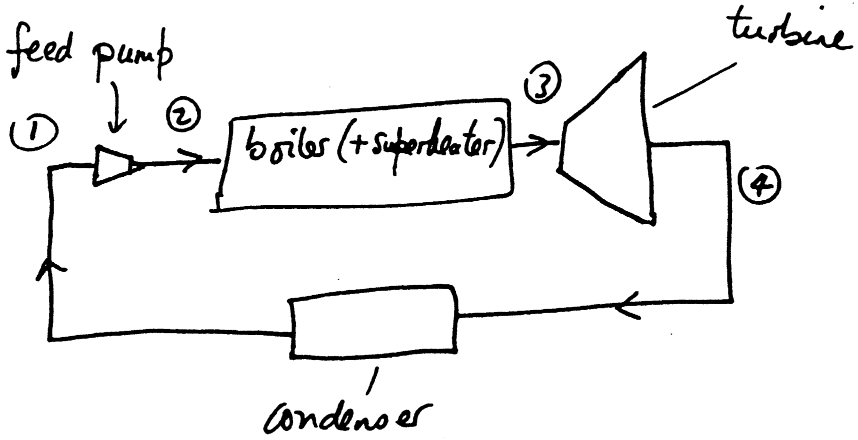
$$\eta = \frac{W_{net}}{Q_{23}}, \quad W_{net} = \oint p\,dv = \text{area inside cycle}$$

Shaded area is subtracted from W_{NET} but Q_{23} not changed

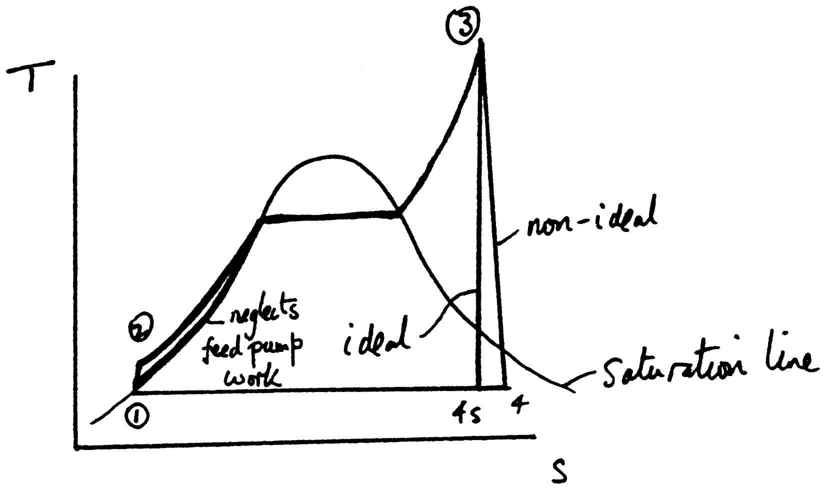
$\Rightarrow \eta$ reduced.

2.

(a)



(b)



(c) From chart: at turbine exit $p_4 = 0.04$, dryness $= .85$

$$\Rightarrow h_4 = 2192 \quad kJ/kg$$

at ideal turbine exit $p_4 = 0.04$, dryness $= .8 \Rightarrow h_{4s} = 2070$

and $s_{4s} = 6.86 \ kJ/kg\,K$

$p_3 = 60$ & $s_3 = s_{4s} \Rightarrow h_3 = 3408$ and $T_3 = 493°C$

$$\eta_{turbine} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{3408 - 2192}{3408 - 2070} = 90.9\%$$

(d) $h_2 \simeq h_1 = 121.4 \ kJ/kg$ (Table 8)

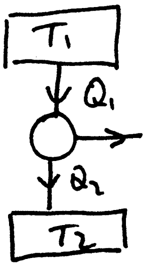
$$\therefore \ \eta_{cycle} = \frac{h_3 - h_4}{h_3 - h_2} = \frac{3408 - 2192}{3408 - 121.4} = 37\%$$

$$\text{Power} = \dot{m}(h_3 - h_4) \Rightarrow \dot{m} = \frac{200 \times 10^6}{1216 \times 10^3} = 164 \ kg/s$$

3\. (a) For a system undergoing a cyclic process, Clausius' Inequality states that

$$\oint \frac{dQ}{T} \le 0$$

where dQ represents heat transfer <u>into</u> the system taking place at temperature T.

