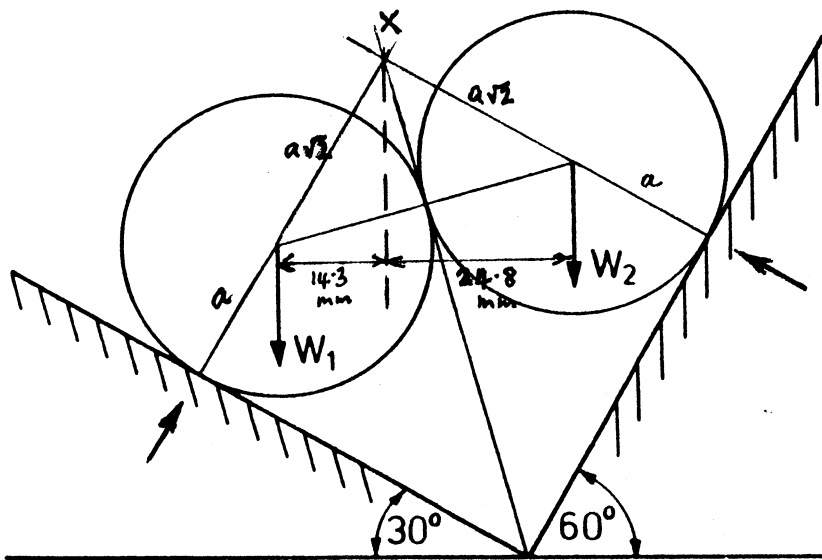


Q1

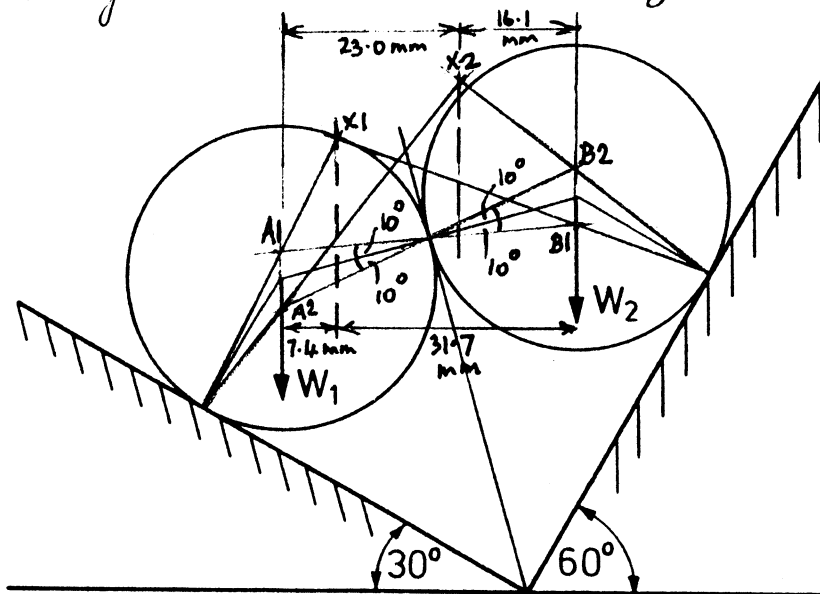


(a) Moments about X for both cylinders

$$W_1 a\sqrt{2} \sin 30^\circ = W_2 a\sqrt{2} \sin 60^\circ \quad \therefore \underline{W_1/W_2 = \sqrt{3}}$$

(or by measurement $W_1 \times 14.3 = W_2 \times 24.8 \quad \therefore W_1/W_2 = 1.73$)

(b) No friction between cylinders, hence reaction forces between cylinders still pass through their centres. The reactions between the planes & cylinders must also pass through the centres and meet again at X



(c) Limiting friction occurs at the cylinder/cylinder contact. Draw in reactions so that the reactions on each cylinder and the weight pass through one point A1 & B1 for one equilibrium state and A2 and B2 for the other. Take moments about X1 and X2 where the reactions intersect.

$$W_1 \times 7.4 = W_2 \times 31.7 \quad \therefore \underline{W_1/W_2 = 4.3} \quad \text{and} \quad W_1 \times 23.0 = W_2 \times 16.1 \quad \therefore \underline{W_1/W_2 = 0.7}$$

2

Alternatively, assume slipping occurs between cylinders.

Take moments about C_1 for cylinder 1

$$W_1 a \sin 30^\circ = R a \sin 45^\circ \pm \mu R (a + a \cos 45^\circ)$$

Take moments about C_2 for cylinder 2

$$W_2 a \sin 60^\circ = R a \sin 45^\circ \mp \mu R (a + a \cos 45^\circ)$$

The signs of the friction terms depend on the direction of slipping

$$\therefore \frac{W_1 (\frac{1}{2})}{W_2 (\frac{\sqrt{3}}{2})} = \frac{\frac{1}{2} \pm \mu (1 + \frac{1}{\sqrt{2}})}{\frac{1}{2} \mp \mu (1 + \frac{1}{\sqrt{2}})} = \frac{1 \pm \mu (\sqrt{2} + 1)}{1 \mp \mu (\sqrt{2} + 1)}$$

$$\mu = \tan 10^\circ = 0.1763$$

With friction direction shown in the figure $\frac{W_1}{W_2} = \sqrt{3} \frac{1 + 0.426}{1 - 0.426} = \underline{4.30}$

With friction in the reverse direction

$$\frac{W_1}{W_2} = \sqrt{3} \frac{1 - 0.426}{1 + 0.426} = \underline{0.697}$$

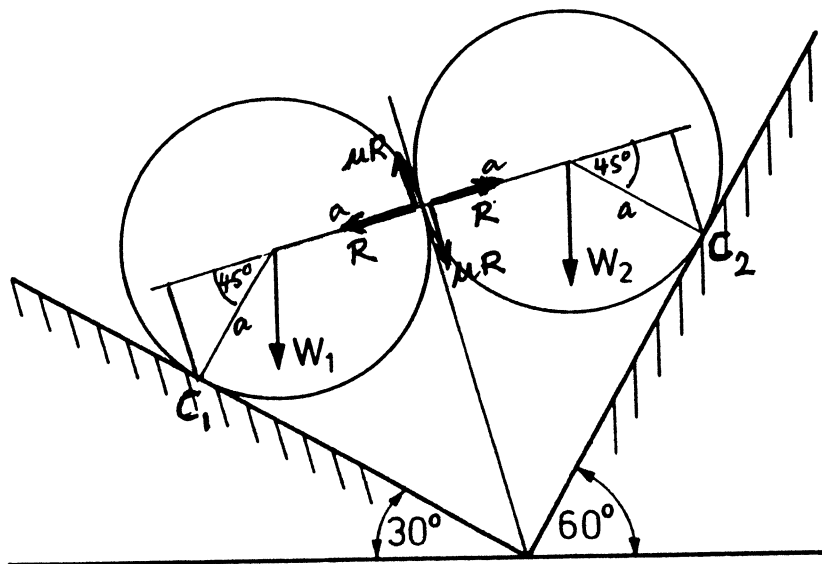
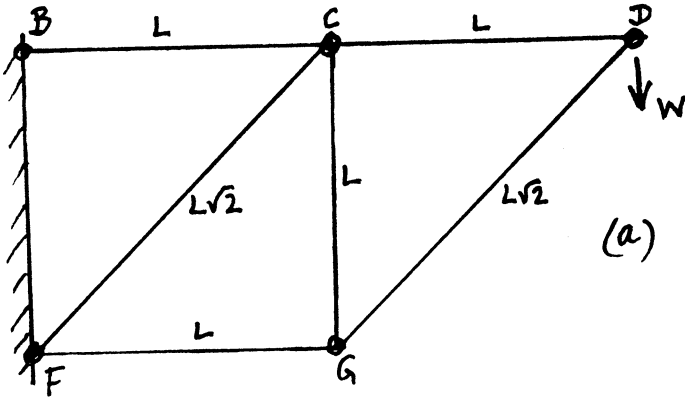


