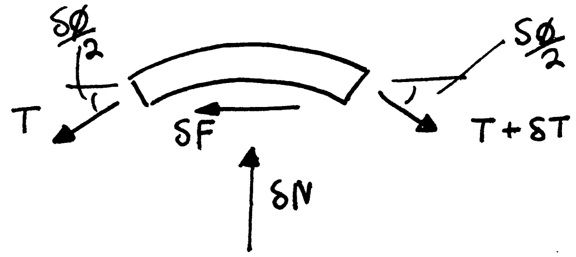
ENGINEERING TRIPOS PART IA
1997
Paper 2: Structures and Materials
SOLUTIONS

1\. (a) Consider a small section of cable subtending an angle $\delta\phi$



Equilibrium $\updownarrow$, $\delta N = T\delta\phi$
$\leftrightarrow$, $\delta F = \delta T$ } neglecting 2nd order terms.

If friction is limiting, $\delta F = \mu\,\delta N$

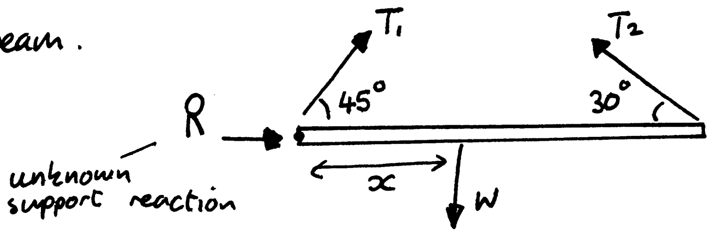
$\delta T = \mu T\,\delta\phi$

As $\delta\phi \to 0$ $\quad \dfrac{dT}{d\phi} = \mu T$

If friction is limiting forward $\displaystyle\int_{T_1}^{T_2}\frac{dT}{T} = \mu\int_0^{\theta} d\phi \Rightarrow T_2 = T_1 e^{\mu\theta}$

If friction is limiting in reverse $\displaystyle\int_{T_2}^{T_1}\frac{dT}{T} = \mu\int_0^{\theta} d\phi \Rightarrow T_2 = T_1 e^{-\mu\theta}$

1(b) Consider the beam.



Moments about left hand end, $T_2 \sin 30^\circ . l = Wx$

$$T_2 = 2W\frac{x}{l}$$

" " right " " , $T_1 \sin 45^\circ . l = W(l-x)$

$$T_1 = \sqrt{2}\,W\frac{(l-x)}{l}$$

Angle θ subtended by cable around peg (in radians)

$$\theta = (30+45) \times \frac{\pi}{180} = 1.309 \quad , \mu = 0.3$$

$$\therefore e^{\mu\theta} = 1.481$$

If $T_2 > T_1$, and friction is limiting, $T_2 = T_1 e^{\mu\theta}$

$$\therefore 2W\frac{x}{l} = \sqrt{2}W\frac{(l-x)}{l} . 1.481$$

$$\therefore 2x = 2.094\,(l-x)$$

$$\therefore \underline{x = 0.511\,l}$$ Maximum for x

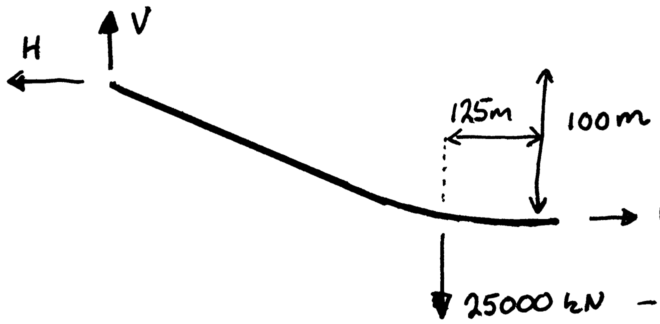
If $T_1 > T_2$ and friction is limiting, $T_1 = T_2 e^{\mu\theta}$

$$\therefore \sqrt{2}W\frac{(l-x)}{l} = 2W\frac{x}{l} . 1.481$$

$$\therefore 1.414(l-x) = 2.962x$$

$$\underline{x = 0.323\,l}$$ Minimum for x

2(a) Consider half cable



25000 kN – resultant of distributed load.

Equilibrium ↕ , $\underline{V = 25000\ \text{kN}}$

" ↻ about left-hand support $H.100 = 25000\,(500-125)$

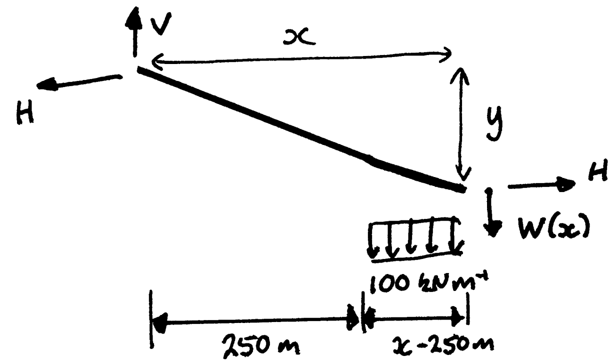
$\underline{H = 93750\ \text{kN}}$

b(i) for $0 \leq x \leq 250\text{m}$, no external loads, constant slope

$$\frac{y}{x} = \frac{V}{H} = 0.267$$

$$\underline{y = 0.267.x}$$

b(ii) for $250 \leq x \leq 500$, draw free body of cable cut at x



Moments about cut

$$V.x = H.y + 100.(x-250).\frac{(x-250)}{2}$$

$$\therefore 25000x = 93750.y + 50(x^2 - 500x + 62500)$$

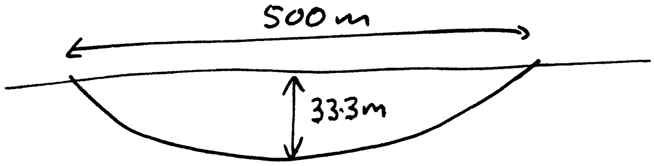
$$\therefore y = \frac{-50}{93750}(x^2 - 1000x + 62500)$$

