CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT

EXAMINATION PAPERS

1996

ENGINEERING TRIPOS: 1A

ENGINEERING TRIPOS PART IA

Tuesday 11 June 1996 1.30 to 4.30

Paper 1

MECHANICAL ENGINEERING

Answer not more than **eight** *questions, of which not more than* **three** *may be taken from section A, not more than* **three** *from Section B and not more than* **two** *from section C.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer not more than **three** *questions from this section.*

1 (a) Explain why the volumetric compression ratio in petrol engines is much less than that used in diesel engines.

(b) The ideal air-standard Otto cycle consists of four processes:-

(1→2) an adiabatic, reversible compression from temperature T_1 through a volumetric compression ratio of r ;

(2→3) heating at constant volume through a temperature ratio of β ;

(3→4) an adiabatic, reversible expansion;

(4→1) cooling at constant volume.

Sketch this cycle on a pressure-volume (p-v) diagram.

(c) Calculate the temperature of the air at the beginning of each of the four processes in terms of T_1, r and β and find the heat transfer per unit mass associated with each process. Hence show that the cycle efficiency is

$$\eta = 1 - \frac{1}{r^{\gamma-1}} .$$

(d) In practice, the expansion stroke is not adiabatic. Show clearly on your sketch the effect that this has on the cycle. Does the cycle efficiency increase or decrease because of this heat transfer ? Justify your answer.

2 (a) A steam power plant consists of a condenser, boiler, feed pump and turbine. Draw a schematic diagram of the plant, clearly marking the components and the order in which they appear.

(b) Sketch an ideal steam power cycle for this plant on a temperature-entropy (T-s) diagram, assuming that the steam leaving the turbine is wet. Indicate on your diagram how the cycle is modified when the flow through the turbine is not reversible.

(c) The pressure at the condenser exit is 0·04 bar and that leaving the boiler is 60 bar. If the dryness fraction of the steam leaving the turbine is 0·85, while that leaving an ideal turbine would have been 0·8, find the temperature of the steam at the turbine inlet and the turbine isentropic efficiency.

(d) Calculate the efficiency of the cycle and the mass flow rate of steam for a power output of 200 MW.

[You may assume that the feed pump work is small enough to be neglected.]

3 (a) State the *Clausius Inequality.* Use it to derive a relationship between the efficiency of a cyclic heat engine operating between two constant-temperature reservoirs and the temperatures of those reservoirs.

(b) A cyclic heat engine, operating between two reservoirs at constant temperatures T_1 and T_2 respectively, is used to power a heat pump which takes a heat flow in from the lower temperature (T_2) reservoir. The heat pump is used to provide a heat flow to a large room at temperature T_3. If the heat input to the engine is Q_1, calculate the maximum amount of heat, Q_3, which can be delivered by the heat pump.

Determine the circumstances under which Q_3 can be greater than Q_1. Does the Clausius Inequality impose any restriction on the relative sizes of $\frac{Q_1}{T_1}$ and $\frac{Q_3}{T_3}$?

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4 Two streams of air are mixed in an insulated pipe of constant cross-sectional area, A, as shown in Fig. 1. The pressure in both streams at P is 10 bar and one stream enters with temperature 1200 K and velocity 300 ms^{-1}, while the other does so with temperature 600 K and velocity 200 ms^{-1}. Show that the ratio of the mass flows in the two streams at P is $9/4$.

If the pressure at a downstream point Q, where the mixing of the streams is complete and the gas properties are uniform, is measured as 9·5 bar, find the temperature and the velocity of the gas at Q.

Find the entropy change per unit mass flow rate associated with this mixing process and describe the physical processes responsible for it.