Fig. 1

Check that friction force at C_1 is $< \mu \times$ normal reaction and similarly at C_2 . For friction as shown at C_1 , e.g.
 Normal reaction is $W_1 \cos 30^\circ + R \cos 45^\circ - \mu R \sin 45^\circ$
 Tangential reaction is $W_1 \sin 30^\circ - R \sin 45^\circ + \mu R \cos 45^\circ$

3

Q2

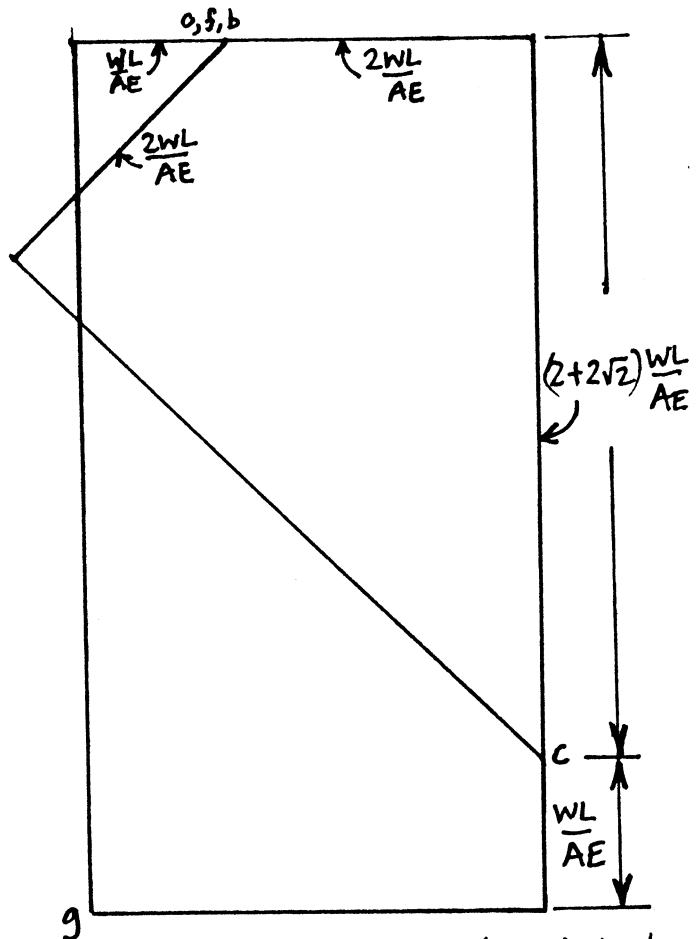


(a) Resolve forces at joints
D, G, C
+ve tension, -ve compression

$$\text{Extension} = \frac{\text{Bar force} \times \text{Bar length}}{AE}$$

Bar	Force	Extension
DG	$-W\sqrt{2}$	$-2WL/AE$
DC	W	WL/AE
GF	$-W$	$-WL/AE$
GC	W	WL/AE
CF	$-W\sqrt{2}$	$-2WL/AE$
CB	$2W$	$2WL/AE$

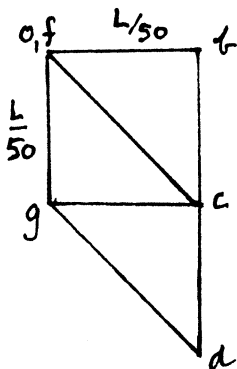
(b) Displacement diagram



or By virtual work
Apply unit loads at G
vertically & horizontally.

(c) No bar extensions.
Structure rotates as a
rigid body about F.

Displacement diagram

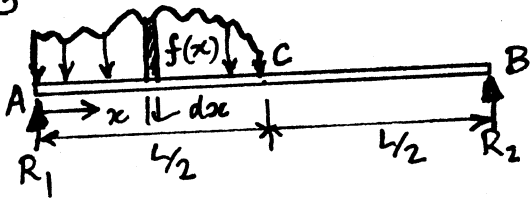


Displacements of G : $(3+2\sqrt{2})\frac{WL}{AE}$ down
and $\frac{WL}{AE}$ horizontally to the left.

If B moves $L/50$ to the right, G has a displacement $L/50$ downwards.

4

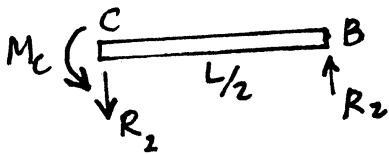
Q3



Consider a load $f(x)$ on the left-hand half of the span.

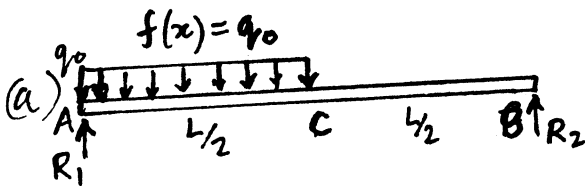
Take moments about A for the whole system.

$$R_2 L = \int_0^{L/2} x f(x) dx$$

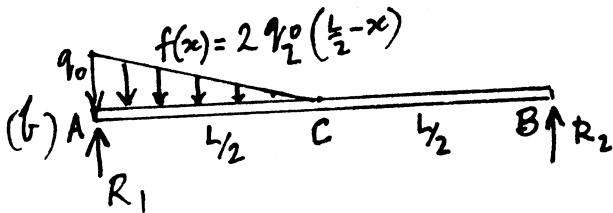


Cut the beam at C and take moments about C for the free body CB

$$R_2 L/2 + M_c = 0$$

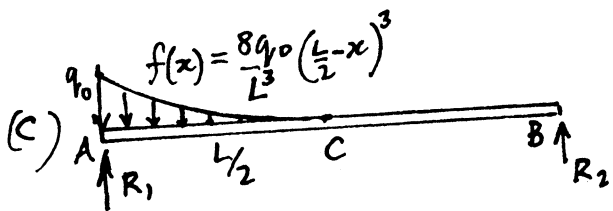


$$M_c = -\frac{1}{2} \int_0^{L/2} x q_0 dx = \underline{\underline{-q_0 L^2 / 16}}$$



$$M_c = -\frac{1}{2} \int_0^{L/2} x \cdot \frac{2q_0}{L} \left(\frac{L}{2} - x\right) dx$$

$$= -\frac{q_0}{L} \left[\frac{L}{2} \frac{x^2}{2} - \frac{x^3}{3} \right]_0^{L/2} = \underline{\underline{-q_0 L^2 / 48}}$$



$$M_c = -\frac{1}{2} \int_0^{L/2} x \cdot \frac{8q_0}{L^3} \left(\frac{L}{2} - x\right)^3 dx$$