Check, at $x = 500\text{m}$, $y = 100\text{m}$ ✓, $\frac{dy}{dx} = 0$ ✓

at $x = 250\text{m}$, $y = 66.7\text{m}$ from (i) and (ii) ✓

2(c) Length for $0 < x < 250$ is $\sqrt{250^2 + 66.7^2} = 258.7\,m$

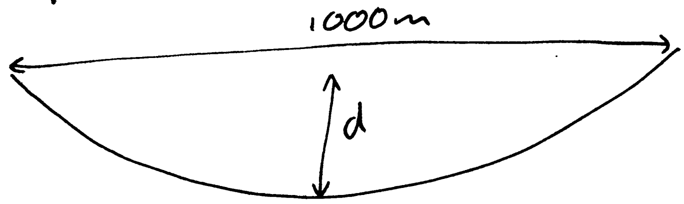
Length for $250 < x < 750$, use data book.



$$L = 500\left(1 + \frac{8 \times 33.3^2}{3 \times 500^2}\right) = 505.9\,m$$

$\therefore$ Total length $= 505.9 + 258.7 \times 2 \simeq \underline{1023m}$

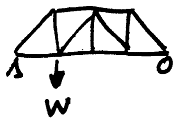
(d) With uniform loading over whole span

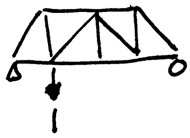


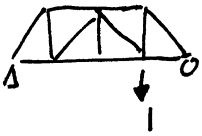
$$1000\left(1 + \frac{8d^2}{3 \times 1000^2}\right) = 1023$$

$\therefore d \simeq 93m$ (without rounding errors, 93.67m)

3. Actual loading — Virtual system (1) — Virtual system (2)







(a)

	Actual Tension T $(\times W)$	Length $(\times \ell)$	Actual Extension e $(\times \frac{W\ell}{AE})$	Virtual Tension(1) T_1^*	Virtual Tension(2) T_2^*	$T_1^* e$ $(\times \frac{W\ell}{AE})$	$T_2^* e$ $(\times \frac{W\ell}{AE})$
AB	$-\frac{3\sqrt{2}}{4}$	$\sqrt{2}$	$-\frac{3}{2}$	$-\frac{3\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{9\sqrt{2}}{8}$	$\frac{3\sqrt{2}}{8}$
FH	$-\frac{\sqrt{2}}{4}$	$\sqrt{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{4}$	$-\frac{3\sqrt{2}}{4}$	$\frac{\sqrt{2}}{8}$	$\frac{3\sqrt{2}}{8}$
AC	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{9}{16}$	$\frac{3}{16}$
GH	$\frac{1}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{16}$	$\frac{3}{16}$
BC	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{9}{16}$	$\frac{3}{16}$
FG	$\frac{1}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{16}$	$\frac{3}{16}$
BD	$-\frac{3}{4}$	1	$-\frac{3}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$	$\frac{9}{16}$	$\frac{3}{16}$
DF	$-\frac{1}{4}$	1	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{16}$	$\frac{3}{16}$
CD	$\frac{\sqrt{2}}{4}$	$\sqrt{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{4}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8}$
DG	$-\frac{\sqrt{2}}{4}$	$\sqrt{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{8}$	$-\frac{\sqrt{2}}{8}$
CE	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
EG	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
DE	0	1	0	0	0	0	0

Arranged in symmetric pairs.

As column (1)

As column (1) with symmetric pairs reversed.

3(b) Virtual work, $\sum F^* \delta = \sum T^* e$

Virtual, equilibrium (F^*, T^*)

Actual, compatible (δ, e).

Using virtual system (1)

$$1 . \delta_c = \sum T_1^* e = \frac{24\sqrt{2}+38}{16} . \frac{Wl}{AE} = \underline{4.50 \frac{Wl}{AE}}$$

Using virtual system (2)

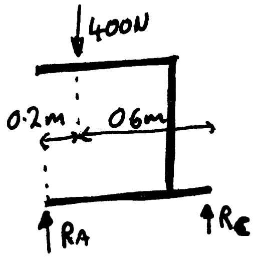
$$1 . \delta_G = \sum T_2^* e = \frac{8\sqrt{2}+26}{16} . \frac{Wl}{AE} = \underline{2.33 \frac{Wl}{AE}}$$