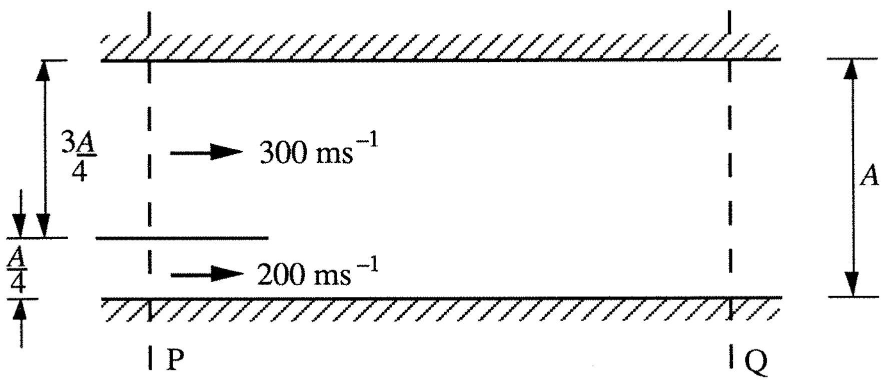

Fig. 1

SECTION B

Answer not more than **three** *questions from this section.*

5 (a) Explain what is meant by the term *moment of momentum* for a particle, and state the conditions under which it is conserved about a line in space.

(b) A thin-walled cone of height $2h$ and half angle θ is held with its axis vertical and its apex downwards, as shown in Fig. 2. A particle of mass $2m$ slides freely on the frictionless inner surface of the cone, and is travelling in a circle at a constant height h above the apex of the cone, in an anticlockwise direction as seen from above. A second particle of mass m is held at the top edge of the cone at the position $(2h \tan\theta, 0, 2h)$, and is released into the cone. Its release is timed such that it strikes the first particle, whereupon the two particles fuse together. Find the velocity vector of the larger mass just prior to impact, and that of the combined mass just after impact.

(c) Describe the subsequent motion of the combined mass qualitatively, and state the principal equations which govern it. Show that, during the subsequent motion, the minimum and maximum heights achieved by the combined mass are approximately $0.524\,h$ and $1.171\,h$ respectively.

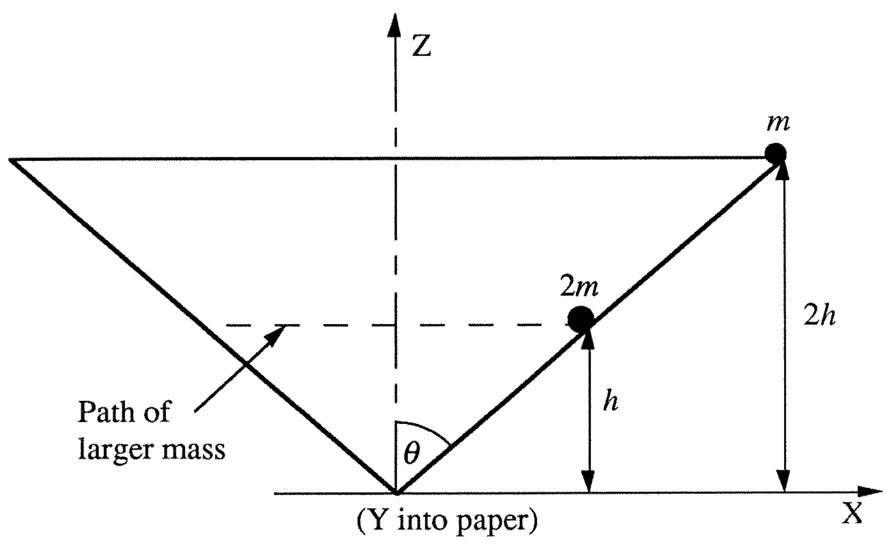

Fig. 2

6 Part of the mechanism for bolting the door of a safe is shown in Fig. 3. The crank AB has length 40 mm, and is hinged to the bar BC which is of length 160 mm. Bar DF also has length 160 mm, and is hinged to bar BC at its midpoint. The bolt EG is attached to bar DF via a slider at E, which is instantaneously midway between C and F.

(a) Draw a Velocity Diagram for the mechanism in the position shown, and find the velocity of G if the crank is rotating clockwise with an angular velocity of 1.25 rad s^{-1}. Find also the angular velocities of the bars CB and DF, and the sliding velocity at E. A suitable scale for the Velocity Diagram is for 1 mm to represent a velocity of 1 mm s^{-1}.

(b) The movement of the bolt is resisted by a force of 20 N. Find the torque required to turn the crank:

(i) if there is no friction in the mechanism, and

(ii) if there is a frictional torque of 1 Nm at hinges B and C, and a sliding resistance of 5 N at E.

