ENGINEERING TRIPOS PART IA

Tuesday 11 June 1996 9 to 12

Paper 2

STRUCTURES AND MATERIALS

Answer not more than **eight** *questions, of which not more than* **four** *may be taken from Section A and not more than* **four** *from Section B.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

*Answer not more than **four** questions from this section.*

1 (a) Two frictionless uniform circular cylinders of the same diameter but different weights W_1 and W_2 rest with their axes horizontal in equilbrium between two frictionless planes as shown in Fig. 1. The plane joining their axes lies at an angle of 15° to the horizontal. Determine, either by a graphical method or by calculation, the ratio W_1/W_2.

(b) Show that if there is friction between the cylinders and the planes, but not between the two cylinders, the ratio W_1/W_2 for equilibrium is unchanged.

(c) If there is friction on all three lines of contact, with a friction angle of 10°, determine the two limiting equilibrium conditions and the corresponding ratios of W_1/W_2.

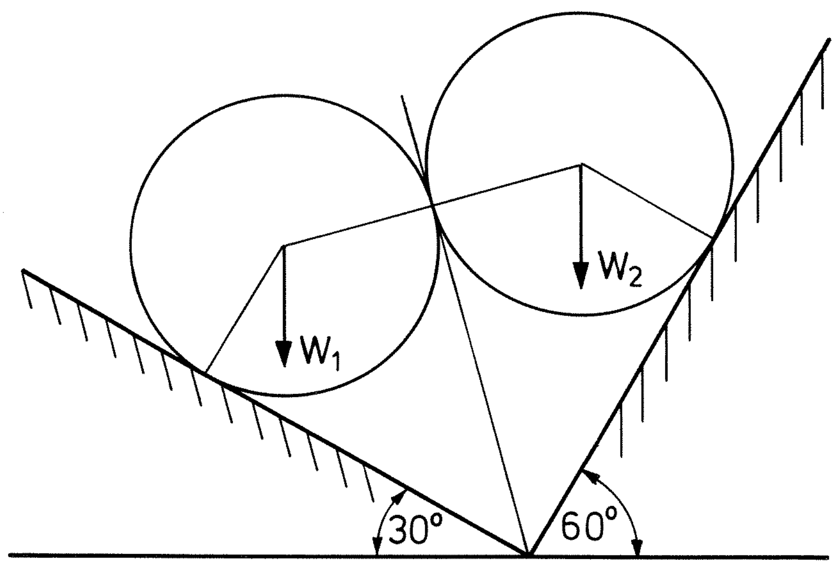

Fig. 1

2 Figure 2 shows a two-dimensional pin-jointed structure in which all the bars are of the same cross-sectional area A, and are made of a linear-elastic material with a Young's modulus E. The bars BC, CD, CG and FG are of length L and bars CF and DG are of length $L\sqrt{2}$. A vertical load W is applied at joint D.

(a) Calculate the bar forces in the structure and hence determine the change in length of each bar caused by the load W.

(b) Determine the vertical and horizontal components of the displacement of joint G.

(c) Find the additional vertical displacement of joint G if the support at B is displaced horizontally by $L/50$.

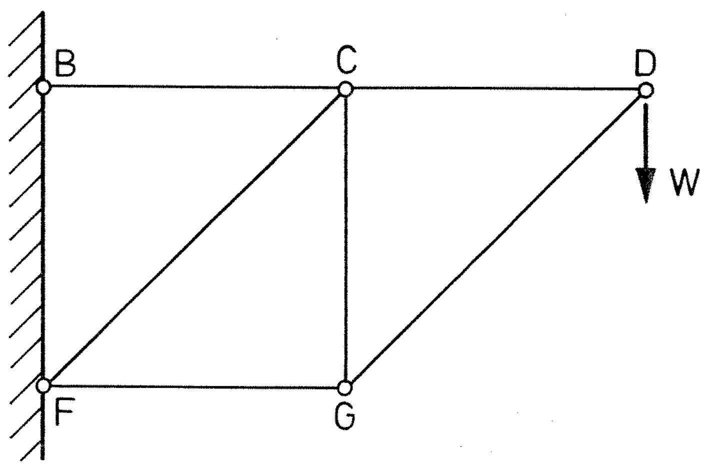

Fig.2

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3 A horizontal beam of length L is simply-supported at its two ends. It is loaded on the left-hand half of its span by a distributed vertical load q per unit length, and the right-hand half of the span is unloaded.