(c) By symmetry, deflection at C due to loading at G only

$$\delta_{C2} = \delta_G = 2.33 \frac{Wl}{AE}$$

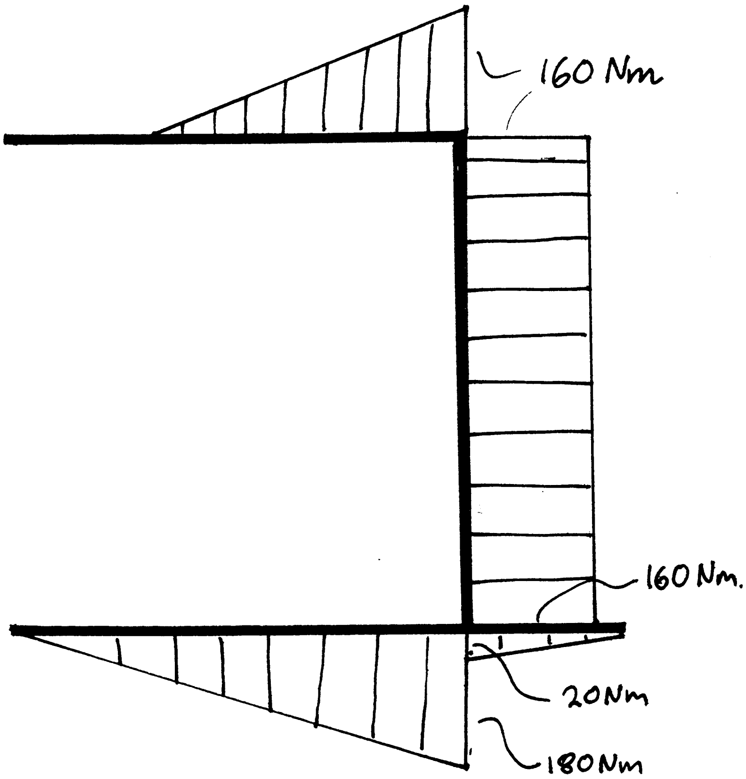
By superposition, total deflection $= \delta_C + \delta_{C2} = \underline{6.83 \frac{Wl}{AE}}$

4(a) Overall equilibrium

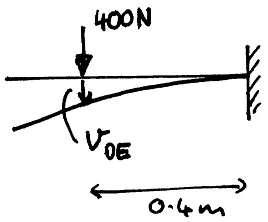


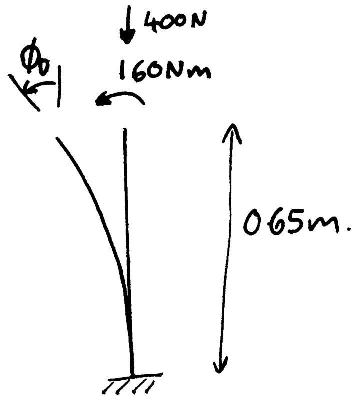
$R_A = 300\ N$
$R_E = 100\ N$

(b) Bending moments



(c) Consider as two cantilevers





From data book

$$\phi_D = \frac{Ml}{EI} = \frac{160 \times 0{\cdot}65}{20 \times 10^3} = 5{\cdot}2 \times 10^{-3}\ \text{rad}$$

$$v_{DE} = \frac{Wl^3}{3EI} = \frac{400 \times 0{\cdot}4^3}{3 \times 20 \times 10^3} = 0{\cdot}427 \times 10^{-3}\ m$$

4(c) cont. Vertical defln. of E $= \phi_D \times 0.4\,m + v_{DE}$

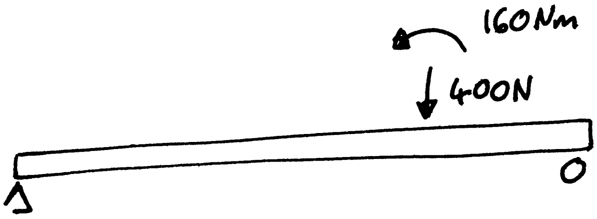
$= \underline{2.51 \times 10^{-3}\ m}$

(d) To calculate additional defln. of E, require

rotn. of B ϕ_B

defln of B v_B

Consider ABC



Could find ϕ_B and v_B in two ways

(i) Use Macauley - must allow for jump in B.M. of 160 Nm at B, jump in S.F. of 400N, at B.

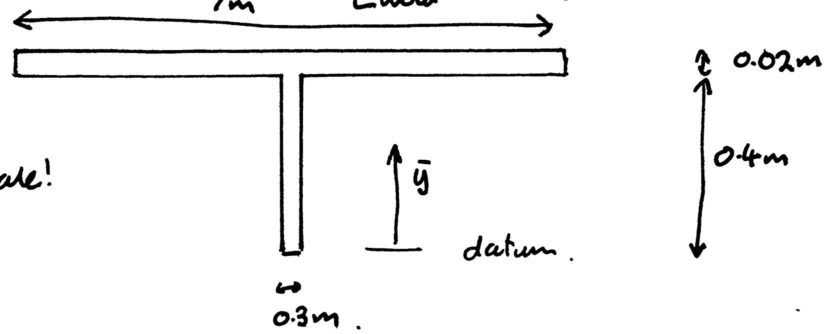
(ii) Consider AB and BC as two back-back cantilevers. Calculate ϕ_B and v_B to return A and C to their supports.

Additional defln. will be $\phi_B \times 0.4\,m + v_B$.

5 (a) Max S.F. at supports $= \dfrac{20\,kNm^{-1} \times 10\,m}{2} = \underline{100\,kN}$

Max B.M. at centre $= \dfrac{wl^2}{8} = \dfrac{20 \times 10^2}{8} = \underline{250\,kNm}$

(b) Transform section to wood, $\dfrac{E_{steel}}{E_{wood}} = \dfrac{210}{9}$



Not to scale!