Change the variable to simplify the integral

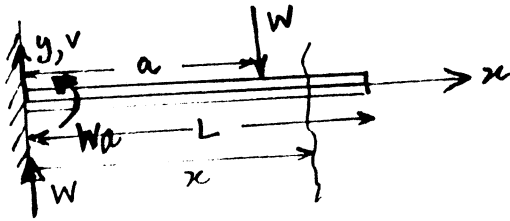
Put $u = L/2 - x$ then

$$M_c = -\frac{1}{2} \int_{L/2}^0 \left(\frac{L}{2} - u\right) \frac{8q_0}{L^3} u^3 (-du) = \frac{4q_0}{L^3} \left[\frac{L}{2} \frac{u^4}{4} - \frac{u^5}{5} \right]_{L/2}^0$$

$$\therefore M_c = \underline{\underline{-q_0 L^2 / 160}}$$

5

Q4



Resolve and take moments for the whole beam.

At the built-in end, the reactions are W and Wa , as shown.

Take the right-hand bay

$$M = -EI \frac{d^2v}{dx^2} = Wa - Wx + W[x-a]$$

$$-EI \frac{dv}{dx} = Wax - \frac{Wx^2}{2} + \frac{W}{2}[x-a]^2 + C_1$$

$$-EI v = \frac{Wax^2}{2} - \frac{Wx^3}{6} + \frac{W}{6}[x-a]^3 + C_2$$

$$\therefore v = -\frac{Wax^2}{2EI} + \frac{Wx^3}{6EI} - \frac{W}{6EI}[x-a]^3$$

At $x=0$
 $\frac{dv}{dx} = 0 \therefore C_1 = 0$
since $[\]$ is -ve,
At $x=0$ $v=0$
 $\therefore C_2 = 0$.

At $x=L$

$$v_L = -\frac{WaL^2}{2EI} + \frac{WL^3}{6EI} - \frac{W}{6EI}[L-a]^3$$

$$= \frac{W}{6EI} (-3aL^2 + L^3 - L^3 + 3La^2 - 3La^2 + a^3)$$

$$= -\frac{Wa^2(3L-a)}{6EI}$$

Note -ve
Downward load, v +ve upwards.

From the Data Book

$$-v_L = \frac{Wa^3}{3EI} + \frac{Wa^2(L-a)}{2EI}$$

$$v_L = -\frac{Wa^2(3L-a)}{6EI}$$

Deflection under load W as a cantilever length a plus slope \times "pointer" length $(L-a)$.

6

Q5 From Data Book $I_{xx} = 5556 \text{ cm}^4 = 5556 \times 10^4 \text{ mm}^4$
 $y = 256/2 = 128 \text{ mm}$

$$\sigma = My/I_{xx} = \frac{75 \times 10^6 \times 128}{5556 \times 10^4} = \underline{\underline{173 \text{ N/mm}^2}} \text{ (or } \text{MN/m}^2 = \text{MPa)}$$

Revised I_{xx} with added plates

$$I'_{xx} = 5556 \times 10^4 + \frac{1}{12} \times 146.4 (276^3 - 256^3) = \underline{\underline{10738 \times 10^4 \text{ mm}^4}}$$

$$\sigma' = \frac{75 \times 10^6 \times 138}{10738 \times 10^4} = \underline{\underline{96 \text{ N/mm}^2}} \text{ since "extreme fibre" } y' = 128 + 10 \text{ mm}$$

$$\tau_t = \frac{FA\bar{y}}{I'_{xx}} = \frac{50 \times 10^3 \times (146.4 \times 10) \times 133}{10738 \times 10^4} = \underline{\underline{90.7 \text{ N/mm}}}$$

Bolt pitch is 300 mm, so force transmitted by

two bolts is $90.7 \times 300 = 27200 \text{ N} = 27.2 \text{ kN}$

and by one bolt $\frac{1}{2}(27.2) = \underline{\underline{13.6 \text{ kN}}}$

ENGINEERING TRIPOS PART IA

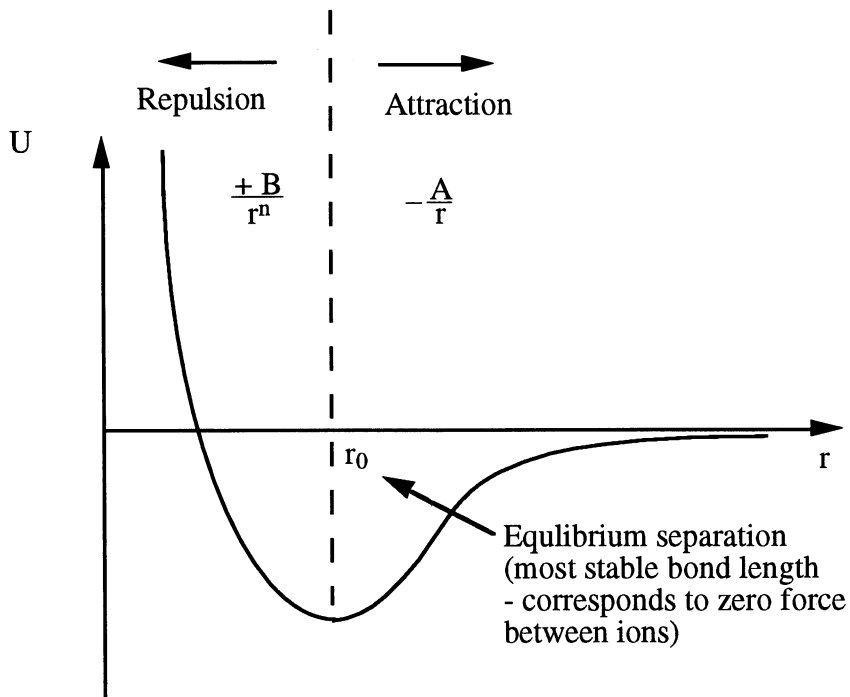
Tuesday 11 June 1996 9.00 to 12.00

Paper 2

MATERIALS SOLUTIONS (FINAL VERSION)

6. Primary bonds have energies of up to 8 eV per atom (1.5 MJ per mole) and are therefore strong. Examples are ionic (NaCl), covalent (diamond) and metallic (copper). Secondary bonds have characteristic energies of 0.1 eV per atom and are much weaker. Examples are Van der Waals (dipole-induced dipole) or hydrogen (permanent dipole-permanent dipole) bonds. Materials are bound on a microscopic level by a mixture of primary and secondary bonds. Primary bonding is driven by the behaviour of the outer, or valence electrons whereas secondary bonding occurs between independent, chemically stable atoms or molecules. [In general elastic properties of metals and ceramics are determined by primary bonds. Secondary bonds account for elastic behaviour of polymers below the glass transition temperature.]

