ENGINEERING TRIPOS PART IA

Tuesday 10 June 1997 9 to 12

Paper 2

STRUCTURES AND MATERIALS

Answer not more than **eight** *questions, of which not more than* **four** *may be taken from Section A and not more than* **four** *from Section B.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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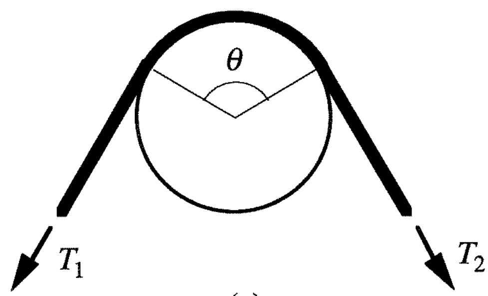
SECTION A

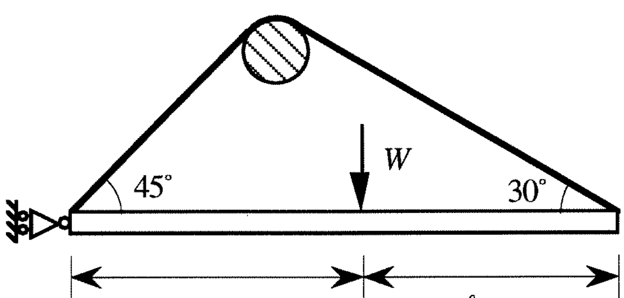
*Answer not more than **four** questions from this section.*

1 (a) Figure 1(a) shows a cable partially wrapped around a fixed peg. The cable in contact with the peg subtends an angle θ. The ends of the cable are subject to tensions T_1 and T_2, and the coefficient of friction between the cable and the peg is μ. From first principles, calculate the maximum and minimum values of T_2 for the cable not to slip, in terms of T_1, θ, and μ. [8]

(b) Figure 1(b) shows a light beam of length ℓ, carrying a weight W at a distance x from its left-hand end. It is supported by a cable which runs over the top of a fixed peg, as shown. A support prevents any horizontal movement of the beam, but allows vertical movement and rotation. Calculate the minimum and maximum values of x for the cable not to slip, if the coefficient of friction between the rope and the peg is 0.3. [12]



(a)



(b)

Fig. 1

2 The cable shown in Fig. 2 is one of the two main cables of a suspension bridge during construction. The partially constructed roadway imposes a load on this cable of 100 kN/m over the central half of the span, while elsewhere the cable is unloaded. The dip of the cable at the centre of the span is 100 m.

(a) Calculate the horizontal and vertical reactions at the supports. [4]

(b) Derive an analytical expression for the shape of the cable, $y(x)$ [8]

(i) for $0 \leq x \leq 250$ m;

(ii) for $250 \text{ m} \leq x \leq 500$ m.

(c) Show that the total length of the cable is approximately 1023 m. [5]

(d) When the bridge is completed, the load of 100 kNm^{-1} will be acting upon the whole span of the cable. Assuming that the length of the cable remains constant, calculate the expected dip of the cable at the centre of the span. [3]

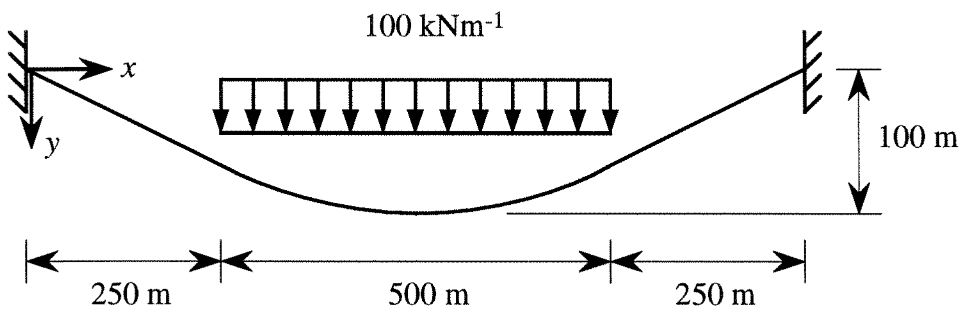


Fig. 2

3 Figure 3 shows a two-dimensional pin-jointed truss structure, in which all the bars have the same cross-sectional area A, and are made of a linear-elastic material with Young's modulus E. A vertical load W is applied at joint C.