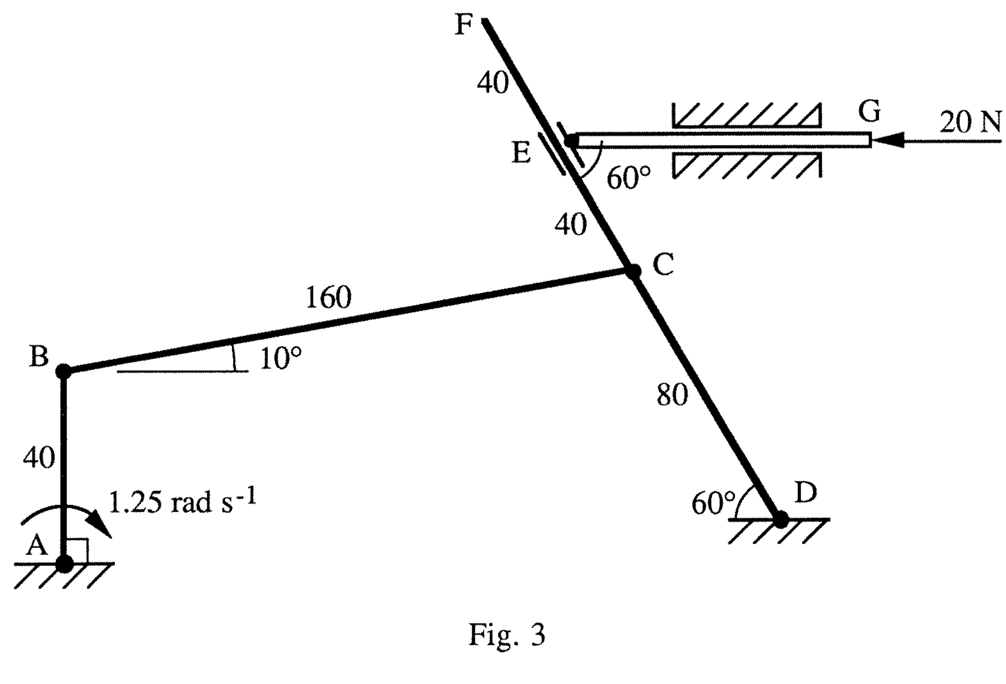

Fig. 3

7 The drum shown in Fig. 4 has radius R and rotates about the origin with constant angular velocity $\dot{\theta}$; and a particle P of mass m is attached to it by means of a light inextensible string wrapped round the drum and fixed at C. Initially, the particle is held against the drum at point A. When $\theta = 0$ the particle is released, and the string starts to unwrap.

(a) Express the subsequent position of P in terms of the unit vectors $\underline{e}_1$ and $\underline{e}_2$, R, and the angle ϕ through which the string has unwrapped. What is the angular velocity of the length of string between B and P?

(b) Differentiate the expression you have obtained, to find expressions for the velocity and acceleration of the particle.

(c) By considering the forces acting on the particle, derive a differential equation relating ϕ to θ, and show that it is satisfied by the relationship $\phi = \theta$. Hence obtain an expression for the torque T needed to drive the drum, in terms of m, R, θ, and $\dot{\theta}$.

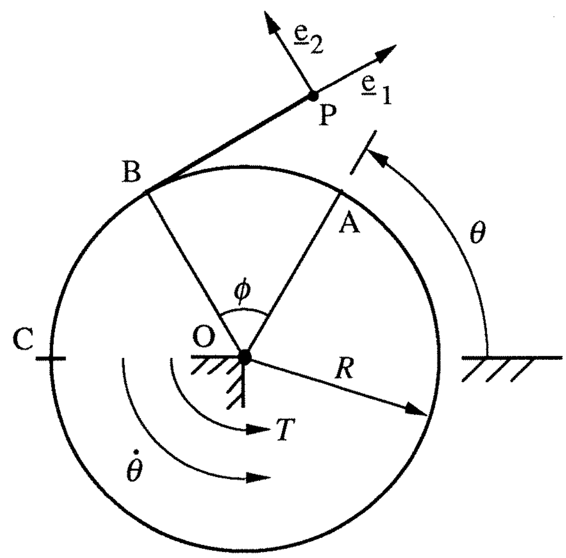

Fig. 4

8 Figure 5 shows two views of an eccentrically mounted circular cam of radius 30 mm rotating at 10 rad s^{-1} which drives a flat-footed circular follower. The follower is free to move vertically, but is prevented from rotating about its axis of symmetry.