Find centroid, $A = 7 \times 0.02 + 0.3 \times 0.4 = 0.26\,m^2$

$A\bar{y} = 0.4 \times 0.3 \times 0.2 + 7 \times 0.02 \times 0.41 = 0.0814\,m^3$

$\therefore \bar{y} = 0.313\,m$

Calculate I using I for rectangle and parallel axis theorem

$$I = \frac{0.3 \times 0.4^3}{12} + 0.3 \times 0.4 \times (0.313 - 0.2)^2$$

$$+ \frac{7 \times 0.02^3}{12} + 7 \times 0.02 \times (0.41 - 0.313)^2$$

$$= \underline{4.45 \times 10^{-3}\,m^4}$$

(c) Maximum tensile stress at bottom of beam, $y = 0.313\,m$

$$\sigma = \frac{My}{I} = \frac{250 \times 10^3 \times 0.313}{4.45 \times 10^{-3}} = \underline{17.6 \times 10^6\,N/m^2 \text{ (tensile)}}$$

Maximum compressive stress at wood - steel interface, $y = (0.4 - 0.313)\,m$

$$\sigma = \frac{My}{I} = \frac{250 \times 10^3 \times 0.087}{4.45 \times 10^{-3}} = \underline{4.89 \times 10^6\,N/m^2 \text{ (compressive)}}$$

5(d) $\frac{\text{Force}}{\text{length}}$ at interface $= \frac{S(A\bar{y})}{I}$

$A\bar{y}$ is 1st moment of excluded area, top flange $= 7 \times 0.02 \times (0.41 - 0.313)$

$= 0.0136 \, m^3$

$\therefore \frac{\text{Force}}{\text{length}} = \frac{100 \times 10^3 \times 0.0136}{4.45 \times 10^{-3}} = 306 \times 10^3 \, N/m$

Stress $= \frac{\text{Force}}{\text{Area}} = \frac{306 \, N/m}{0.3 \, m} = 1.02 \times 10^6 \, N/m^2$

ENGINEERING TRIPOS PART IA

Tuesday 10 June 1997 9.00 to 12.00

Paper 2

MATERIALS SOLUTIONS

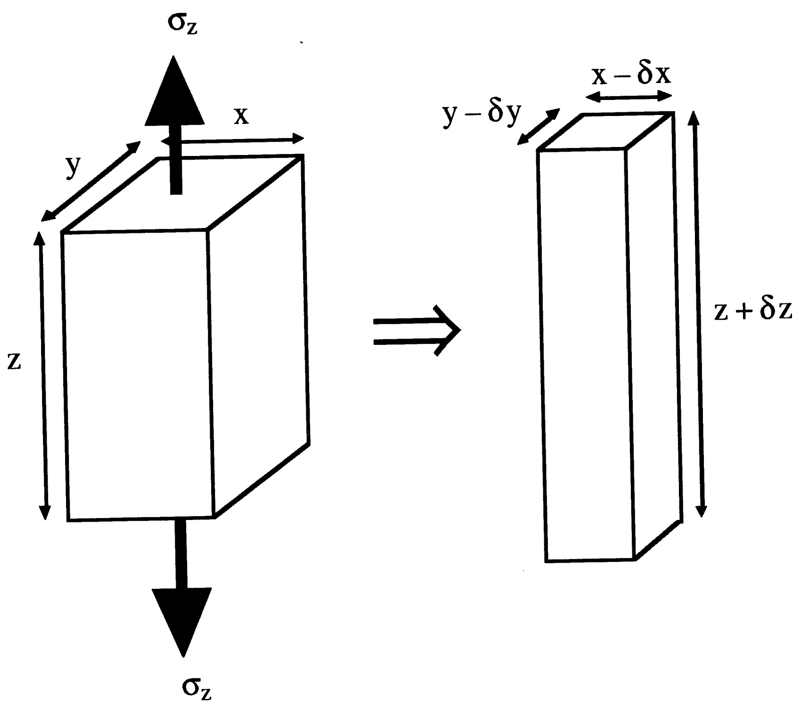
6. (a) (i) The dilatation is defined as the change in volume ($V - V_0$) divided by the original volume (V_0) during elastic deformation;

$$\Delta = \frac{V - V_0}{V_0}$$

(ii) Poisson's ratio is defined as the negative ratio of lateral and axial strains that result from an applied axial stress during elastic deformation of a test specimen. For an isotropic material with the stress applied along the z direction this may be stated as :

$$\upsilon = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z}$$

[Equations with all the terms defined are acceptable definitions.]



(b) Before deformation : volume $V_0 = xyz$

After deformation : volume $V = (x - \delta x) \times (y - \delta y) \times (z + \delta z)$

Expand and neglect terms in $(\delta)^2$: $V = (x - \delta x) \times (yz + y\delta z - z\delta y)$

$$V = xyz + xy\delta z - xz\delta y - yz\delta x$$

i.e. $$\Delta = \frac{V - V_0}{V_0} = \frac{(xyz + xy\delta z - xz\delta y - yz\delta x) - xyz}{xyz}$$

$$\Delta = \frac{\delta z}{z} - \frac{\delta y}{y} - \frac{\delta x}{x}$$

Strain along z is tensile and therefore positive. i.e. $\varepsilon_z = \frac{\delta z}{z}$

Strains along x and y are compressive and therefore negative. i.e. $\varepsilon_x = -\frac{\delta x}{x}$, $\varepsilon_y = -\frac{\delta y}{y}$

Therefore $\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z$, as required.