(a) Calculate the bar forces due to the load at C, and hence calculate the change in length of each bar caused by this load. [8]

(b) Find the vertical deflections of joints C and G due to the load at C. [8]

(c) A vertical load W is now applied to G in addition to that already applied at C. Determine the total vertical deflection of C due to this loading. [4]

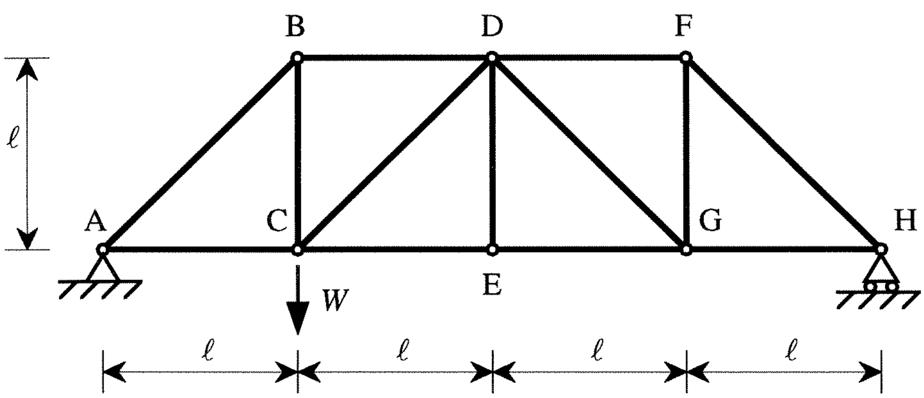


Fig. 3

4 A desk is supported by two identical frames, one of which is shown in Fig. 4. These frames have a uniform bending stiffness EI of $20\ \text{kNm}^2$. When someone sits on the desk, a vertical load of 400 N is applied to each of the frames at E, as shown in Fig. 4. The weight of the frame and the desk may be neglected.

(a) Calculate the reactions at A and C. [2]

(b) Show the bending moment distribution on a sketch of the frame by plotting the bending moments on the tension side of each member. Indicate salient values on your plot. [6]

(c) By initially assuming that the beam ABC is rigid, calculate the vertical deflection of the point E. [6]

(d) Without performing any additional calculations, explain how you could calculate the additional deflection of E due to the flexibility of the beam ABC. [6]