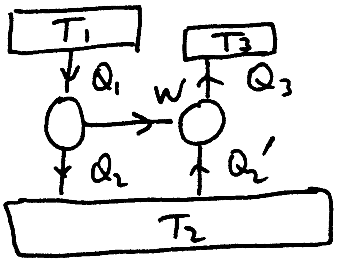


Clausius (applied to engine as the system) $\Rightarrow \dfrac{Q_1}{T_1} - \dfrac{Q_2}{T_2} \le 0$

i.e. $\dfrac{Q_2}{Q_1} \geqslant \dfrac{T_2}{T_1}$

$$\therefore \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \le 1 - \frac{T_2}{T_1}$$

(b)



For maximum work output from the engine

$$\frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

For maximum heat pumped $\dfrac{Q_3}{T_3} = \dfrac{Q_2'}{T_2}$

Thus $Q_3 = W + Q_2' = Q_1\left(1 - \dfrac{T_2}{T_1}\right) + \dfrac{T_2}{T_3} Q_3$

i.e. $$Q_3 = \frac{1 - \frac{T_2}{T_1}}{1 - \frac{T_2}{T_3}} Q_1 \qquad (1)$$

$Q_3 > Q_1 \Rightarrow 1 - \dfrac{T_2}{T_3} > 1 - \dfrac{T_2}{T_1}$ i.e. $T_3 < T_1$

Clausius has nothing <u>directly</u> to say about $\dfrac{Q_1}{T_1}$ & $\dfrac{Q_3}{T_3}$

(If system is engine plus heat pump $\dfrac{Q_1}{T_1} - \dfrac{Q_3}{T_3} + \dfrac{Q_2' - Q_2}{T_2} \le 0$ $* Q_2 \ne Q_2'$)

Examiner's Note:

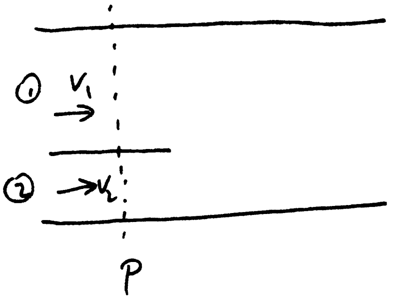
A variety of answers were accepted for this last part:—

(i) Eqn (1) which was derived using Clausius gives:-

$\dfrac{Q_3}{T_3} \le \dfrac{T_1 - T_2}{T_3 - T_2} \dfrac{Q_1}{T_1}$ which is a valid way of interpreting the quest

(ii) Some attempts were $\dfrac{Q_1}{T_1} - \dfrac{Q_3}{T_3} \le \dfrac{Q_2 - Q_2'}{T_2}$ which isn't wrong!

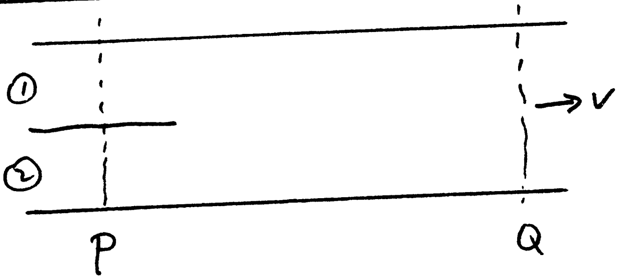
4.



Mass flow in stream ① $= \rho_1 V_1 A_1 = \frac{P_1 V_1 A_1}{RT_1}$

Similarly for stream ②

$\therefore$ Ratio $= \frac{P_1 V_1 A_1}{RT_1} \frac{RT_2}{P_2 V_2 A_2} = \frac{300 \times 3A/4}{1200} \times \frac{600}{200 \times A/4} = \frac{9}{4}$