Sketch the form of the loading and determine the bending moment at mid-span for the following three forms of distributed loading :

(a) $q = q_o$ (constant)

(b) $q = 2 \dfrac{q_o}{L} \left(\dfrac{L}{2} - x \right)$

(c) $q = 8 \dfrac{q_o}{L^3} \left(\dfrac{L}{2} - x \right)^3$

where x is measured horizontally from the left-hand support.

4 A uniform cantilever beam of bending stiffness $B = EI$ and length L carries a transverse concentrated load W at a distance a from the built-in end, as shown in Fig. 3. Use Macaulay's method to obtain a general expression for the bending moment and hence, by integration, derive an expression for the transverse deflection v of the beam at a distance x along the beam, measured from an origin at its left hand end, in terms of W, B, a, L and x.

Obtain an expression for the deflection at the free end of the cantilever, and express this in its simplest form. Compare your answer with that derived from the results tabulated in the Data Book.

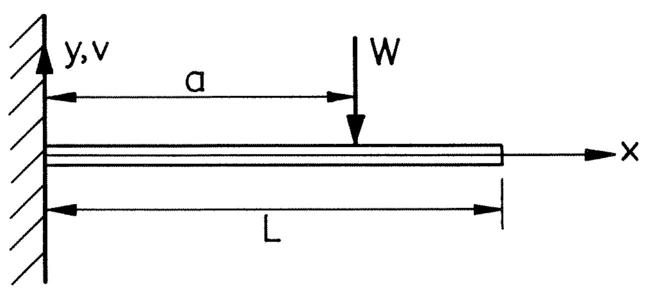

Fig. 3

5 A 254 mm × 146 mm × 37 kgm^{-1} Universal Beam (Data Book page 10) carries a bending moment of 75 kNm about its major axis at a particular cross-section. Calculate the maximum longitudinal bending stress at this section assuming that the beam remains elastic and is stress-free before the bending moment is applied.

Two long steel plates, each of cross-section 146.4 mm × 10 mm, are bolted to the outer surfaces of the flanges of the Universal Beam, as shown in Fig. 4, by pairs of bolts at a pitch of 300 mm along the length of the beam. Determine the second moment of area of the new cross-section about the major axis, and calculate the maximum longitudinal bending stress caused by the same bending moment of 75 kNm.

With the beam mounted horizontally a vertical shearing force of 50 kN is applied symmetrically to the cross-section. Calculate the shearing force carried by each bolt.

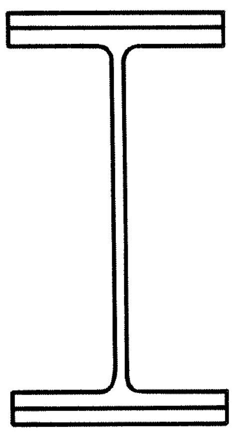

Fig. 4

SECTION B

Answer not more than **four** *questions from this section.*

Material properties not specifically quoted should be taken from the Materials Data Book.

6 Explain carefully the difference between primary and secondary bonds and give two examples of each.

The potential energy U between two isolated positive and negative ions varies with their separation r according to the following equation :

$$U(r) = -\frac{A}{r} + \frac{B}{r^n}$$

where A, B and n are constants. Sketch $U(r)$ as a function of r and label any important features.

S_0 is the bond stiffness at the equilibrium separation r_0. Derive an expression for S_0 in terms of A, B, n and r_0.

Calculate the equilibrium separation of sodium and chlorine ions, assuming $A = 1.5 \times 10^{-10}$ N nm^2, $B = 7.0 \times 10^{-16}$ N nm^9 and $n = 8$ for this pair of ions.