Potential energy :
$$U = -\frac{A}{r} + \frac{B}{r^n}$$



$$F = \frac{dU}{dr} = \frac{A}{r^2} - \frac{nB}{r^{n+1}}$$

$F = 0$ at equilibrium separation (i.e. $r = r_0$).

$$\text{i.e.} \quad \frac{A}{r_0^2} = \frac{nB}{r_0^{n+1}} \quad \Rightarrow \quad r_0 = \left(\frac{A}{nB} \right)^{\frac{1}{1-n}}$$

Bond stiffness is defined by the gradient of the force-displacement curve i.e. ;

$$\text{Modulus of elasticity :} \quad S_0 = \left(\frac{dF}{dr} \right)_{r=r_0} = \left(\frac{d^2U}{dr^2} \right)_{r=r_0}$$

$$\frac{dF}{dr} = -\frac{2A}{r^3} + \frac{n(n+1)B}{r^{(n+2)}}$$

$$\left(\frac{dF}{dr} \right)_{r=r_0} = S_0 = -\frac{2A}{r_0^3} + \frac{n(n+1)B}{r_0^{(n+2)}}$$

$$\left[S_0 = -\frac{2A}{\left(\frac{A}{nB} \right)^{\frac{3}{1-n}}} + \frac{n(n+1)B}{\left(\frac{A}{nB} \right)^{\frac{n+2}{1-n}}} \right]$$

$$r_0 = \left(\frac{A}{nB} \right)^{\frac{1}{1-n}} = \left(\frac{1.5 \times 10^{-10}}{8 \times 7 \times 10^{-16}} \right)^{\frac{-1}{7}} = 0.23 \text{ nm} = 2.3 \text{ \AA}$$

7. The three methods of increasing the strength of ductile metal alloys are (i) work hardening, (ii) solid solution hardening and (iii) precipitate hardening.

(i) Work Hardening : Plastic deformation arises as a result of movement of dislocations along slip planes in crystalline lattices. Ease of dislocation movement varies between crystallographic planes and between crystal structures. Dislocations moving along intersecting planes can interact and thus obstruct each other, which is the basis of work hardening. This behaviour is apparent in the stress - strain curve as a rising portion between σ_y and the necking point.

(ii) Solid Solution Hardening : Achieved by adding dissolved impurities to the metal to be hardened. The effect of adding impurities of a different size to the host lattice species is to create a mis-fit lattice strain which distorts the long range crystallographic order of the dislocation slip plane. This inhibits the movement of dislocations through the lattice. The process is often likened to a 'roughening' of the slip plane.

(iii) Precipitate Hardening : Strength of base material may increased significantly by adding a fine distribution of second phase material which impedes the movement of dislocations. This can be achieved by a combination of alloying and heat treatment. Alternatively, an oxide dispersoid can be mixed with a powdered metal, pressed and sintered to form a strong composite mixture.

Work hardening is responsible for the observed increase in flow stress with plastic strain.

$$\sigma_t = \sigma_0 \epsilon_t^n$$

Need also the equations relating true stress to nominal stress and true strain to nominal strain :

$$\epsilon_t = \ln (1 + \epsilon_n) \qquad \sigma_t = \sigma_n (1 + \epsilon_n)$$

Substituting these in the equation given :

$$\sigma_0 [\ln (1 + \epsilon_n)]^n = \sigma_n (1 + \epsilon_n) \qquad \text{i.e.} \qquad \sigma_n = \frac{\sigma_0 [\ln (1 + \epsilon_n)]^n}{(1 + \epsilon_n)} \qquad (1)$$

Evaluate σ_n from equation (1) for values of ϵ_n between 0.05 and 0.4 : [N.B. one of these points would be sufficient to identify the alloy although at least two are required for full marks]

ϵ_n	σ_n (MPa)
0	0
0.05	194
0.10	257
0.15	297
0.20	324
0.25	343
0.30	357
0.35	368

Comparing this data with the graph on page 16 of the Data Book, identifies the alloy as **annealed 70/30 brass** (70% copper and 30% zinc).

$$\text{Nominal Strain, } \epsilon_n = \frac{u}{l_0} = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1$$

$$\text{From conservation of volume : } A_0 l_0 = A l \quad \text{i.e.} \quad \frac{A_0}{A} = \frac{l}{l_0}$$

$$\frac{A_0}{A} = \frac{\pi (0.55)^2}{\pi (0.5)^2} = 1.21 \quad \text{i.e.} \quad \epsilon_n = 1.21 - 1 = 0.21$$

From equation 1, $\sigma_n = 328$ MPa. This corresponds to a true stress, $\sigma_t = \sigma_n (1 + \epsilon_n)$, of 397 MPa.

$$\text{Thus } F < \sigma_t A = 397 \times \pi r^2 = 397 \times \pi \times (0.005)^2 = 0.031 \text{ MN} = 31 \text{ kN.}$$

i.e. **F < 31 kN.**

8. The vessel is subjected to repeated applications of internal pressure so mechanical failure may be by fatigue. The initial size of the crack, a_0 , and the magnitude of the internal pressure, p , will determine whether the vessel will fail by fast fracture, fatigue followed by fast fracture or by yielding. The distance from the crack tip, r , the specimen thickness, t , the critical stress intensity factor, K_{IC} , the yield stress, σ_y , and the rate at which the crack length increases with number of cycles, $\frac{da}{dN}$, are the variables which determine the mode of failure. These are related by $K_{IC} = \frac{p r}{t} \sqrt{\pi a}$ and $\sigma_y = \frac{p r}{t}$.

Fast fracture is caused by rapid growth (up to the speed of sound) of cracks in the material that become unstable. Failure by this mechanism is catastrophic and often **explosive** in the case of pressure vessels. The stress intensity factor K is proportional to \sqrt{a} and indicates how a remote load is amplified across a given crack. Sub-critical length cracks in the vessel grow by fatigue on repeated pressure cycles - hence the vessel will fail by fast fracture below its yield strength when the crack becomes large enough to cause the stress intensity factor to exceed a critical value K_{IC} (characteristic of the metal in question).

Plastic failure (yielding) occurs when the effective applied stress becomes equal to the yield stress of the vessel material and is independent of crack growth. This level of stress will only be achieved if the stress intensity factor due to the crack does not exceed K_{IC} (i.e. lies below the critical value for failure by fast fracture). Failure by yielding is unlikely in practical pressure vessels since it is easy to design against and would happen during commissioning.