Let $\dot{m}_2 = \dot{m} \Rightarrow \dot{m}_1 = \frac{9\dot{m}}{4}$

Conservation of mass between P & Q

$\Rightarrow \dot{m}|_Q = \frac{13\dot{m}}{4} = \frac{pvA}{RT}$ Ⓐ

S.F.E.E. applied to pipe between P & Q $(\dot{Q} = \dot{W}_x = 0)$ Ⓑ

$\Rightarrow \dot{m}_1\left(c_pT_1 + \frac{V_1^2}{2}\right) + \dot{m}_2\left(c_pT_2 + \frac{V_2^2}{2}\right) = (\dot{m}_1 + \dot{m}_2)\left(c_p T + \frac{v^2}{2}\right)$

i.e. $c_pT + \frac{v^2}{2} = 1.063 \times 10^6$

Ⓐ $\Rightarrow \frac{pvA}{RT} = \frac{13}{4}\frac{P_2 V_2}{RT_2}\frac{A}{4} \Rightarrow \frac{v}{T} = \frac{13}{4}\frac{P_2}{p}\frac{V_2}{T_2}\cdot\frac{1}{4} = \frac{13}{4}\frac{10}{9.5}\frac{200}{600}\cdot\frac{1}{4} = .285$

Eliminating $T \Rightarrow v^2 + 7.09 \times 10^3 v - 2.126 \times 10^6 = 0$

i.e. $v = 288$ m/s (other root –ve) and $T = 1010$ K

Change in entropy $= \dot{m}s - \dot{m}_1 s_1 - \dot{m}_2 s_2 \Rightarrow$ change/unit mass $= s - \frac{\dot{m}_1}{\dot{m}}s_1 - \frac{\dot{m}_2}{\dot{m}}s_2$

$\Rightarrow \Delta s = c_p \ln T - R\ln p - \frac{9}{13}(c_p \ln T_1 - R\ln p_1) - \frac{4}{13}(c_p \ln T_2 - R\ln p_2)$

$= 56.0$ J/K

This comes about because of the effect of viscosity and heat conduction through finite temperature differences during mixing.

Examiners Note (a) Most candidates found the algebra difficult under exam conditions and that aspect was marked very leniently.

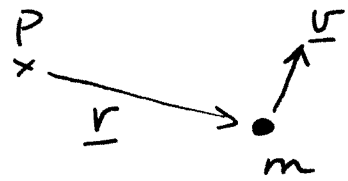
(b) A variety of answers to the physical processes responsible were accepted:-

e.g. (i) Mixing increases the randomness of the system and entropy is a direct function of randomness.

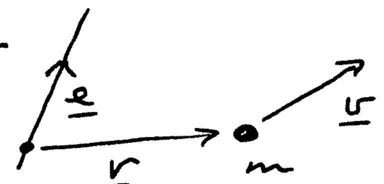
(ii) "The rise is due to mixing" (iii) Mixing is fundamentally an irreversi process and entropy rises in an irreversible process, etc.

5a) Moment of momentum:

About a point: $\underline{r} \times m\underline{v}$

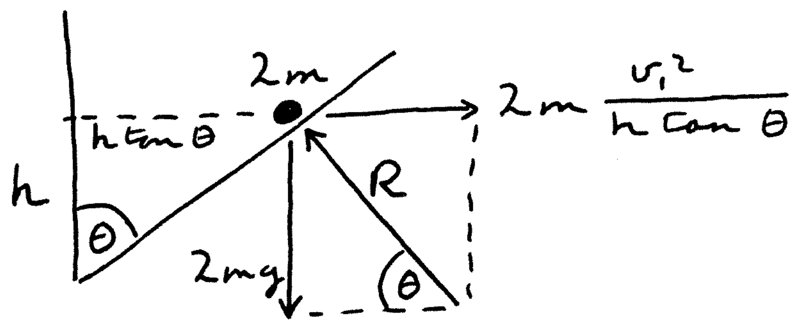


About a line: $(\underline{r} \times m\underline{v}) \cdot \underline{e}$



Conserved about a line if resultant force on particle has no moment about the line, i.e. it acts through, or parallel to the line.

b)



For constant height, total acceleration must be ⊥ to cone surface,

i.e. $\tan\theta = \dfrac{gh\tan\theta}{v_1^2} \Rightarrow v_1 = \sqrt{gh}$

$\therefore$ $\underline{v}_1$ just before impact $= \sqrt{gh}\,\underline{j}$

For smaller mass, P.E. → K.E., i.e.

$$\tfrac{1}{2} m v_2^2 = mgh$$

so $\underline{v}_2 = \sqrt{2gh}\,(-\sin\theta\,\underline{i} - \cos\theta\,\underline{k})$

Linear momentum is conserved on impact so final velocity is given by:

5 (b) (cont.)

$$\underline{v}_3 = \frac{\sqrt{gh}}{3} \begin{bmatrix} -\sqrt{2} \sin\theta \\ 2 \\ -\sqrt{2}\cos\theta \end{bmatrix}$$

c) Mass continues to travel round cone, with its height oscillating between a fixed maximum and minimum.