7 Describe three methods of increasing the yield strength of metals and alloys.

The true stress σ_t in an annealed metal alloy is observed to vary with true strain ε_t according to the approximate equation :

$$\sigma_t = \sigma_0 \, \varepsilon_t^{\,m}$$

where m is a positive constant. Identify the mechanism for the observed increase in σ_t with increasing ε_t.

A tensile test on the alloy involves measuring the nominal stress σ_n and the nominal strain ε_n. Derive an expression for σ_n in terms of σ_0, ε_n and m

The stress-strain behaviour of the alloy may be described by the above relation with σ_0 = 895 MPa and m = 0.49 . Use Fig. 2.6 on page 16 of the Materials Data Book to identify the alloy.

A rod of the alloy having a circular cross-section is to be reduced in diameter by drawing through a die, as shown in Fig. 5. Estimate the maximum applied force F that could be used for this purpose without causing the rod to yield downstream of the die.

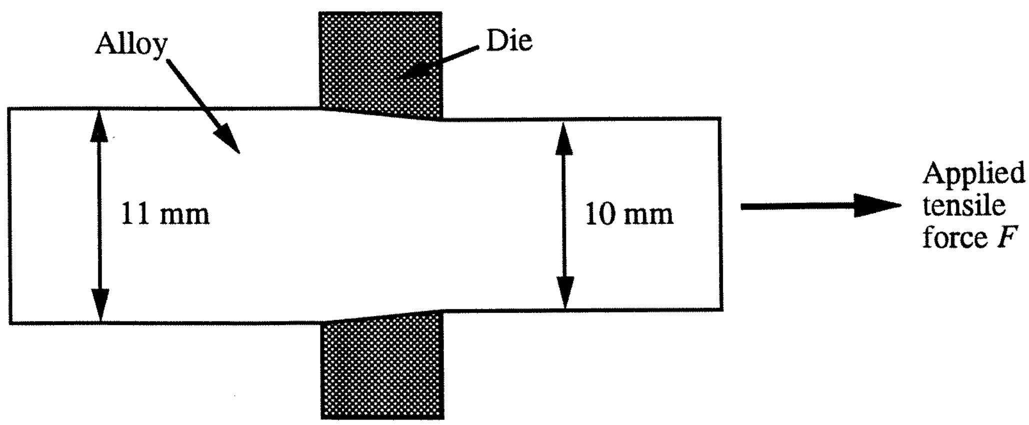

Fig. 5

8 Discuss the factors which determine whether a pressure vessel will fail gradually by leaking or by fast fracture when subjected to repeated applications of internal pressure.

The rate at which a crack of length a grows due to fatigue is given by :

$$\frac{da}{dN} = A\,(\Delta K)^n$$

where N is the number of stress cycles, A and n are constants, and ΔK is the tensile stress intensity range.

Use the above equation and the expression for the stress intensity factor in the Materials Data Book to show that :

$$N_f = \frac{1}{AY^n \pi^{n/2} (\Delta\sigma)^n} \times \frac{1}{1 - n/2} \times \left[a_c^{(1 - n/2)} - a_0^{(1 - n/2)} \right]$$

where N_f is the number of cycles to failure, Y is a dimensionless geometry-dependent parameter, $\Delta\sigma$ is the range of applied tensile stress, a_0 is the initial flaw length, a_c is the critical crack length for fast fracture and $n/2 \neq 1$.

A die fabricated from pressure vessel steel (HY130) is to withstand tensile hoop stresses of between 0 and 400 MPa. Prior to use it has been determined that the length of the largest crack in the steel is 1 mm. Determine the minimum number of times the die may be used before failing by fatigue, assuming $A = 1.0 \times 10^{-12}\,\mathrm{MPa^{-3}m^{-1/2}}$, $n = 3$ and $Y = 1.13$.

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9 Identify the three stages of creep that occur in metals and alloys under constant load at an elevated temperature. Use a sketch of creep strain against time to illustrate your answer.