The rate at which crack length a increases due to fatigue with number of stress cycles N is given by :

$$\frac{da}{dN} = A (\Delta K)^n \quad (1)$$

The expression for the stress intensity factor in the Materials Data Book is :

$$K = Y \sigma \sqrt{\pi a}$$

Hence,
$$\Delta K = Y \Delta \sigma \sqrt{\pi a} \quad (2)$$

Rearranging (1) gives :

$$N_f = \int_0^{N_f} dN = \int_{a_0}^{a_c} \frac{da}{A (\Delta K)^n}$$

Substituting for ΔK :

$$N_f = \int_{a_0}^{a_c} \frac{da}{A (Y \Delta \sigma \sqrt{\pi a})^n}$$

$$N_f = \frac{1}{A Y^n \pi^{n/2} \Delta \sigma^n} \int_{a_0}^{a_c} \frac{da}{a^{n/2}} = \frac{1}{A Y^n \pi^{n/2} \Delta \sigma^n} \times \frac{1}{1 - \frac{n}{2}} \times \left[(a_c)^{1 - \frac{n}{2}} - (a_0)^{1 - \frac{n}{2}} \right]$$

i.e. $N_f = \frac{B}{Y^n (\Delta \sigma)^n} \times \frac{1}{1 - \frac{n}{2}} \times \left[(a_c)^{1 - \frac{n}{2}} - (a_0)^{1 - \frac{n}{2}} \right]$ with $B = \frac{1}{A \pi^{n/2}}$

For the die fabricated from pressure vessel steel :

First need to calculate a_c from K_{1C} given in Materials Data Book. i.e. $K_{1C}(\text{HY130}) = 170 \text{ MNm}^{-3/2}$.

$$a_c = \frac{1}{\pi} \left(\frac{K_{1C}}{\sigma Y} \right)^2 = \frac{1}{\pi} \left(\frac{170}{1.13 \times 400} \right)^2 = 0.045 \text{ m}$$

For $n = 3$ equation (3) becomes :

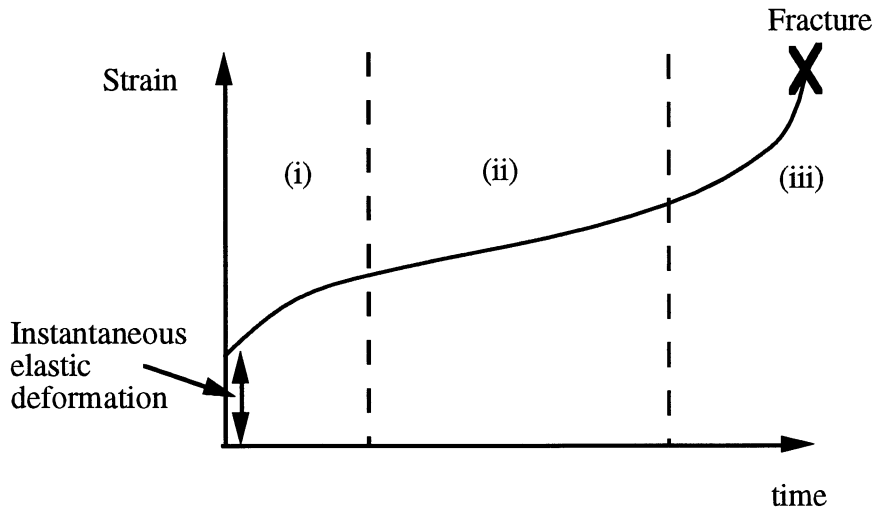
$$N_f = \frac{2}{A Y^3 \pi^{3/2} \Delta \sigma^3} \times \left[(a_0)^{-\frac{1}{2}} - (a_c)^{-\frac{1}{2}} \right]$$

i.e. (with $A = 1.0 \times 10^{-12} \text{ MPa}^{-3} \text{ m}^{-1/2}$, $a_0 = 0.001 \text{ m}$, $Y = 1.13$ and $\Delta \sigma = 400 \text{ MPa}$ - units cancel),

$$N_f = \frac{2}{1.0 \times 10^{-12} \times 1.13^3 \times \pi^{3/2} (400)^3} \times \left[\frac{1}{\sqrt{0.001}} - \frac{1}{\sqrt{0.045}} \right] = 104661$$

$$\approx 10^5 \text{ cycles}$$

9. [Identification of the functional form of each stage is required, not a description of the underlying mechanisms.] The three stages of creep that occur in metal alloys under constant load at constant elevated temperature are (i) primary or transient creep, (ii) secondary creep and (iii) tertiary creep.



(i) Primary creep is characterised by a continually decreasing creep rate - i.e. the slope decreases with increasing time. [This suggests that the material is work hardening (deformation becomes more difficult as the material is strained).]

(ii) In secondary, or steady state, creep the plot of strain against time becomes linear and the creep rate is constant. [This is usually the region of longest duration. The constant creep rate is explained by a balance between competing processes of hardening and recovery.]

(iii) Tertiary creep is a region prior to fracture where the creep rate accelerates significantly.

$$\dot{\epsilon}_s = A \sigma^n \exp\left(-\frac{Q}{RT}\right)$$

Need to take logs to base 10 to read from graph :

i.e. $\log_{10} \dot{\epsilon} = n \log_{10} \sigma + \log_{10} A + \log_{10} (\exp-Q/RT)$

Gradient of log - log plot of $\dot{\epsilon}$ vs. stress = n.

Hence, gradient of log - log plot of stress (ordinate) vs. $\dot{\epsilon}$ (abscissa) = 1/n.

From the 700K graph :

σ (Pa)	$\dot{\epsilon}$ per hour
4×10^7	3×10^{-8}
1×10^8	3×10^{-6}

($\dot{\epsilon}$ % per 1000 hours $\times 10^{-5} = \dot{\epsilon}$ per hour)

$$n = \frac{\log_{10} \dot{\epsilon}_1 - \log_{10} \dot{\epsilon}_2}{\log_{10} \sigma_1 - \log_{10} \sigma_2} = \frac{\log_{10} 3 \times 10^{-8} - \log_{10} 3 \times 10^{-6}}{\log_{10} 4 \times 10^7 - \log_{10} 10^8} \approx 5$$

Hence power law creep is responsible for deformation of the alloy at these temperatures. [$n \sim 1$ for diffusion creep].

Read creep rate from the graphs at constant stress (e.g. 5×10^7 Pa). i.e.

σ (Pa)	$\dot{\epsilon}$ per hour	T (K)
5×10^7	9×10^{-8}	700
5×10^7	5.5×10^{-6}	800

From equation given : $\ln \dot{\epsilon} = \ln A \sigma^n - \frac{Q}{RT}$

But $\ln A \sigma^n = \text{constant}$, i.e. $\ln \left(\frac{\dot{\epsilon}_1}{\dot{\epsilon}_2} \right) = \frac{Q}{800 R} - \frac{Q}{700 R}$

$$Q = \frac{8.31 (\ln 0.0095 - \ln 0.55)}{\frac{1}{800} - \frac{1}{700}} = \mathbf{188872 \text{ J per mol.}}$$

From $\dot{\epsilon}_s = A \sigma^n \exp \left(-\frac{Q}{RT} \right)$ $A = \frac{\dot{\epsilon}}{\sigma^5 \exp \left(\frac{-Q}{RT} \right)}$

5.5×10^{-6} % per hour = $1.53 \times 10^{-9} \text{ s}^{-1}$ i.e. ;

$$A = \frac{1.53 \times 10^{-9}}{(5 \times 10^7)^5 \exp\left(\frac{-188872}{8.31 \times 800}\right)} = 1.07 \times 10^{-35} \text{ Pa}^{-5} \text{ s}^{-1}$$

(Equivalently, $A = 3.8 \times 10^{-29} \text{ Pa}^{-5} \text{ khour}^{-1}$).