(a) Find expressions in terms of α (as defined in Fig. 5) for:

(i) the vertical velocity of the follower;

(ii) the sliding velocity at the point of contact.

(b) The follower is now freed so that it can rotate without friction about its vertical axis of symmetry. Because of friction at the contact point, it rotates such that the sliding velocity is minimised. What is the angular velocity of the follower when $\alpha = 30°$?

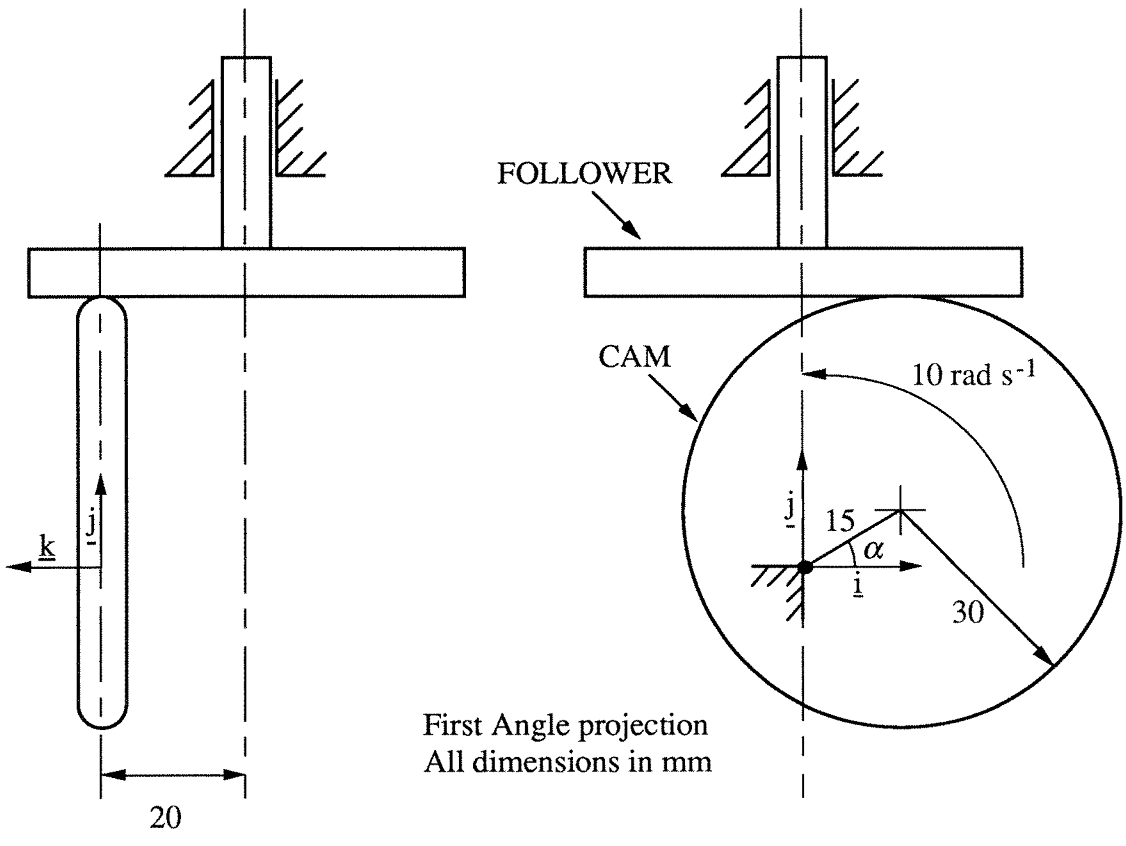

Fig. 5

SECTION C

Answer not more than **two** *questions from this section.*

9 (a) The circuit shown in Fig. 6 is running in a steady state when a voltmeter with an internal resistance of 300 kΩ is connected across the terminals. Derive an expression for the subsequent reading of the voltmeter, and draw a sketch of the voltage against time. What is the time constant of the circuit, and how long does it take before the recorded voltage falls to 65 V?

(b) As soon as the voltage falls to 65 V, the voltmeter is removed. How much longer is it before the voltage across the terminals of the circuit returns to within 5 V of its original value?

