ENGINEERING TRIPOS PART IA

Tuesday 10 June 1997 1.30 to 4.30

Paper 1

MECHANICAL ENGINEERING

Answer not more than **eight** *questions, of which not more than* **three** *may be taken from Section A, not more than* **three** *from Section B and not more than* **two** *from Section C.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer not more than **three** *questions from this section.*

1 (a) Starting from the equation [7]

$$Tds = dh - vdp ,$$

show that for a perfect gas undergoing a reversible process the change in specific entropy is

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right) .$$

Explain why this equation is also valid for an irreversible process.

(b) Air flows steadily along a horizontal duct of varying cross-sectional area. [13]
The temperature in the duct is uniform at 127 °C.

At a section A, the pressure is 1.5 bar and the velocity is 400 ms^{-1}. At another section B, the pressure is 1.8 bar and the velocity is 300 ms^{-1}.

Assuming that the flow is from A to B:

(i) Find the ratio of the cross-sectional areas at A and B;

(ii) Calculate the heat transferred (per kg of air) between A and B;

(iii) Calculate the change in specific entropy of the air between A and B.

Is the assumption that the flow is from A to B correct?

2 (a) It is comparatively easy to measure the pressure and temperature of a fluid. Explain why measurements of pressure and temperature are not always sufficient to establish the thermodynamic state of a fluid. [8]

Steam is bled from a turbine at a pressure of 1 MPa. It flows steadily and adiabatically through a *throttling calorimeter*. The pressure and temperature downstream of the throttling calorimeter are 0.05 MPa and 100 °C. The change in the steam's kinetic energy is negligible across the throttle. The process is shown schematically in Fig. 1 below.

(i) Find the specific enthalpy of the steam bled from the turbine.

(ii) Find the dryness fraction of the steam bled from the turbine.

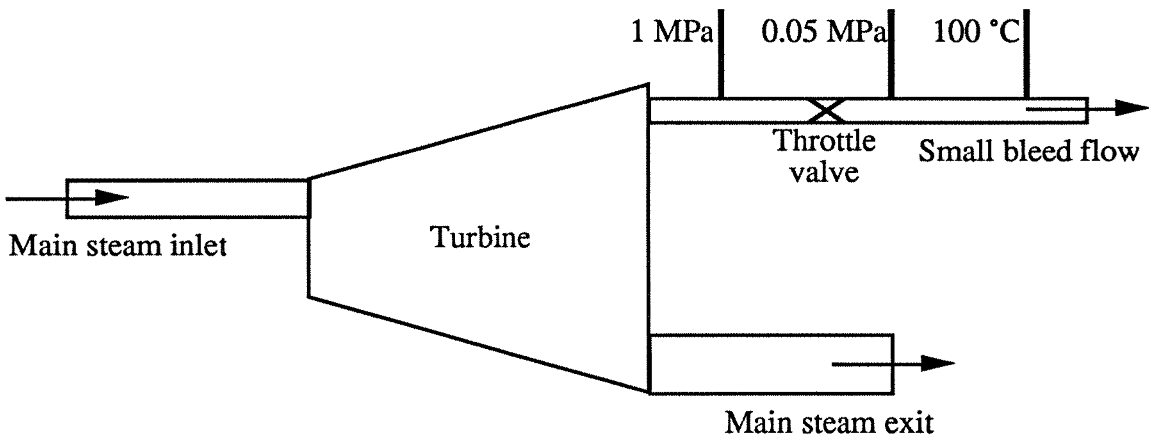

Fig. 1

(b) Discuss the similarities and differences between four-stroke petrol and diesel engines. Your answer should cover the following topics: [12]

(i) The four-stroke cycle;

(ii) The combustion process;

(iii) How power output is controlled;

(iv) Compression ratio limitations.

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3 (a) Starting from the differential form of the 1st Law of Thermodynamics [4]

$$dq - dw = du \ ,$$

show that the heat transferred to a system (not necessarily a perfect gas) during a constant pressure process is equal to the change of enthalpy of that system during the process.

(b) 1 kg of steam is confined in a cylinder behind a piston and executes the following *reversible* cycle: [16]

I. isentropic compression while the pressure rises from 0.01 MPa to 1 MPa at the end of which the steam is *dry saturated*;

II. isothermal expansion while the pressure falls to 0.01 MPa;

III. a constant pressure process during which the volume returns to its value at the start of process I.

(i) Find the heat exchanged in each of the three processes.

(ii) Calculate the thermal efficiency of the cycle.

(iii) Sketch the cycle on a T-s diagram, identifying key values.

You are encouraged to use the h-s chart to obtain values which cannot be read directly from Tables.