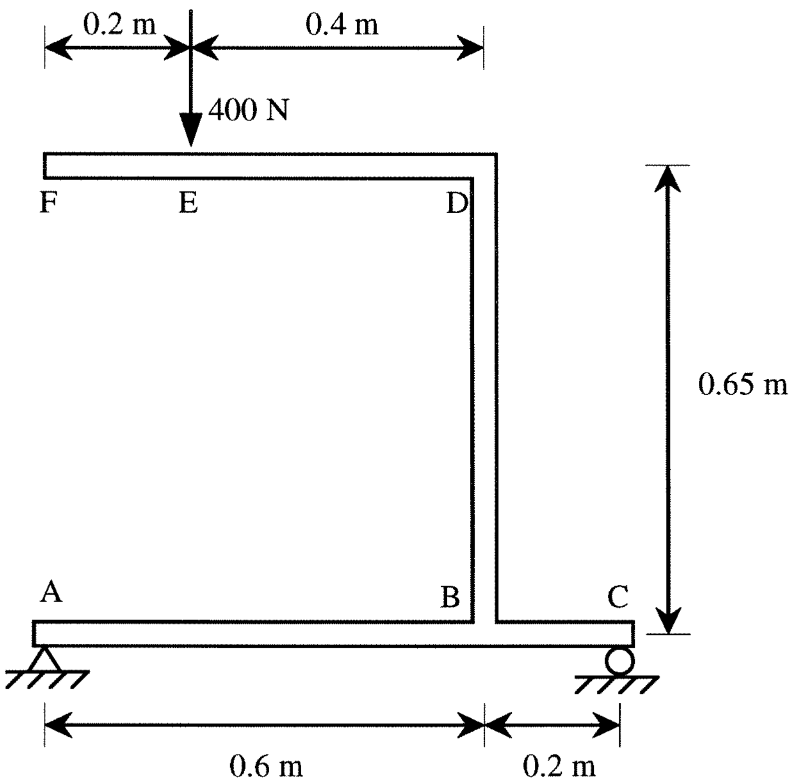


Fig. 4

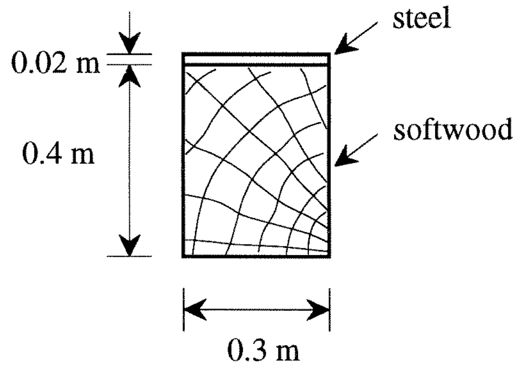
5 Figure 5(a) shows a cross-section of a composite softwood-and-steel beam which has been constructed by firmly gluing together the wood and steel. A 10 m length of this beam is simply supported with the steel on the upper face, and is subject to a uniformly distributed load of 20 kNm^{-1}, as shown in Fig. 5(b).

(a) Calculate the maximum bending moment and shear force carried by the beam. [4]

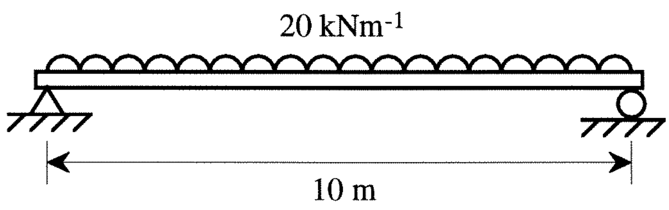
(b) Transform the cross-section into an equivalent cross-section made entirely of *either* wood *or* steel, and calculate the second moment of area of this transformed cross-section. [6]

(c) Find both the maximum tensile stress *and* the maximum compressive stress carried by the wood. [4]

(d) Find the maximum shear stress carried by the glue at the wood-to-steel interface. [6]



(a)



(b)

Fig. 5

SECTION B

Answer not more than **four** *questions from this section.*

Material properties not specifically quoted should be taken from the Materials Data Book.

6 (a) Application of a tensile stress to a metallic test specimen causes it to elongate elastically. Define (i) the dilatation Δ and (ii) Poisson's ratio υ for the specimen. [4]

(b) Show that for small strains

$$\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where ε_x, ε_y and ε_z are the strains along the principal axes of the specimen. Hence, or otherwise, determine the value of υ for an isotropic material if its volume is conserved during elastic deformation. [8]

(c) A tensile stress is applied parallel to the axis of a pure copper cylinder which has an initial diameter of 10 mm. Determine the magnitude of the load required to produce a 1 μm change in diameter if the deformation is entirely elastic. Use the Materials Data Book to show that this load is insufficient to cause the rod to deform plastically. [8]

[Poisson's ratio for copper is 0.35].

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7 (a) Explain carefully the mechanism responsible for power law creep in metals. [5]

(b) The steady state creep rate $\dot{\varepsilon}_{ss}$ varies with applied stress σ according to the following equation

$$\dot{\varepsilon}_{ss} = B \sigma^n$$

where B and n are constants. State how B varies with temperature and give a typical range of values for n for a metal at about 0.3 of its melting temperature in the power law creep regime. [3]

(c) Figure 6 shows the steady state creep behaviour of 12% Cr steel at 500 °C. Determine B and n for this material. [5]

(d) A 12% Cr steel turbine blade of length 10 cm runs with an initial clearance of 100 μm between its tip and the housing in a generator operating at 515 °C. Determine the lifetime of the blade if it runs continuously, assuming a uniform tensile stress of 60 MPa is applied along its axis. [7]

[The creep activation energy for 12% Cr steel is 103 kJ mol^{-1}].