For isotropic material and conservation of volume, $\varepsilon_x = \varepsilon_y$ and $\Delta = 0$

$$0 = 2\varepsilon_x + \varepsilon_z \; (=2\varepsilon_y + \varepsilon_z) \quad \Rightarrow \quad \varepsilon_z = -2\varepsilon_x = -2\varepsilon_y$$

$$\nu = -\frac{\text{lateral strain}}{\text{tensile strain}} = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z} = \frac{1}{2}$$

(c) For cylindrical pure copper rod assume original length = l_1, new length = l_2, original diameter = d_1 and new diameter = d_2.

$$\Delta d = d_2 - d_1 \qquad \Delta l = l_2 - l_1$$

If tensile stress is applied along the z direction, reduction in diameter is along x (or equivalently y) direction.

$$\varepsilon_x = \frac{\Delta d}{d_1} = \frac{-1 \times 10^{-6}}{10 \times 10^{-3}} = -1 \times 10^{-4} \qquad \text{(negative since diameter reduced)}$$

Use Poisson's ratio to calculate axial strain : $\varepsilon_z = -\frac{\varepsilon_x}{\upsilon} = -\frac{(-1 \times 10^{-4})}{0.35} = 2.86 \times 10^{-4}$

$$\sigma = \varepsilon_z E = 2.86 \times 10^{-4} \times 124 \times 10^{9} = 3.54 \times 10^{7}\ \text{Pa} \qquad (35.4\ \text{MPa})$$

(Young's Modulus for copper is 124 GPa, from Data Book).

$$\text{Force} = \sigma A_0, \qquad A_0 = \pi\left(\frac{d_1}{2}\right)^2 = \pi\left(5 \times 10^{-3}\right)^2 = 7.85 \times 10^{-5}\ \text{m}^2$$

$$\text{Force} = 3.54 \times 10^{7} \times 7.85 \times 10^{-5} = 2780\ \textbf{N}$$

Yield stress of copper = 60 MPa (Data Book) which is greater than the applied stress so rod will deform elastically.

7. (a) Power law or dislocation creep occurs when diffusion allows dislocations to pass pinning centres, such as precipitates, in crystalline materials (see Ashby and Jones, page 172). On straining a material dislocations glide until they encounter an obstacle. The stresses at the pinned dislocation act as a driving force and cause atoms at the bottom of the half plane to diffuse away to enable the dislocation to move out of its slip plane and, ultimately, climb round the pinning centre. Dislocation creep is continuous and progressive and only occurs when the temperature is above $0.3T_m$, since it requires diffusion. [c.f. plastic flow which is the movement of dislocations through a material via the breaking and re-forming of bonds, which is not a diffusion effect].

Two diffusion-based mechanisms account for the movement of dislocations in the power law creep regime. (1) Core diffusion ($0.3 < T/T_m < 0.5$); atoms diffuse along the dislocation cores [cores act as channels with relatively high diffusion rates due to the distortion of the crystal structure]. (2) Bulk diffusion ($0.5 < T/T_m < 1$); atoms diffuse away from pinned dislocations through the bulk of the grains.

(b) B varies exponentially with temperature : $B = A\exp\left(-\frac{Q}{RT}\right)$ where Q is the activation energy for creep (Data Book, page 39).

n typically lies between 3 and 8 for a metal under high stress at 0.3 of its melting temperature.

(c) Reading data from the graph gives :

σ (MPa)	ε	time (minutes)
255	0.01	0
255	0.04	15 000
205	0.005	0
205	0.02	27 500

$\dot{\varepsilon} = B\sigma^n$ i.e. $\frac{\Delta\varepsilon}{\Delta t} = B\sigma^n$

$\frac{0.03}{15\times10^3} = B\left(255\times10^6\right)^n$ and $\frac{0.015}{27.5\times10^3} = B\left(205\times10^6\right)^n$

$\frac{0.03\times27.5}{0.015\times15} = \left(\frac{255}{205}\right)^n$ i.e. $n = \frac{\text{Log } 3.667}{\text{Log } 1.244} = \frac{0.564}{0.095} = 5.94$ [n=6 O.K.]

Therefore, $$\frac{0.03}{15 \times 10^3 \times 60} = B\left(255 \times 10^6\right)^{5.94}$$

Hence, $B = 3.87 \times 10^{-58}\ Pa^{-5.94}s^{-1}$ [$B = 1.21 \times 10^{-58}\ Pa^{-6}s^{-1}$ with n = 6]
($\equiv 2.32 \times 10^{-56}\ Pa^{-5.94}min^{-1} \equiv 1.01 \times 10^{-20}\ MPa^{-5.94}s^{-1} \equiv 1.69 \times 10^{-22}\ MPa^{-5.94}min^{-1}$)

[N.B. $B = 3.18 \times 10^{-58}\ Pa^{-5.95}s^{-1} \equiv 1.91 \times 10^{-56}\ Pa^{-5.95}min^{-1}$ if n = 5.95 is used]

Power law is $\dot{\varepsilon} = 2.32 \times 10^{-56}\,\sigma^{5.94}$ at 500°C ($B = 2.32 \times 10^{-56}\ Pa^{-5.94}min^{-1}$)

$\dot{\varepsilon} = 3.87 \times 10^{-58}\,\sigma^{5.94}$ at 500°C ($B = 3.87 \times 10^{-58}\ Pa^{-5.94}s^{-1}$)

[B in either $Pa^{-5.94}s^{-1}$, $Pa^{-5.94}min^{-1}$, $MPa^{-5.94}s^{-1}$ or $MPa^{-5.94}min^{-1}$ is acceptable].