Two main equations are:
1) K.E. + P.E. = constant
2) Moment of momentum about Z axis = const.

If subsequent velocity is $\underline{v}$ and height is x, then

1) $\Rightarrow \frac{3}{2} m v^2 + 3 m g x = 3m\left(\frac{1}{2} v_3^2 + gh\right)$

Now $v_3^2 = \dfrac{2gh}{3}$, so $\frac{3}{2} v^2 + 3gx = 4gh$ ③

At maximum & minimum heights, velocity is horizontal, so

2) $\rightarrow 3 m v x \tan\theta = 2m\sqrt{gh}\, h\tan\theta$

$$\Rightarrow v = \frac{2h}{3x}\sqrt{gh} \quad ④$$

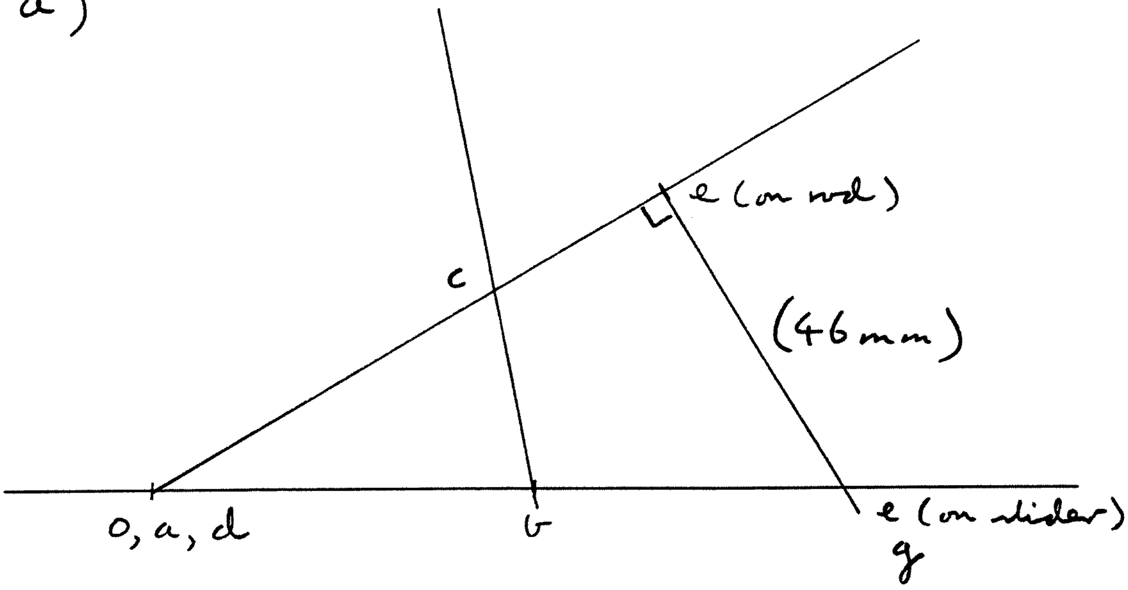
Combining ③ and ④,

$$\frac{2}{3} \frac{h^2}{x^2} gh + 3gx = 4gh$$

$$\Rightarrow 2h^3 + 9x^3 - 12hx^2 = 0$$

which has roots at $\begin{cases} x = 0.524h \\ x = 1.171h \end{cases}$

6. a)



From diagram, velocity of G = $\underline{\underline{91}}$ mm/s to the right.

Angular velocity of CB = $\frac{26}{160}$ = $\underline{\underline{0.1625}}$ rad/s (anticlockwise)

Angular velocity of DF = $\frac{52}{80}$ = $\underline{\underline{0.65}}$ rad/s (clockwise)

b) Let torque = Q, use virtual work:

i) $1.25\,Q = 20 \times 91 \times 10^{-3}$

$\Rightarrow \underline{\underline{Q = 1.46 \text{ Nm}}}$

ii) Sliding velocity is ~ $\underline{\underline{46 \text{ mm/s}}}$

$\therefore 1.25\,Q = 20 \times 91 \times 10^{-3} + 5 \times 46 \times 10^{-3} + 1 \times (1.25 + 0.1625 + 0.65 + 0.1625)$