4 (a) A closed-cycle gas turbine plant consists of a turbine, a compressor, a heater and a cooler. Draw a schematic diagram of the plant, marking clearly the components and the order in which they appear. [2]

(b) The compressor, which has an isentropic efficiency of 90%, receives air at 1.0 bar and 100 °C and compresses it adiabatically to 6.0 bar. Show that the temperature of the air leaving the compressor is 377 °C and find the work input to the compressor per kg of air flow. [5]

(c) Air enters the turbine at a temperature of 1200 °C and a pressure of 6.0 bar. The turbine is adiabatic, has an isentropic efficiency of 92% and an exhaust pressure of 1.0 bar. Show that the temperature of the air leaving the turbine is 657 °C and find the work output from the turbine per kg of air flow. [5]

(d) If the turbine drives both the compressor and a load of 50 MW, determine the flow rate of air required. [2]

(e) What is the thermal efficiency of this cycle? [3]

(f) Explain how the thermal efficiency of the cycle can be improved through the use of an additional heat exchanger. [3]

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SECTION B

Answer not more than **three** *questions from this section.*

5 An emergency buffer system for runaway railway trucks is shown schematically in Fig. 2. The system essentially consists of a buffer block of mass 5000 kg, connected to a spring of stiffness 250 kNm^{-1}. The coefficient of friction between the buffer block and the surface upon which it slides is 1.5. When a truck runs into the buffer, it locks onto the buffer block such that there is no subsequent movement between the two bodies, and such that the truck exerts no vertical force on the buffer block.

The system is initially set up with no force in the spring, and is struck by a truck of mass 10,000 kg rolling without friction at a speed of 30 ms^{-1}.

(a) How much kinetic energy is lost during the initial impact? [4]

(b) What is the maximum tensile force and the maximum compressive force that the spring will encounter during the subsequent motion? [12]

(c) What is the maximum acceleration that the truck will experience, once it has locked onto the buffer block? [4]

Take the acceleration due to gravity (g) as 10 ms^{-2}.

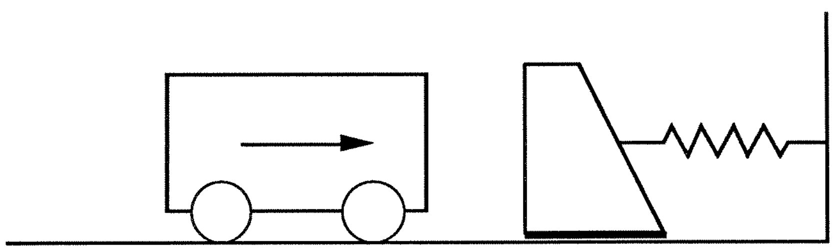

Fig. 2

6 A planar mechanism is shown diagrammatically in elevation in Fig. 3. The crank AB is 20 mm long, and rotates in a clockwise direction at 5 rad s^{-1}. It is connected to a rigid link BD which is 100 mm long and drives a second rigid link, EF, by means of a slider at D. In addition, the link BD slides freely through a swivel at C which can rotate about a stationary point at the same height as A.

(a) For the configuration shown, draw a velocity diagram for the crank AB and the link BD, and use it to find the angular velocity of BD, and the sliding velocity at C. A suitable scale for the velocity diagram is to let 1 mm represent 1 mm s^{-1}. [8]

(b) Complete your velocity diagram to include the link EF, and use it to find the angular velocity of EF, and the sliding velocity at D. [6]

(c) The motion of the point F is resisted by a horizontal force of 20 N, as shown in the figure. Assuming that the mechanism is light and that there is no friction, what torque Q is required to drive the crank in the position shown? [6]