(d) Substitute numbers into expression for $\dot{\varepsilon}$ at end of calculation to avoid rounding errors.

$$B = A\exp\left(-\frac{Q}{RT}\right)$$

$$A = \frac{B}{\exp\left(-\frac{Q}{RT}\right)} = \frac{0.03}{15 \times 10^3 \times \left(255 \times 10^6\right)^{5.94} \times \exp\left(-\frac{103\,000}{8.31 \times 773}\right)}$$

[$A = 2.13 \times 10^{-49}\ Pa^{-5.94}min^{-1}$ at 500°C (773K) or $1.76 \times 10^{-49}\ Pa^{-5.95}min^{-1}$ if n = 5.95 is used]

$$\dot{\varepsilon} = A\sigma^n\exp\left(-\frac{Q}{RT}\right)$$

$$\dot{\varepsilon} = \frac{0.03 \times \left(60 \times 10^6\right)^{5.94} \times \exp\left(-\frac{103\,000}{8.31 \times 788}\right)}{15 \times 10^3 \times \left(255 \times 10^6\right)^{5.94} \times \exp\left(-\frac{103\,000}{8.31 \times 773}\right)} \quad \text{at } 515\ °C$$

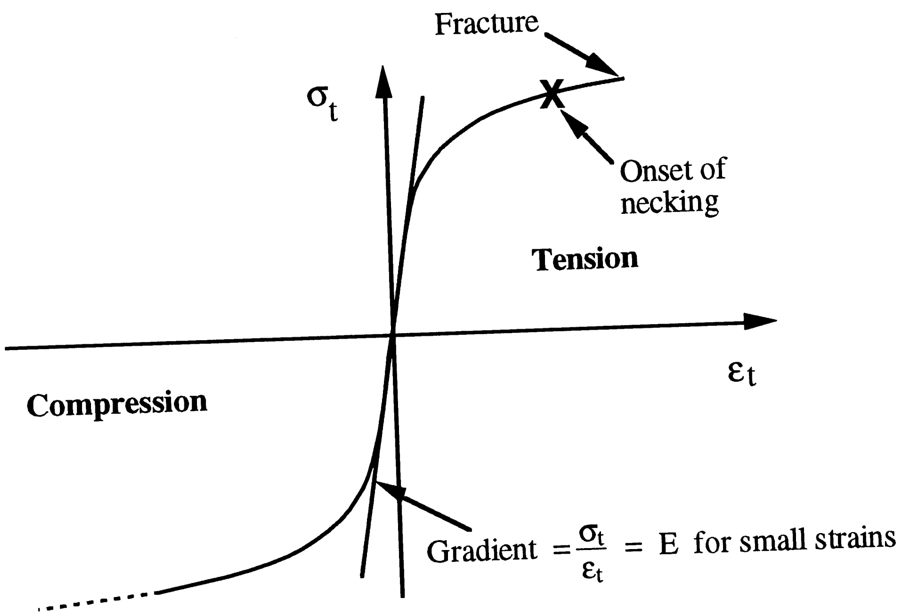
$$\dot{\varepsilon} = \frac{0.03}{15 \times 10^3} \times \left(\frac{60}{255}\right)^{5.94} \times \exp\left[\frac{103\,000}{8.31} \times \left(\frac{1}{773} - \frac{1}{788}\right)\right]$$

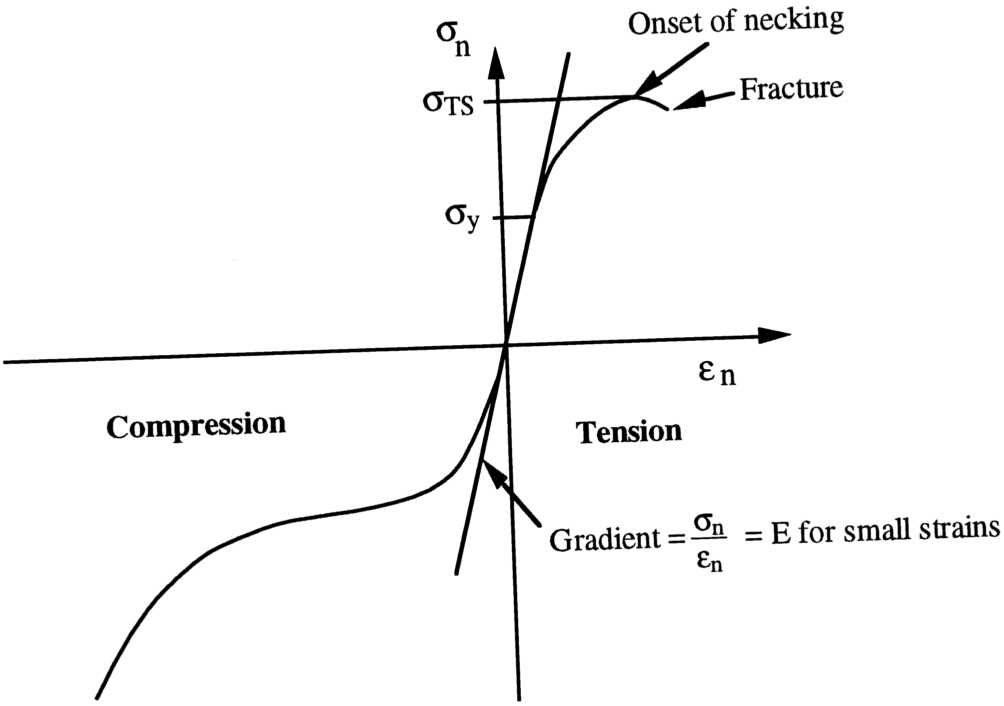
i.e. $\dot{\varepsilon} = 5 \times 10^{-10}\ min^{-1}$ at 515 °C. [same answer to 1 s.f. for $5.94 < n < 6$]

$$\text{Maximum possible strain} = \frac{100 \times 10^{-6}}{10 \times 10^{-2}} = 0.001 = 0.1\%$$

$$\text{Lifetime of blade} = \frac{0.001}{5 \times 10^{-10}} = 2 \times 10^6 \text{ minutes} = \mathbf{3.8\ years}$$