$\Rightarrow \underline{\underline{Q = 3.42 \text{ Nm}}}$

7 a) $\underline{P} = R\,\underline{e}_2 + R\phi\,\underline{e}_1$

Angular velocity of BP is $(\dot{\theta} + \dot{\phi})$

so $\dot{\underline{e}}_1 = (\dot{\theta} + \dot{\phi})\,\underline{e}_2$ and $\dot{\underline{e}}_2 = -(\dot{\theta} + \dot{\phi})\,\underline{e}_1$

b) $$\begin{aligned}\dot{\underline{P}} &= R\dot{\underline{e}}_2 + R\dot{\phi}\,\underline{e}_1 + R\phi\,\dot{\underline{e}}_1 \\ &= -R(\dot{\theta} + \dot{\phi})\underline{e}_1 + R\dot{\phi}\,\underline{e}_1 + R\phi(\dot{\theta} + \dot{\phi})\,\underline{e}_2 \\ &= \underline{\underline{-R\dot{\theta}\,\underline{e}_1 + R\phi(\dot{\theta} + \dot{\phi})\,\underline{e}_2}}\end{aligned}$$

$$\begin{aligned}\ddot{\underline{P}} &= -R\dot{\theta}\,\dot{\underline{e}}_1 + R\dot{\phi}(\dot{\theta} + \dot{\phi})\underline{e}_2 + R\phi\ddot{\phi}\,\underline{e}_2 + R\phi(\dot{\theta} + \dot{\phi})\,\dot{\underline{e}}_2 \\ &= R(-\dot{\theta}^2 - \dot{\theta}\dot{\phi} + \dot{\phi}\dot{\theta} + \dot{\phi}^2 + \phi\ddot{\phi})\,\underline{e}_2 - R\phi(\dot{\theta} + \dot{\phi})^2\underline{e}_1 \\ &= \underline{\underline{R(\phi\ddot{\phi} + \dot{\phi}^2 - \dot{\theta}^2)\,\underline{e}_2 - R\phi(\dot{\theta} + \dot{\phi})^2\,\underline{e}_1}}\end{aligned}$$

c) Since all forces on the particle act in the $\underline{e}_1$ direction, there will be no acceleration in the $\underline{e}_2$ direction.

$$\therefore \quad \phi\ddot{\phi} + \dot{\phi}^2 - \dot{\theta}^2 = 0$$

which is clearly satisfied by $\phi = \theta$, since $\ddot{\theta} = 0$.

Then $$\begin{aligned}T &= -mR(\ddot{\underline{P}} \cdot \underline{e}_1) \\ &= mR^2\phi(\dot{\theta} + \dot{\phi})^2 \\ &= \underline{\underline{4mR^2\theta\dot{\theta}^2}} \quad \text{if } \phi = \theta\end{aligned}$$

8 a) Let P be point on cam currently in contact with the follower.

$$\underline{v}_P = \underline{\omega} \times \underline{r}_P = 10\,\underline{k} \times (15\cos\alpha\,\underline{i} + (30+15\sin\alpha)\,\underline{j})$$
$$= -(300+150\sin\alpha)\,\underline{i} + 150\cos\alpha\,\underline{j}$$

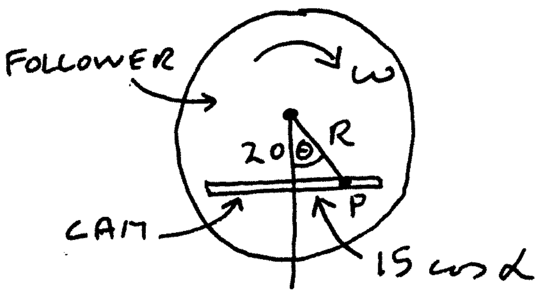
Follower must have the same vertical velocity, i.e. $\underline{\underline{150\cos\alpha \text{ mm/sec}}}$

Since follower has no horizontal velocity, sliding velocity = horizontal vel. of cam