Equation becomes $\dot{\epsilon}_s \text{ (s}^{-1}\text{)} = 1.07 \times 10^{-35} \sigma^5 \exp\left(-\frac{188872}{8.31 T}\right)$

The mass of the low carbon-nickel alloy bar is negligible in this test ($\rho = 9 \text{ gcm}^{-3}$, volume = .98 cm³, mass ~ 9g = 0.02% of load). This is not asked for in the question but deserves 'bonus' credit if demonstrated.

$$\text{Stress in test bar} = \sigma = \frac{F}{A} = \frac{40 \times 9.81}{\pi (2.5 \times 10^{-3})^2} = 2 \times 10^7 \text{ Pa}$$

i.e. $\dot{\epsilon}_s = 1.07 \times 10^{-35} (2 \times 10^7)^5 \exp\left(-\frac{188872}{8.31 \times 900}\right) = 3.67 \times 10^{-10} \text{ per second}$

(= 1.33×10^{-6} per hour)

$$\text{Strain for elongation of } 50\mu\text{m} = \frac{50 \times 10^{-6}}{50 \times 10^{-3}} = 0.001 \text{ (= 0.1 \%)}$$

$$\text{Hence time taken} = \frac{0.001}{1.33 \times 10^{-6}} = 752 \text{ hours}$$

10. Aqueous corrosion occurs as a result of an electrochemical reaction. Metal atoms oxidise by losing electrons and pass into solution as ions (anodic reaction). The electrons are then conducted through the metal to reduce a second chemical species (cathodic reaction) which results in a net loss of material from the surface of the metal. The electrode potential corresponds to the potential that would have to be applied to the metal to stop the reaction - i.e. its electronegativity. Hence the more negative the electrode potential, the greater the driving force for corrosion when exposed to oxygenated water.

Methods of protecting metals against aqueous corrosion are :

(i) Application of protective surface. Painting is an effective way of isolating a metal surface from a corroding electrolyte. This simply involves placing a physical barrier between the metal and the environment. Its main limitation is its behaviour in service. Exposure to high temperatures or abrasive wear particularly limits the effectiveness of organic coatings. Dip coating by relatively inert metals is also an example of this method of protection (e.g. Cr plate, Ni plate, Cd plate etc.).

(ii) Cathodic Protection involves lowering the electrical potential of a metal. This simply supplies electrons to the metal which reverses the anodic reaction and stops corrosion. One way of achieving this is by galvanic protection. This involves connecting the metal to be protected (usually steel) to another, more electronegative metal (usually magnesium or zinc) in a given environment. The second metal, which may be chosen by reference to the Galvanic Series, then becomes the (sacrificial) anode and corrodes whereas the protected cathode remains intact. Alternatively, a negative voltage may be applied directly to the metal to be protected from an external d.c. voltage source, with the positive terminal connected to an inert or sacrificial anode (e.g. graphite or steel). A protective coating is applied to the primary metal in order to reduce the amount of sacrificial anode or power required.

(iii) Galvanising involves applying a layer of zinc to steel by hot dipping which provides a protective coating. A stable oxide layer forms on the surface of the zinc which resists corrosion. Zinc is more electronegative than steel and offers cathodic protection in the event of surface damage. Corrosion of the zinc under these conditions will be extremely slow due to the large ratio of anode to cathode area.

(iv) Alloying. Alloy elements can help to provide a protective film. For example a protective layer of Cr_2O_3 forms on stainless steel. This layer has the added advantage of being self-healing when damaged. [This method is less convincing than the first three since alloying effectively changes the material. Equal credit to be awarded, however.]

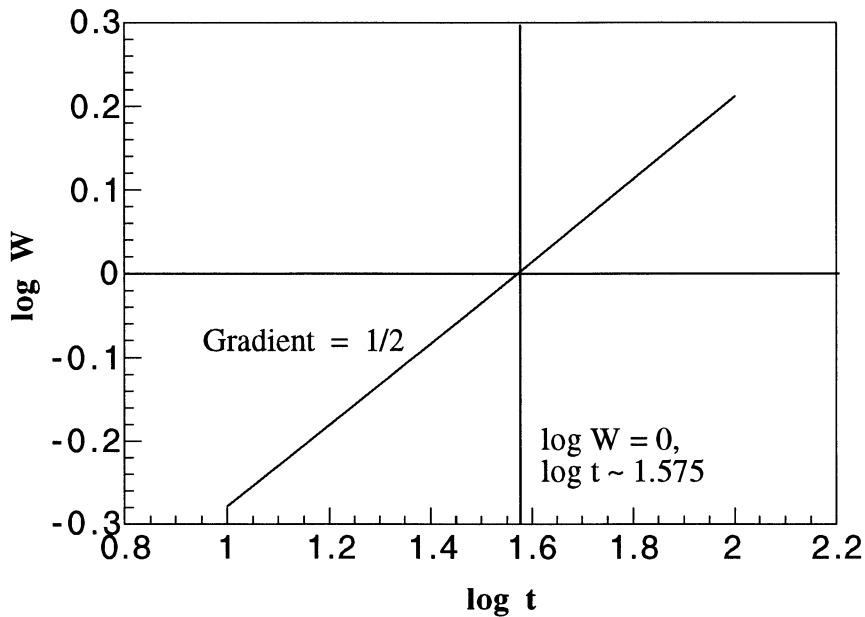
The rate expression for the oxidation kinetics of nickel is **parabolic** i.e.:

$$W^2 = K_1t$$

where K_1 is a constant. This may be verified by determining K_1 from at least two of the data points given.

Alternatively the relationship can be derived from a log-log plot of W against t :

$$W^n = K_1 t; \quad n \log W = \log (K_1 t) \quad \text{hence} \quad \log W = 1/n \log t + 1/n \log K_1$$



Therefore $n = 2$ and $\log t \sim 1.575 = -\log K_1$

Hence $K_1 = 0.0266$. $W^2 = 0.0266 t$ (W in mg cm^{-2} and t in minutes)

For $t = 600$ minutes, $W^2 = 0.0266 \times 600 = 16.0$

i.e. $W = 4 \text{ mg cm}^{-2}$

Metals are ranked in the Galvanic Series given on page 4 of the Materials Data Book in terms of their **observed** corrosivity in sea water.

i.e. in order of increasing rate of corrosion : Nickel, tin, cadmium.

Table 1.2, on the other hand, simply ranks the metals in terms of the relative driving force for wet oxidation in pure water and does not account for kinetic and other environmental changes that occur in sea water (the presence of other ions such as Cl^-).