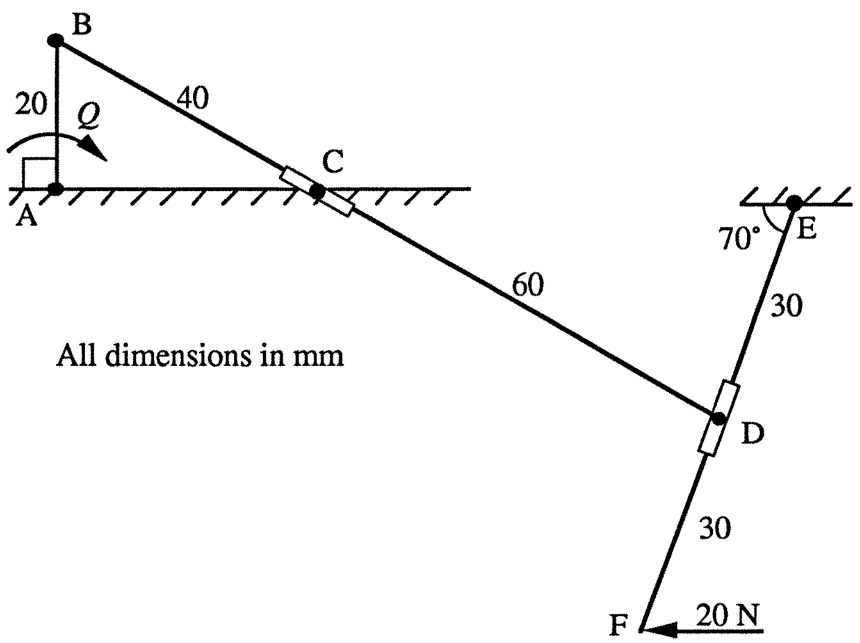

Fig. 3

7 (a) Two coplanar laminas are in contact and free to rotate about separate, fixed axes perpendicular to their plane. One lamina is then driven by the rotation of the other. Explain the significance of the *pitch point* in this situation by stating: [8]

(i) where it is located;

(ii) what can be said about the relative motion of the two laminas when it is coincident with the point of contact between them;

(iii) the significance of it remaining in the same place as the two laminas rotate;

(iv) how it can be used to determine whether the two laminas will rotate in the same, or opposite, directions.

(b) (i) Figure 4 shows a follower driven by a cam. Initially, the wheel on the follower is held so that it cannot rotate relative to the follower arm. For the configuration shown, find the angular velocity of the follower arm, and the sliding velocity at the point of contact. [12]

(ii) The wheel on the follower is now allowed to rotate freely. What will its angular velocity be, if it rotates so as to eliminate sliding at the contact point? State clearly whether the velocity you give is the absolute angular velocity of the wheel, or its angular velocity relative to the follower arm.

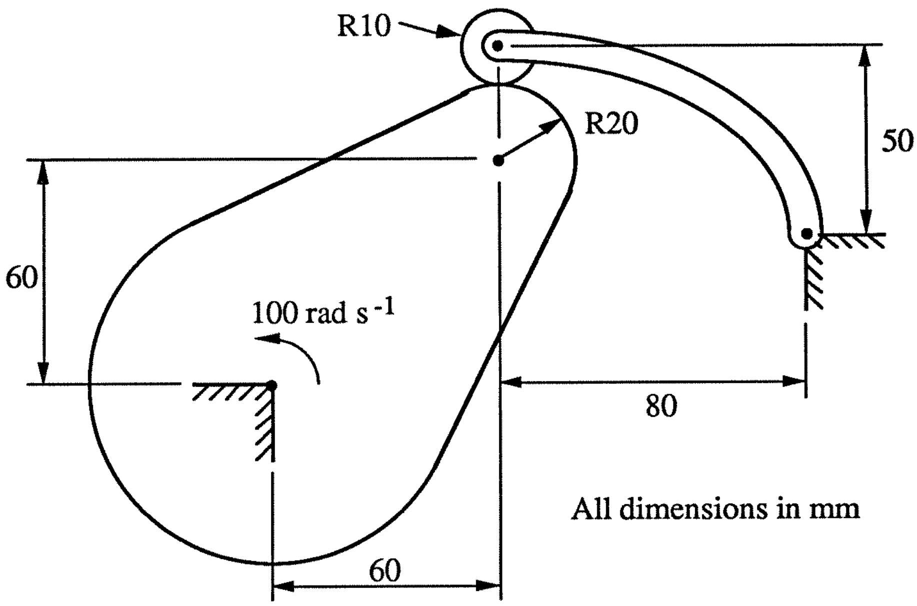

Fig. 4

8 A satellite of mass m orbits the earth in a circular orbit of radius r.

(a) Find an expression for the time taken to complete each orbit in terms of r, the universal gravitational constant G, and the earth's mass M. [4]

(b) The satellite has a head-on collision with a piece of space debris which reduces its velocity by 25% but does not instantaneously alter its direction of travel. In the subsequent orbit, what is the largest and smallest distance (in terms of r) that the satellite will travel from the centre of the earth? [14]

(c) Without further calculation, state whether you would expect the satellite's orbit time to decrease or increase as a result of the collision. Give brief reasons for your answer. [2]

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SECTION C

Answer not more than **two** *questions from this section.*

9 (a) A thermometer is in thermal equilibrium with a bath of water at 20 °C. It is then rapidly transferred into another bath of agitated water at 100 °C. The temperature measured by the thermometer varies with time as follows: [12]

Time / s	0	0.2	0.4	0.6	0.8	1.0	1.6
Temperature θ / °C	20	46	64	76	84	89	97

Show that these data are consistent with the thermometer reading θ responding to the bath temperature ϕ as a first-order linear system governed by the differential equation:

$$T\frac{\mathrm{d}\theta}{\mathrm{d}t} + \theta = \phi ,$$

and estimate the time constant T.