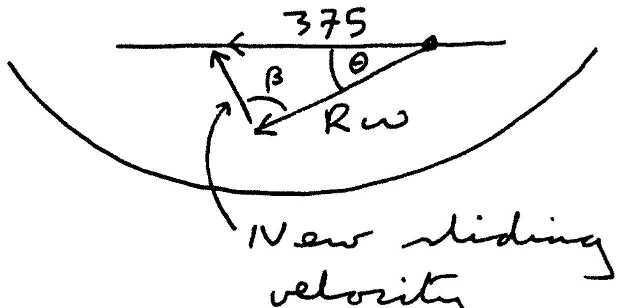
$\therefore$ sliding velocity = $\underline{\underline{300+150\sin\alpha \text{ mm/sec}}}$

b) When $\alpha = 30°$, horizontal velocity of contact point on cam = 375 mm/sec

View from above:



Velocities:



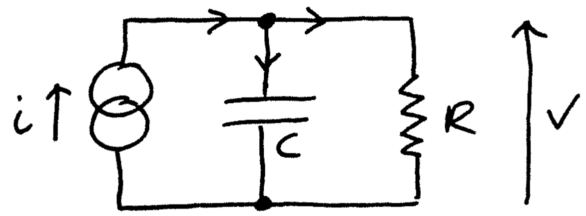
By inspection $R = \dfrac{20}{\cos\theta}$, $\theta = \tan^{-1}\left(\dfrac{15\cos\alpha}{20}\right) = 33°$

For min. sliding velocity, $\beta = 90°$

i.e. $\omega R = 375\cos\theta$

$$\therefore \omega = \frac{375\cos^2\theta}{20} = \underline{\underline{13.19 \text{ rad/sec}}}$$

9 a)



$$i = C\frac{dV}{dt} + V/R \quad \text{where} \quad \frac{1}{R} = \left(\frac{1}{200\times10^3} + \frac{1}{300\times10^3}\right)$$

$$\rightarrow R = 120\times10^3$$

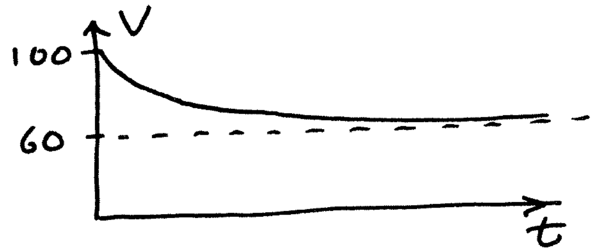
$$\therefore CR\dot{V} + V = iR \quad \text{or} \quad 12\dot{V} + V = 60$$

$$\Rightarrow V = 60 + Ae^{-t/T} \quad \text{where} \quad \underline{\underline{T = CR = 12\ \text{sec}}}$$

@ $t = 0$, $V = 0.5\times10^{-3}\times200\times10^{3} = 100\ \text{v}$

@ $t = \infty$, $V = 0.5\times10^{-3}\times120\times10^{3} = 60\ \text{v}$

So $V = 60 + 40e^{-t/12}$



$V = 65$ when $40e^{-t/12} = 5$

i.e. $e^{t/12} = 8$

$\therefore t = 12\ln 8 = \underline{\underline{24.95\ \text{sec}}}$

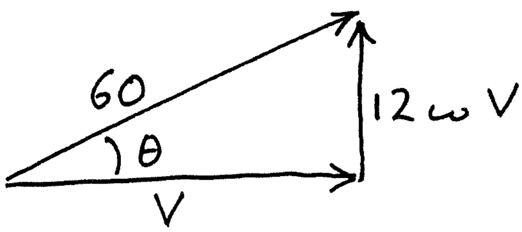
b) $R' = 200\times10^3 \Rightarrow T' = CR' = 20\ \text{sec}$

$\Rightarrow V = 100 - 35e^{-t'/20}$

$V = 95$ when $35e^{-t'/20} = 5 \Rightarrow e^{t'/20} = 7$

$\therefore t' = 20\ln 7 = \underline{\underline{38.9\ \text{sec}}}$

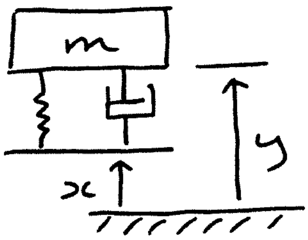
c) $v = Ve^{j\omega t}$ where $\frac{2\pi}{\omega} = 60 \Rightarrow \omega = 0.1047$



$\theta = \tan^{-1}(12\omega) = \underline{\underline{51.5°}}$

$V = 60\cos\theta = \underline{\underline{37.4\ \text{V}}}$

10



$$m\ddot{y} = \lambda(\dot{x} - \dot{y}) + k(x - y)$$

$$m\ddot{y} + \lambda\dot{y} + ky = \lambda\dot{x} + kx$$

$\Rightarrow$ case (c)

For max $Y/X = 2$, $c = 0.3$ from data book.