ENGINEERING TRIPOS PART IA

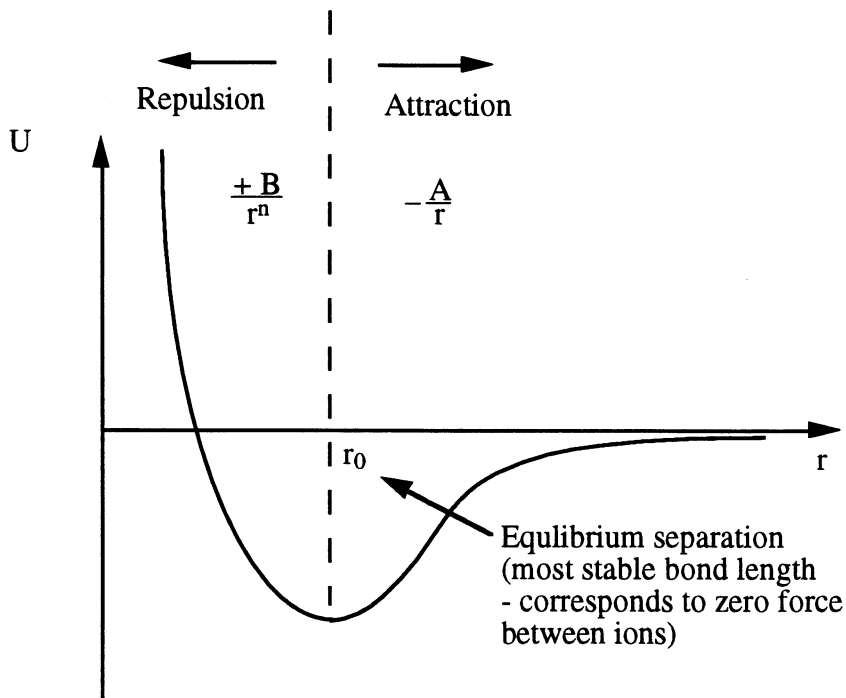
Tuesday 11 June 1996 9.00 to 12.00

Paper 2

MATERIALS SOLUTIONS (FINAL VERSION)

6. Primary bonds have energies of up to 8 eV per atom (1.5 MJ per mole) and are therefore strong. Examples are ionic (NaCl), covalent (diamond) and metallic (copper). Secondary bonds have characteristic energies of 0.1 eV per atom and are much weaker. Examples are Van der Waals (dipole-induced dipole) or hydrogen (permanent dipole-permanent dipole) bonds. Materials are bound on a microscopic level by a mixture of primary and secondary bonds. Primary bonding is driven by the behaviour of the outer, or valence electrons whereas secondary bonding occurs between independent, chemically stable atoms or molecules. [In general elastic properties of metals and ceramics are determined by primary bonds. Secondary bonds account for elastic behaviour of polymers below the glass transition temperature.]

Potential energy :
$$U = -\frac{A}{r} + \frac{B}{r^n}$$



7. The three methods of increasing the strength of ductile metal alloys are (i) work hardening, (ii) solid solution hardening and (iii) precipitate hardening.

(i) Work Hardening : Plastic deformation arises as a result of movement of dislocations along slip planes in crystalline lattices. Ease of dislocation movement varies between crystallographic planes and between crystal structures. Dislocations moving along intersecting planes can interact and thus obstruct each other, which is the basis of work hardening. This behaviour is apparent in the stress - strain curve as a rising portion between σ_y and the necking point.

(ii) Solid Solution Hardening : Achieved by adding dissolved impurities to the metal to be hardened. The effect of adding impurities of a different size to the host lattice species is to create a mis-fit lattice strain which distorts the long range crystallographic order of the dislocation slip plane. This inhibits the movement of dislocations through the lattice. The process is often likened to a 'roughening' of the slip plane.

(iii) Precipitate Hardening : Strength of base material may increased significantly by adding a fine distribution of second phase material which impedes the movement of dislocations. This can be achieved by a combination of alloying and heat treatment. Alternatively, an oxide dispersoid can be mixed with a powdered metal, pressed and sintered to form a strong composite mixture.

Work hardening is responsible for the observed increase in flow stress with plastic strain.

$$\sigma_t = \sigma_0 \varepsilon_t^n$$

Need also the equations relating true stress to nominal stress and true strain to nominal strain :

$$\varepsilon_t = \ln (1 + \varepsilon_n) \qquad \sigma_t = \sigma_n (1 + \varepsilon_n)$$

Substituting these in the equation given :

$$\sigma_0 [\ln (1 + \varepsilon_n)]^n = \sigma_n (1 + \varepsilon_n) \qquad \text{i.e.} \qquad \sigma_n = \frac{\sigma_0 [\ln (1 + \varepsilon_n)]^n}{(1 + \varepsilon_n)} \qquad (1)$$

8. The vessel is subjected to repeated applications of internal pressure so mechanical failure may be by fatigue. The initial size of the crack, a_0 , and the magnitude of the internal pressure, p , will determine whether the vessel will fail by fast fracture, fatigue followed by fast fracture or by yielding. The distance from the crack tip, r , the specimen thickness, t , the critical stress intensity factor, K_{IC} , the yield stress, σ_y , and the rate at which the crack length increases with number of cycles, $\frac{da}{dN}$, are the variables which determine the mode of failure. These are related by $K_{IC} = \frac{p r}{t} \sqrt{\pi a}$ and $\sigma_y = \frac{p r}{t}$.

Fast fracture is caused by rapid growth (up to the speed of sound) of cracks in the material that become unstable. Failure by this mechanism is catastrophic and often **explosive** in the case of pressure vessels. The stress intensity factor K is proportional to \sqrt{a} and indicates how a remote load is amplified across a given crack. Sub-critical length cracks in the vessel grow by fatigue on repeated pressure cycles - hence the vessel will fail by fast fracture below its yield strength when the crack becomes large enough to cause the stress intensity factor to exceed a critical value K_{IC} (characteristic of the metal in question).

Plastic failure (yielding) occurs when the effective applied stress becomes equal to the yield stress of the vessel material and is independent of crack growth. This level of stress will only be achieved if the stress intensity factor due to the crack does not exceed K_{IC} (i.e. lies below the critical value for failure by fast fracture). Failure by yielding is unlikely in practical pressure vessels since it is easy to design against and would happen during commissioning.