8. (a) Curves of true stress σ_t against true strain ε_t and nominal stress σ_n against nominal strain ε_n for a ductile metal under compressive and tensile loads:





(b) From Data Book ; $\sigma_t = \frac{F}{A}$, $\sigma_n = \frac{F}{A_0}$, $\varepsilon_t = \ln\left(\frac{l}{l_0}\right)$, $\varepsilon_n = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1$. Substituting $A = \frac{A_0 l_0}{l}$ into expression for σ_t and using equations for ε_n and σ_n gives

$$\sigma_t = \sigma_n (1 + \varepsilon_n)$$

Similarly, substituting $l = \frac{A_0 l_0}{A}$ into expression for ε_t and using equation for ε_n gives

$$\varepsilon_t = \ln (1 + \varepsilon_n), \text{ as required.}$$

To show $\frac{d\sigma_t}{d\varepsilon_t} = \sigma_t$ (1) and $\frac{d\sigma_n}{d\varepsilon_n} = 0$ (2) are equivalent conditions for the onset of necking:

Hence $$\frac{d\varepsilon_t}{d\varepsilon_n} = \frac{1}{1+\varepsilon_n}$$

Substituting into (1) ; $$\frac{\dfrac{d[\sigma_n(1+\varepsilon_n)]}{d\varepsilon_n}}{(1+\varepsilon_n)} = \sigma_n(1+\varepsilon_n)$$

i.e. $$\sigma_n = \frac{d[\sigma_n(1+\varepsilon_n)]}{d\varepsilon_n} = \frac{d\sigma_n}{d\varepsilon_n}(1+\varepsilon_n) + \sigma_n$$

$$\frac{d\sigma_n}{d\varepsilon_n}(1+\varepsilon_n) = 0$$

Hence either $\frac{d\sigma_n}{d\varepsilon_n} = 0$ or $\varepsilon_n = -1$.

$\frac{d\sigma_n}{d\varepsilon_n} = 0$ is the relevant solution. Q.E.D.

(c) For a ductile metal ; $\sigma_t = \sigma_0(\varepsilon_t)^m$ (5)

Substituting (5) into (1) gives ;

$$\frac{d[\sigma_0(\varepsilon_t)^m]}{d\varepsilon_t} = m\sigma_0(\varepsilon_t)^{m-1} = \sigma_t$$

i.e. $m\sigma_0(\varepsilon_t)^{m-1} = \sigma_0(\varepsilon_t)^m$ hence $\boldsymbol{\varepsilon_t} = \mathbf{m}$ at onset of necking.

(d) The metal under test has $\sigma_0 = 880$ MPa and starts to neck at a true strain of 0.18. Need to convert this to a nominal strain and determine the associated nominal stress.

Therefore, $\varepsilon_t = 0.18 = m$, hence $\sigma_t = 880(\varepsilon_t)^{0.18}$ MPa.

At necking $\sigma_t = 880(0.18)^{0.18} = 646$ MPa

From (3) $\varepsilon_n = e^{\varepsilon_t} - 1 = e^{0.18} - 1 = 0.20$

From (4) $\sigma_n = \frac{\sigma_t}{1+\varepsilon_n} = \frac{646}{1+0.2} = 538$ MPa

By inspection of Fig. 2.6 in the Materials Data Book, the point $(\sigma_n, \varepsilon_n) = (538\text{MPa}, 0.20)$ corresponds to the maximum in the stress strain curve for **mild steel**. [Note $\sigma_t = 880(\varepsilon_t)^{0.18}$ MPa is only an *approximate* fit to the actual curve for mild steel and some ambiguity may arise by evaluating the stress and strain at points other than the necking point. Additional credit given for comments along these lines.]

9. G is the strain energy release rate and K is the stress intensity factor. Fast fracture occurs when G and K reach critical values, G_{IC} and K_{IC} which are material properties. K and G are measures of the crack tip loading in terms of stresses and energy (G is the energy required to create unit area of crack) and have units of Jm^{-2} and $Nm^{-3/2}$, respectively.

For constrained strain, G_{IC} and K_{IC} are related by :

$$E' G_{IC} = K_{IC}^2 \text{ where } E' = \frac{E}{1 - \nu^2} \text{ and } \nu = \text{Poisson's ratio.}$$

[Brittle materials, for which appreciable plastic deformation is not possible in front of an advancing crack have low K_{IC} values and are vulnerable to catastrophic failure.]

Experimental measurement of K_{IC} : A sharp crack of known dimension, a, is machined into a test specimen. Stress is then applied until the material fails (by fast fracture) at σ_c. K_{IC} can be calculated from $K_{IC} = Y \sigma_c \sqrt{\pi a}$.

Assume a stress-free circle around the crack and that the stress in the rest of the material is equal to the applied stress. As the crack advances by δa at either end, the change in strain energy of the whole specimen is given, approximately, by the change in strain energy of the annulus of volume $2 \pi a t \delta a$.

Release of elastic strain energy per unit volume when stress is removed $= \frac{\sigma \varepsilon}{2} = \frac{\sigma^2}{2E}$

[N.B. This is the area under the stress - strain curve].