Then $Y/X = 0.25$ when

$$\frac{\sqrt{1+(0.6r)^2}}{\sqrt{(1-r^2)^2+(0.6r)^2}} = 0.25 \quad \text{where } r = \frac{\omega}{\omega_n}$$

$$16(1 + 0.36r^2) = 1 - 2r^2 + r^4 + 0.36r^2$$

$$r^4 - 7.4r^2 - 15 = 0$$

$$r^2 = \frac{7.4 \pm \sqrt{54.76+60}}{2} = 9.056$$

$$\omega/\omega_n = 3.001 \Rightarrow \omega_n = \frac{200\pi}{3.001} = \underline{208.79}$$

$$\therefore\ k/3 = 208.79^2 \Rightarrow \underline{\underline{k = 130.8 \text{ kN/m}}}$$

$$\frac{2c}{\omega_n} = \frac{0.6}{208.79} = \frac{\lambda}{k} \Rightarrow \underline{\underline{\lambda = 375.8 \text{ Ns/m}}}$$

Now $X = \frac{1}{2} \times 2$ mm $= 0.001$ m, and $\ddot{Y} = \omega^2 Y$

Resonance is at $\omega_n(1-c^2) = 0.932\,\omega_n$

$\Rightarrow$ Max accn $= 2 \times 0.001 \times (0.932\,\omega_n)^2$

$= \underline{\underline{75.7 \text{ m/s}^2}}$

At running speed, max acceleration

$= 0.25 \times .001 \times (200\pi)^2 = \underline{\underline{98.1 \text{ m/s}^2}}$

11 a) In matrix form, equations of motion are:

$$\begin{bmatrix} 3J & 0 \\ 0 & 2J \end{bmatrix}\begin{bmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix} + \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ T\cos\omega t \end{bmatrix}$$

↖ mass matrix ↖ stiffness matrix

b) For resonant frequencies, let $\begin{cases} T = 0 \\ \ddot\theta = -\omega^2\theta \end{cases}$

Then $$\begin{bmatrix} 3k - 3J\omega^2 & -k \\ -k & k - 2J\omega^2 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (1)$$

or $[A][\theta] = [0]$

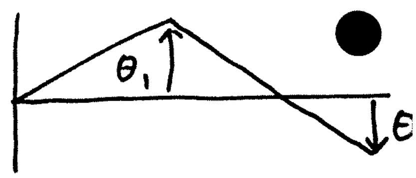
Solutions are when $|A| = 0$, i.e.

$$3k^2 - 9kJ\omega^2 + 6J^2\omega^4 - k^2 = 0$$
$$6J^2\omega^4 - 9kJ\omega^2 + 2k^2 = 0$$
$$\omega^2 = \frac{k}{J}\left(\frac{9 \pm \sqrt{81-48}}{12}\right) = \begin{matrix} 1.2287 & k/J \\ 0.2713 & k/J \end{matrix}$$

Substituting in (1),

(i) $\omega = 1.108\sqrt{k/J}$

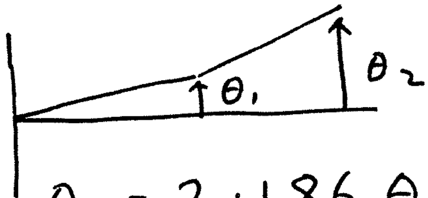
$$\begin{bmatrix} -.686 & -1 \\ -1 & -1.4574 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$\theta_2 = -0.686\,\theta_1$

(ii) $\omega = 0.521\sqrt{k/J}$

$$\begin{bmatrix} 2.1861 & -1 \\ -1 & 0.4574 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$\theta_2 = 2.186\,\theta_1$

c) write $[A]\begin{bmatrix} \Theta_1 \\ 0 \end{bmatrix}\cos\omega t = \begin{bmatrix} 0 \\ T \end{bmatrix}\cos\omega t$

whence $\underline{\underline{\omega = \sqrt{k/J}}}$ and $\underline{\underline{\Theta_1 = T/k}}$