The steady state creep rate $\dot{\varepsilon}_s$ varies with applied stress σ and temperature T according to the following equation :

$$\dot{\varepsilon}_s = A\,\sigma^n \exp\left(-\frac{Q}{R\,T}\right)$$

where Q is the activation energy for creep, A and n are constants and R is the universal gas constant.

Figure 6 shows a log-log plot of $\dot{\varepsilon}_s$ against σ for a nickel-steel alloy at two temperatures. Use this figure to determine the value of n in the above equation and hence identify whether power law or diffusion creep is responsible for the creep deformation of the alloy.

Estimate the activation energy for creep Q in J mol^{-1} and the value of A .

A mass of 40 kg is suspended from a bar of the above alloy of length 50 mm and diameter 5 mm in a creep test at 627°C. Determine the time taken for the test bar to elongate by 50 μm.

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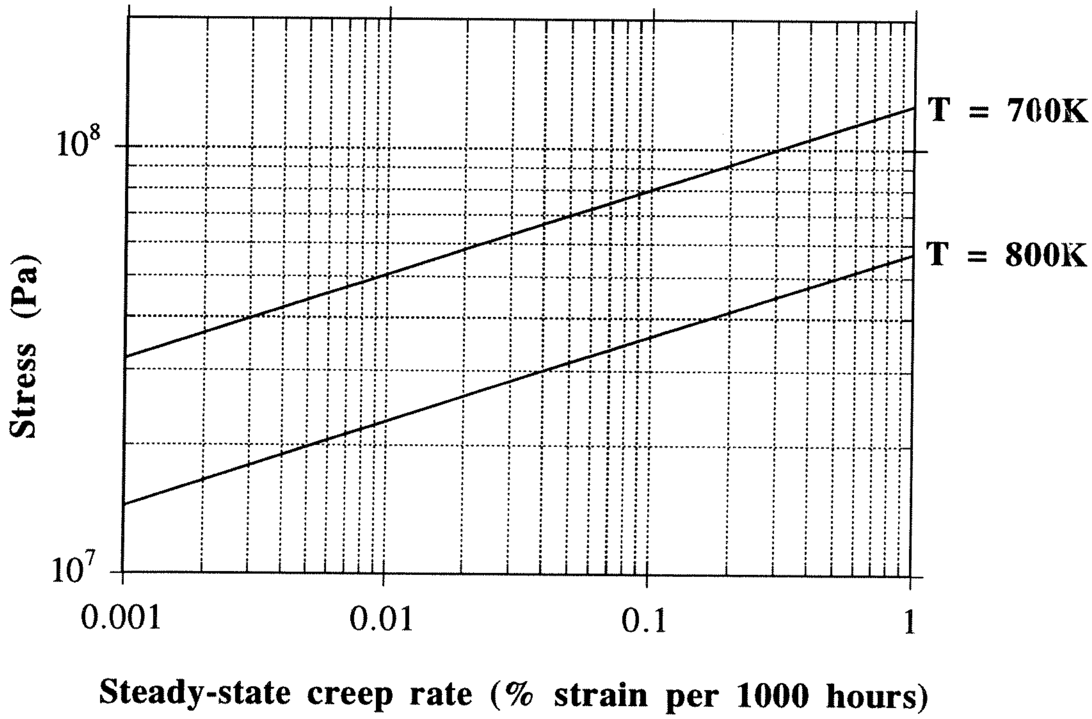

Fig. 6

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10 Explain how metals corrode when exposed to aerated water and outline the significance of the electrode potential in this respect. Describe carefully three methods of protecting metals such as steel against corrosion of this nature.

The oxidation of nickel at elevated temperature in air results in the following gain in weight with time :

Weight Gain ($mg\ cm^{-2}$)	Time (Minutes)
0.527	10
0.899	30
1.627	100

Use these data to derive a simple relationship describing the oxidation kinetics of nickel. Hence estimate the gain in weight per unit of exposed area under these conditions after 600 minutes.

Plates of cadmium, nickel and tin are exposed to sea water of constant temperature and composition. Rank these metals in order of increasing rate of corrosion. Why do these metals appear in a different order in Table 1.2 in the Materials Data Book?

END OF PAPER