Engineering Tripos Part IA 1997

Paper 1: Mechanical Engineering

ANSWERS

Section A

1. (b) (i) 0.90 (ii) -35 kJ/kg (iii) -52.3 J/kg K; yes

2. (a) (i) 2683 kJ/kg (ii) 0.954

3. (b) (i) 0; 1.013 MJ/kg; -755 kJ/kg (ii) 25.5%

4. (b) 280 kJ/kg (c) 548 kJ/kg (d) 187 kg/s (e) 32.3%

Section B

5. (a) 1.5 MJ (b) 1.152 MN; 1.002 MN (c) 81.8 m/s^2

6. (a) 1.25 rad/s clockwise; 87 mm/s (b) 2.40 rad/s anticlockwise; 90 mm/s
(c) 0.54 Nm

7. (b) (i) 75 rad/s clockwise; 11 m/s (ii) 1175 rad/s clockwise (absolute)

8. (a) $2\pi r\sqrt{\frac{r}{MG}}$ (b) r and $0.391r$ (c) decrease

Section C

9. (a) 0.5 s (approx) (b) Amplitude error 1.15° C; Phase error 17.4°

10. (c) (i) 0 (c) V_E

11. (b) $\sqrt{\frac{2k}{m}}$ and $\sqrt{\frac{7k}{m}}$ (c) $2\sqrt{\frac{k}{m}}$

Part 1A Engineering Tripos 1997

Paper 2 Section A, Structures

Answers

1 (a) max. $T_2 = T_1 e^{\mu\theta}$, min. $T_2 = T_1 e^{-\mu\theta}$

(b) max. $x = 0.511\ell$, min. $x = 0.323\ell$

2 (a) vertical 25000 kN, horizontal 93750 kN

(b) (i) $y = 0.267x$

(ii) $y = \dfrac{-50}{93750}\left(x^2 - 1000x + 62500\right)$

(d) 94 m

3 (b) $\delta_C = 4.50\dfrac{W\ell}{AE}$, $\delta_G = 2.33\dfrac{W\ell}{AE}$

(c) $\delta_C = 6.83\dfrac{W\ell}{AE}$

4 (a) $R_A = 300N$, $R_C = 100N$

(c) $\delta_E = 2.51$ mm

5 (a) max. bending moment 250 kNm, max. shear force 100 kN

(b) $I_{\text{wood}} = 4.45\times10^{-3}$ m^4 $\left(I_{\text{steel}} = 1.91\times10^{-4}\text{ m}^4\right)$

(c) max. tensile stress 17.6×10^6 Nm^{-2}

max. compressive stress 4.89×10^6 Nm^{-2}

(d) max. shear stress 1.02×10^6 Nm^{-2}

ENGINEERING TRIPOS PART IA 1997

Tuesday 10 June 1997 9.00 to 12.00

PAPER 2, SECTION B (MATERIALS): NUMERICAL SOLUTIONS

6. (c) 2780 N.
Yield stress of copper = 60 MPa (Data Book) which is greater than the applied stress so rod will deform elastically.

7. (b) $B = A \exp\left(-\frac{Q}{RT}\right)$ $3 < n < 8$

(c) $B \sim 4 \times 10^{-58}\ Pa^{-5.94}s^{-1}$ $n \sim 6$

(d) Lifetime of blade = 3.8 years

8. (d) $(\sigma_n, \varepsilon_n) = (538MPa, 0.20)$ corresponds to the maximum in the stress strain curve for in Fig 2.6in the Materials Data Book for mild steel.

9. $K_{IC} \sim 26MNm^{-3/2}$, $G_{IC} \sim 8.5\ kJm^{-2}$, suggesting that the material is aluminium alloy.

Answers to the numerical questions
Part IA Paper 3 Electrical and Information Engineering June 1997

Section A

Q.1 c. resistance is 0.11 ohm
d. frequency is 1.1 kHz

Q.2 b. input impedance is 10 Mohm
gain is 0.91
output impedance is 455 ohm
d. frequency is 2.4 kHz

Q.3 b. current is 1.1 A
c. current is 1 A
power is 0.5 W
d. current is now 0.588 A

Q.4 a. turns ratio is 8 : 1; impedance is 640 + j320 ohm
c. X_O is 2.77 kohm; X_t is 15 ohm
d. power factor is 0.82 lag; current is 0.42 A; power is 82.5 W
e. capacitance is 3.2 μF