The rate at which crack length a increases due to fatigue with number of stress cycles N is given by :

$$\frac{da}{dN} = A (\Delta K)^n \quad (1)$$

The expression for the stress intensity factor in the Materials Data Book is :

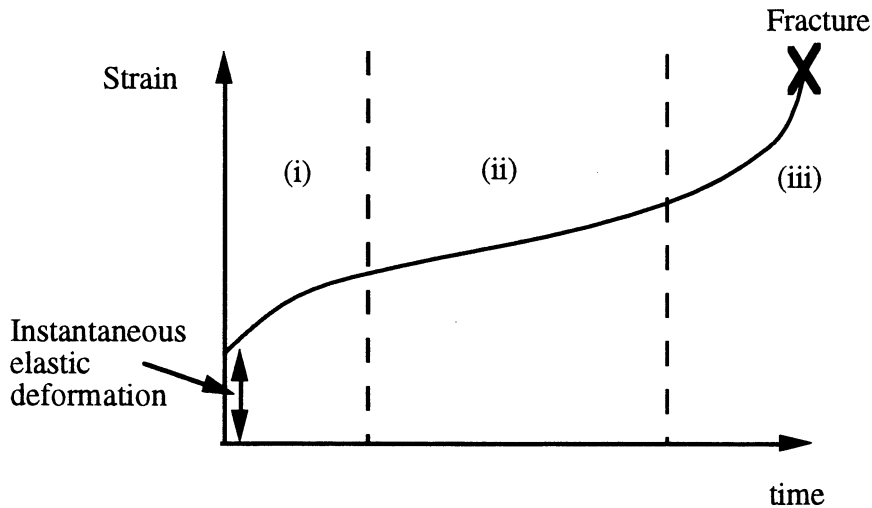
$$K = Y \sigma \sqrt{\pi a}$$

Hence, $\Delta K = Y \Delta \sigma \sqrt{\pi a}$ (2)

Rearranging (1) gives :

$$N_f = \int_0^{N_f} dN = \int_{a_0}^{a_c} \frac{da}{A (\Delta K)^n}$$

9. [Identification of the functional form of each stage is required, not a description of the underlying mechanisms.] The three stages of creep that occur in metal alloys under constant load at constant elevated temperature are (i) primary or transient creep, (ii) secondary creep and (iii) tertiary creep.



(i) Primary creep is characterised by a continually decreasing creep rate - i.e. the slope decreases with increasing time. [This suggests that the material is work hardening (deformation becomes more difficult as the material is strained).]

(ii) In secondary, or steady state, creep the plot of strain against time becomes linear and the creep rate is constant. [This is usually the region of longest duration. The constant creep rate is explained by a balance between competing processes of hardening and recovery.]

(iii) Tertiary creep is a region prior to fracture where the creep rate accelerates significantly.

$$\dot{\epsilon}_s = A \sigma^n \exp\left(-\frac{Q}{RT}\right)$$

Need to take logs to base 10 to read from graph :

i.e. $\log_{10} \dot{\epsilon} = n \log_{10} \sigma + \log_{10} A + \log_{10} (\exp-Q/RT)$

Gradient of log - log plot of $\dot{\epsilon}$ vs. stress = n.

Hence, gradient of log - log plot of stress (ordinate) vs. $\dot{\epsilon}$ (abscissa) = 1/n.

$$A = \frac{1.53 \times 10^{-9}}{(5 \times 10^7)^5 \exp\left(\frac{-188872}{8.31 \times 800}\right)} = 1.07 \times 10^{-35} \text{ Pa}^{-5} \text{ s}^{-1}$$

(Equivalently, $A = 3.8 \times 10^{-29} \text{ Pa}^{-5} \text{ khour}^{-1}$).

$$\text{Equation becomes } \dot{\epsilon}_s \text{ (s}^{-1}\text{)} = 1.07 \times 10^{-35} \sigma^5 \exp\left(-\frac{188872}{8.31 T}\right)$$

The mass of the low carbon-nickel alloy bar is negligible in this test ($\rho = 9 \text{ gcm}^{-3}$, volume = .98 cm³, mass ~ 9g = 0.02% of load). This is not asked for in the question but deserves 'bonus' credit if demonstrated.

$$\text{Stress in test bar } = \sigma = \frac{F}{A} = \frac{40 \times 9.81}{\pi (2.5 \times 10^{-3})^2} = 2 \times 10^7 \text{ Pa}$$

$$\text{i.e. } \dot{\epsilon}_s = 1.07 \times 10^{-35} (2 \times 10^7)^5 \exp\left(-\frac{188872}{8.31 \times 900}\right) = 3.67 \times 10^{-10} \text{ per second}$$

(= 1.33×10^{-6} per hour)

$$\text{Strain for elongation of } 50\mu\text{m} = \frac{50 \times 10^{-6}}{50 \times 10^{-3}} = 0.001 \text{ (= 0.1 \%)}$$

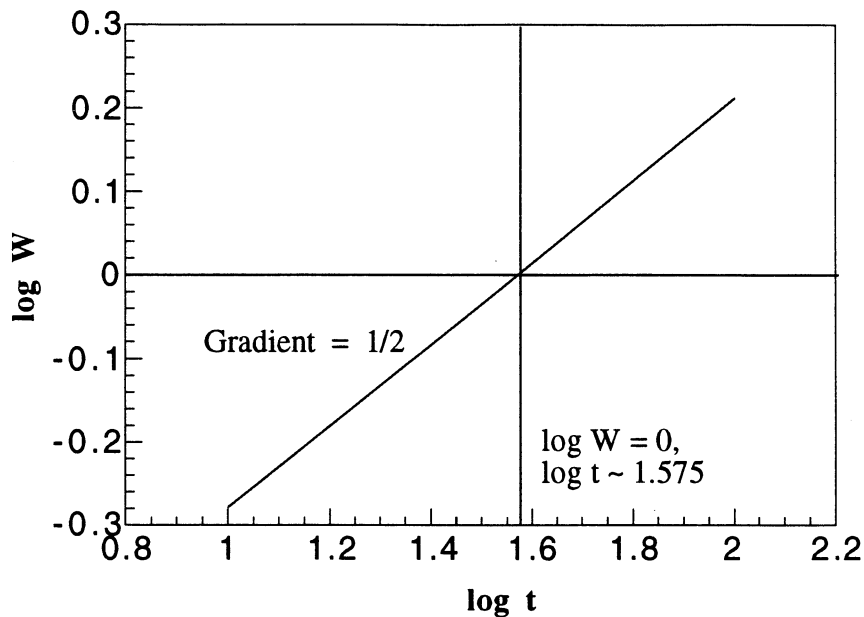
$$\text{Hence time taken} = \frac{0.001}{1.33 \times 10^{-6}} = 752 \text{ hours}$$

10. Aqueous corrosion occurs as a result of an electrochemical reaction. Metal atoms oxidise by losing electrons and pass into solution as ions (anodic reaction). The electrons are then conducted through the metal to reduce a second chemical species (cathodic reaction) which results in a net loss of material from the surface of the metal. The electrode potential corresponds to the potential that would have to be applied to the metal to stop the reaction - i.e. its electronegativity. Hence the more negative the electrode potential, the greater the driving force for corrosion when exposed to oxygenated water.

Methods of protecting metals against aqueous corrosion are :

Alternatively the relationship can be derived from a log-log plot of W against t :

$$W^n = K_1 t; \quad n \log W = \log (K_1 t) \quad \text{hence} \quad \log W = 1/n \log t + 1/n \log K_1$$



Therefore $n = 2$ and $\log t \sim 1.575 = -\log K_1$

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