(b) The thermometer is used in agitated water to measure a temperature which fluctuates sinusoidally with amplitude 25 °C and period 10 s. Draw the phasor diagram for this system, and hence, or otherwise, find the error in amplitude and phase of the temperature recorded by the thermometer. [8]

10 (a) In the circuit shown in Fig. 5, current is flowing steadily through the inductance L from the DC voltage source V_E, with the switch S closed. The switch is then opened. Show that the differential equation governing the response of the circuit after the switch is opened may be written in the form [4]

$$A\frac{d^2V}{dt^2} + B\frac{dV}{dt} + V = V_E ,$$

where V is the voltage across the capacitor. Express A and B in terms of R, L and C.

(b) If $R = 400\ \Omega, L = 10$ H and $C = 250\ \mu$F, show that the circuit can be said to be *critically damped*. [4]

(c) (i) What is the value of V immediately after the switch is opened? [5]

(ii) Show that, immediately after the switch is opened, $\frac{dV}{dt} = \frac{V_E}{RC}$.

(iii) What is the voltage across the capacitor long after the switch is opened?

(d) Show, using sketches where appropriate, that the responses on pages 6 and 7 of the Mechanics Data Book are *not* consistent with these conditions. [4]

(e) Sketch the variation of V with time after the switch is opened, if $V_E = 40$ V. You are *not* required to solve the differential equation. [3]

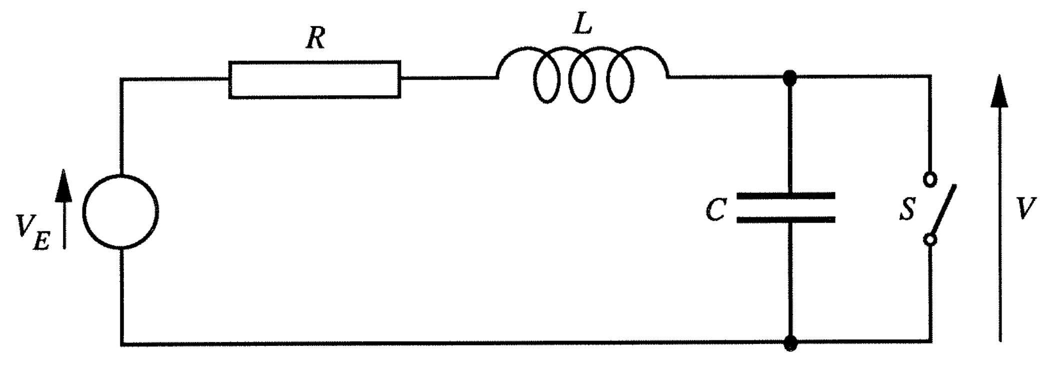

Fig. 5

11 Two equal blocks of mass m are linked together by springs of stiffness k, $2k$ and $4k$ respectively, as shown in Fig. 6. The blocks are constrained to vibrate in a straight line and their displacements are x and y as shown in the figure.

(a) Obtain the mass and stiffness matrices for the vibrating system. [8]

(b) Calculate the natural frequencies and show that the corresponding mode shapes are $\{2, 1\}$ and $\{1, -2\}$. [7]

(c) The system is harmonically excited by the application of a force $f = F \cos \omega t$ to the left-hand mass. By considering the equilibrium of the right-hand mass, or otherwise, determine the angular frequency ω at which both masses oscillate in phase and with the same amplitude. [5]

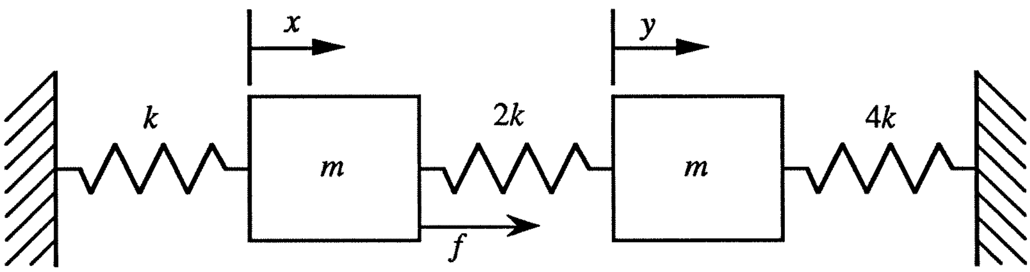

Fig. 6

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