Section B

5. $Y_3 = \overline{X}_3(X_0 + X_1 + X_2) + \overline{X}_0\overline{X}_1\overline{X}_2X_3$
$Y_2 = \overline{X}_0\overline{X}_1X_2 + \overline{X}_2(X_0 + X_1)$
$Y_1 = X_2\overline{X}_3 + \overline{X}_2X_3$

6.(a) With outputs Q and Q' and inputs $\overline{S}$ and $\overline{R}$:
$Q = \overline{\overline{S}.Q'} = S + \overline{Q'}$
$Q' = \overline{\overline{R}.Q} = R + \overline{Q}$
Hence the state $\overline{R} = \overline{S} = 0$ *or* $R = S = 1$ is illegal

(b) $J_B = Q_CQ_D$, $K_B = Q_A + Q_CQ_D$
$J_C = Q_AQ_B + Q_D$, $K_C = Q_D$

(c) Counter enters normal cycle after 4 clock pulses.

7.(a) S_0 is the least significant bit.

(b) 4KΩ, 1KΩ. 75/16 V = 4.6875 V.

(c) S_0 is the least significant bit.
3KΩ, 3KΩ, 3KΩ.

8.(b) (i) ACCA - unchanged. ACCB becomes $02.
C becomes 1, Z becomes 0, N becomes 0.

(ii) ACCB - unchanged. ACCA becomes $00.
C becomes 1, Z becomes 1, N becomes 0.

(iii) ACCB - unchanged. ACCA becomes $80.
C unaffected (0), Z becomes 0, N becomes 1.

Section C

Q.9 a. capacitance is $4 \pi \varepsilon_o r_s$

c. force is 1.8×10^{-8} N

d. angle is 10^o with respect to the y-axis; and $2\ ^1/_2\ ^o$ when the diameter is doubled

Q.10 b. electric field is 17 kVm^{-1}

Q.11 c. force is 6 N; (magnetic flux density is 0.5 T)
with 1 amp the force is 24 N

PAPER 4 **Numerical Answers to 1997 Part 1A, "Mathematical methods"**

Question 1

Expression for $\underline{X}_p$ is: $\underline{X}_p = \underline{X}_0 + \alpha_1\underline{X}_1 + \alpha_2\underline{X}_2$

Coordinates of the Intersection: $X_p = (2\ 2\ 2)$

Question 2

(b) $\lim_{x\to 0} = \frac{1}{2}$

Question3

General solution: $y = e^{-x}(A\cos x + B\sin x - 8\sin 2x)$

Solution to fit boundary cond.: $y = 8e^{-x}(2\sin x - \sin 2x)$

Question 4

General solution : $u_i = A + B\left(\frac{d+c}{d-c}\right)^i$

Solution: $u_i = \frac{1-\left(\frac{d+c}{d-c}\right)^{i-1}}{1-\left(\frac{d+c}{d-c}\right)^{I-1}}$

Question 5

Values are: $r_{12} = \bar{a}_2 \cdot \bar{q}_1 / \bar{q}_1 \cdot \bar{q}_1$

$r_{13} = \bar{a}_3 \cdot \bar{q}_1 / \bar{q}_1 \cdot \bar{q}_1$

$r_{23} = \bar{a}_3 \cdot \bar{q}_3 / \bar{q}_2 \cdot \bar{q}_2$

Question 6

a) $\text{p(first and last)} = \left(\frac{n}{n+m}\right)\left(\frac{n-1}{n+m-1}\right)$

b) $= n\frac{n!m!}{(n+m)!}$

c) $= \frac{(m!)^2}{(m-n)!(m+n)!}$

Question 7

Direction of steepest ascent if ∇_z, and at $x = y = 1$, $\nabla_z = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

Question 8

a) step response: $= 2 - 2e^{-t/2}$

b) response: $= -\left(2t^2 + 8t + 16\right)e^{-t} + 16e^{-t/2}$

Question 9

y is continuous for both odd and even functions but $\frac{dy}{dx}$ is discontinuous for the even function so the odd function converges more rapidly.

$\pi b_n = 0$ if n odd

$\pi b_n = \frac{8n}{\pi\left(n^2 - 1\right)^2}$ if n is even

Question 10

a) $\frac{1}{6}\sinh 2t - \frac{1}{3}\sinh t$

b) $\frac{1}{4} - \frac{1}{4}e^{-2t} + \frac{1}{2}e^{-t}\sin t$