(c) The voltmeter is now reconnected, and the current generator is set to deliver a sinusoidal alternating current of amplitude 0.5 mA with a period of 1 minute. What will be the eventual amplitude of the voltage reading, and the phase angle between the current generator and the voltmeter?

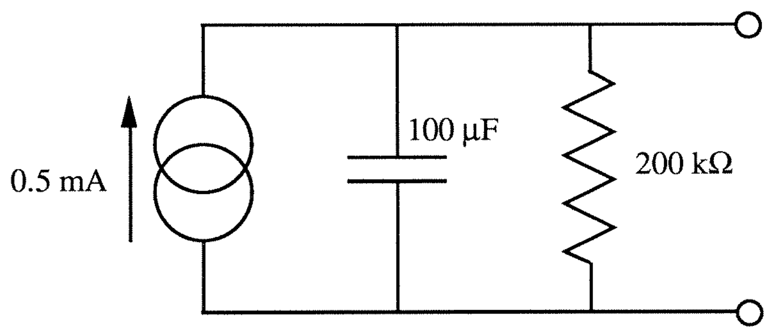

Fig. 6

10 A controller is to be mounted on a heavy machine tool whose predominant frequency of vibration during operation is 100 Hz. There is concern about the effect of the vibration on the controller, and it is proposed to mount it using a spring and dashpot system as shown in Fig. 7. The mass of the controller is 3 kg, and it is required that the amplitude of its vibration y should be one quarter that of the machine tool x at the principal operating frequency, and not more than twice that of the machine tool at any other frequency. Find suitable values for the spring stiffness k, and the damper rate λ.

If the peak-to-peak amplitude of the machine tool's vibration is 2 mm at any frequency, what is the maximum acceleration of the controller at the machine's operating frequency, and at resonance?

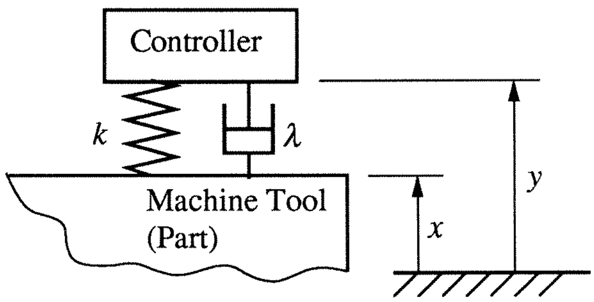

Fig. 7

11 Part of a gearbox is shown schematically in Fig. 8. Two gears with polar moments of inertia $2J$ and $3J$ are connected to each other and to ground by shafts of torsional stiffness k and $2k$ respectively. The smaller gear is excited by a sinusoidal torque of magnitude T.

(a) Write down the equations of angular motion for the two gears in matrix form, and identify the mass matrix and the stiffness matrix.

(b) Use the equation you have written to find the resonant frequencies of the system. Sketch the corresponding modal shapes, showing clearly the ratios of the amplitudes of the two gears for each mode.

(c) At what excitation frequency will the amplitude of the response of the smaller gear be zero, and what is the corresponding amplitude of vibration of the larger gear?

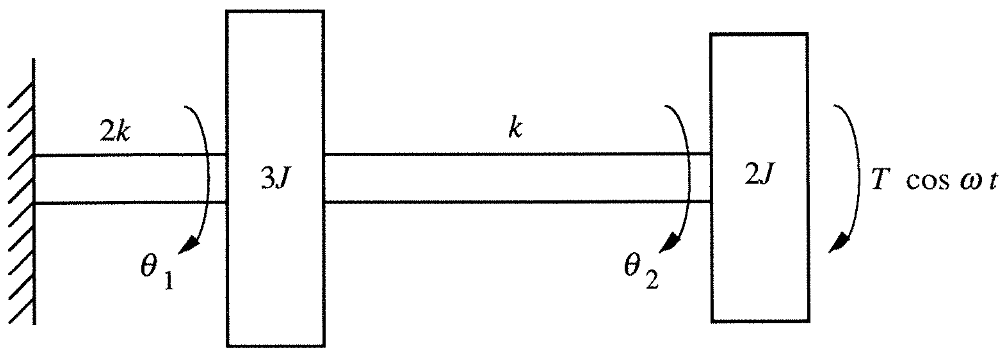

Fig. 8

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