Total strain energy release rate = volume of annulus × strain energy release per unit volume

$$= \frac{2\pi a t \delta a \sigma^2}{2E} = \frac{\pi a t \delta a \sigma^2}{E}$$

Area of crack created for an advance of $\delta a = 2 t \delta a$ (N.B. 2 ends to crack)

Strain energy release rate, G, per unit crack area advance $= \frac{\pi a t \delta a \sigma^2}{E} \times \frac{1}{2 t \delta a} = \frac{\pi a \sigma^2}{2E}$

$$\text{Hence } G = \frac{Y^2 \sigma^2 \pi a}{E} \quad \text{with} \quad Y^2 = \frac{1}{2} \quad (\text{i.e. } Y = \frac{1}{\sqrt{2}} = 0.707)$$

Necessary to calculate G_{IC} and K_{IC} and compare these values (and E) with those given in the Data Book to identify a possible subject of the test. The relationship between K_{IC} and G_{IC} on page 37 of the Data Book involves $E' = \frac{E}{1 - \nu^2}$ rather than E since it is necessary to consider strains out of the plane in the full analysis of this problem. Constraining the plate to constant thickness, as this analysis assumes, effectively amplifies E by $\frac{1}{1-\nu^2}$ [see 1996/97 examples paper 1 question 1d(ii)].

For an edge crack : $G_{IC} = \dfrac{Y^2 \sigma_c^2 \pi a}{E'}$ (from equations in Data Book)

For $\sigma_c = 130$ MPa, $a_c = 1$ cm, E = 70 GPa, $\nu = 0.33$ and Y = 1.12 :

$$G_{IC} = \frac{1.12^2 \times (130 \times 10^6)^2 \times \pi \times 1 \times 10^{-2}}{70 \times 10^9/(1 - 0.33^2)} = 8.5 \text{ kJm}^{-2}$$

$G_{IC} = 9.5$ kJm^{-2} if E rather than E' is used in equation for G_{IC}.

Data Book candidates on basis of E and G_{IC} are values :

Material	E (GPa)	G_{IC}(kJm^{-2})	K_{IC}(MNm$^{-3/2}$)
*Al alloy	69 – 79	8 – 30	23 – 45
*CFRP	70 – 200	5 – 30	32 – 45
Zn Alloys	43 – 96	–	–
Soda Glass	69	0.013 (too low)	
Alkali Halides	15 – 68	–	

130 MPa is significantly less than the yield stress of the above candidate materials * (CFRP is brittle and fails at 670 MPa).

[Also $E' G_{IC} = K_{IC}^2$ (or alternatively $K_{IC} = Y \sigma_c \sqrt{\pi a_c}$)

Hence $K_{IC} = \sqrt{\dfrac{70 \times 10^9 \times 8.5 \times 10^3}{1 - 0.33^2}} \approx 26$ MNm$^{-3/2}$ ($\approx$ 24 MNm$^{-3/2}$ if E is used)]

This suggests that the material is likely to be **aluminium alloy** [CFRP is also acceptable].

[Full marks for identifying candidate materials on basis of E and G_{IC}.]

10. The factors which determine the rate at which a metal corrodes when exposed to oxygen are electrochemical in nature. These are (i) the thermodynamic driving force for the reaction (i.e. electrode potential), (ii) the temperature (rate effects) and (iii) the ability of the metal to form a protective oxide layer on its surface.

(i) Zinc-plated steel is stable against corrosion in an aqueous environment, even when the surface is scratched to exposed the underlying steel metal.

Zinc is anodic relative to iron (i.e. has a lower electrode potential) and will cathodically protect steel in the event of surface damage. Cathodic protection involves supplying electrons to the metal to be protected which lowers its electrode potential and immunises it from corrosion. Corrosion of the zinc under these conditions will be extremely slow due to the large ratio of anode to cathode area.

(ii) Car exhausts made of mild steel corrode much more quickly than those made of stainless steel.

Car exhausts corrode predominantly from the inside out since their interior is exposed continually to the damp gases of the combustion process. Significant external corrosion also occurs from road water which often contains chloride ions from rock salt (this speeds up the corrosion process). Chromium is more electronegative than iron in stainless steels and preferentially reacts with oxygen at the exposed surface of the metal. A resulting thin and highly adhesive layer of Cr_2O_3 subsequently forms on the inner and outer surfaces of the exhaust to form a physical barrier against further corrosion. This is reflected in the length of guarantee with which stainless and mild steel exhausts are sold – lifetime and 12 months respectively!

(iii) Mild steel radiators in domestic central heating systems undergo little corrosion after several years service.

Radiator systems initially corrode quickly as their inner surface reacts with oxygen dissolved in the re-circulating water charge. This oxygen, however, is eventually used up since the system is closed and the rate of corrosion decreases rapidly. Evidence of this initial corrosion is the discolouring of radiator water observed on bleeding. Corrosion from the outside is usually prevented by paint which forms a protective physical barrier.

(iv) Titanium has a much lower electrode potential than lead but appears well above it in the galvanic series.

Metals are ranked in the galvanic series (page 4 of the Materials Data Book) in terms of their observed corrosivity in sea water. This describes the rate at which these metals corrode, which depends strongly on the stability of the protective film in sea water. The electrode potential, on the other hand, simply ranks the metals in terms of the relative driving force for corrosion and does not take account of the formation of any protective film on the metal surface, which will slow down the rate of reaction significantly. The protective film which forms on titanium in sea water is a better barrier to corrosion than the film which forms on lead, which explains why lead appears below titanium in the galvanic series.

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27/6/97