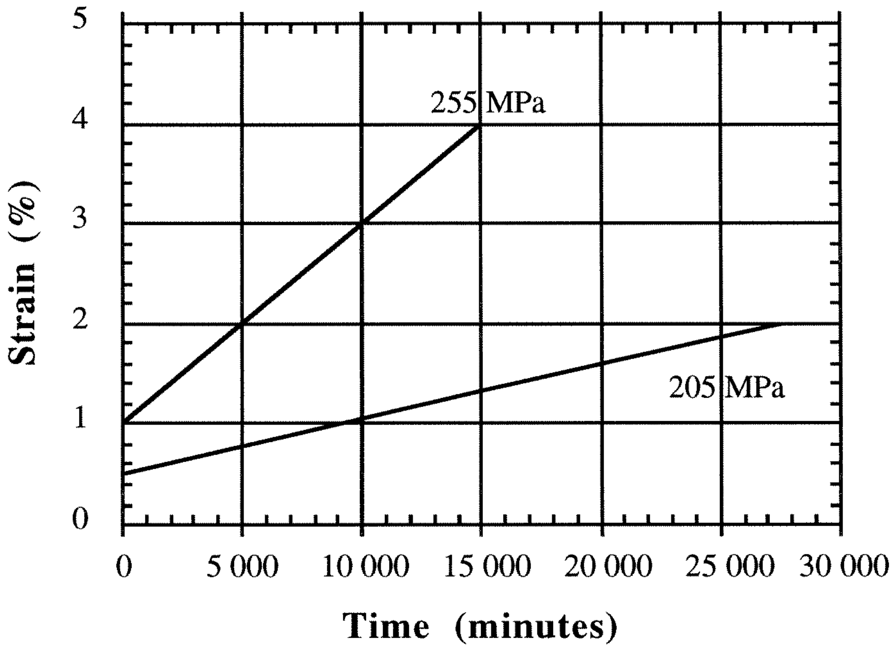

Fig. 6

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8 (a) Sketch curves of true stress σ_t against true strain ε_t and nominal stress σ_n against nominal strain ε_n for a ductile metal under compressive and tensile loads. Indicate any important features on your sketches. [6]

(b) Use the definitions of true and nominal stress and strain to derive the following relationships [7]

$$\varepsilon_t = \ln(1 + \varepsilon_n) \qquad \text{and} \qquad \sigma_t = \sigma_n(1 + \varepsilon_n)$$

Hence show that

$$\frac{d\sigma_t}{d\varepsilon_t} = \sigma_t \qquad \text{and} \qquad \frac{d\sigma_n}{d\varepsilon_n} = 0$$

are equivalent conditions for the onset of necking in ductile metals.

(c) The relationship between true stress and true strain for a ductile metal is given by

$$\sigma_t = \sigma_0\, \varepsilon_t^m$$

where σ_0 and m are positive constants. Verify that necking begins when $\varepsilon_t = m$. [3]

(d) A particular metal for which $\sigma_0 = 880$ MPa starts to neck in a tensile test at a true strain of 0.18. Use this information and Fig. 2.6 in the Materials Data Book to suggest the identity of the metal. [4]

9 Distinguish carefully between the toughness G_{IC} and the fracture toughness K_{IC} of an engineering material. Indicate briefly how K_{IC} can be measured experimentally. [4]

Figure 7 shows a centre crack of length $2a$ in an infinite plate of thickness t (measured perpendicular to the page) under an applied stress σ. By using a simple model of the stresses around the crack, show that the strain energy release rate G is given approximately by [8]

$$G = \frac{Y^2 \sigma^2 \pi a}{E}$$

where E is Young's modulus and Y is a dimensionless constant $= \frac{1}{\sqrt{2}}$ for this geometry.

It has been determined in a tensile test that a material with Young's modulus of 70 GPa and Poisson's ratio of 0.33 fractures under an applied stress of 130 MPa from an *edge* crack of length 1 cm. Use the equations for fast fracture on page 37 of the Data Book to suggest the identity of the material under test. [8]

[Y = 1.12 for edge crack geometry.]

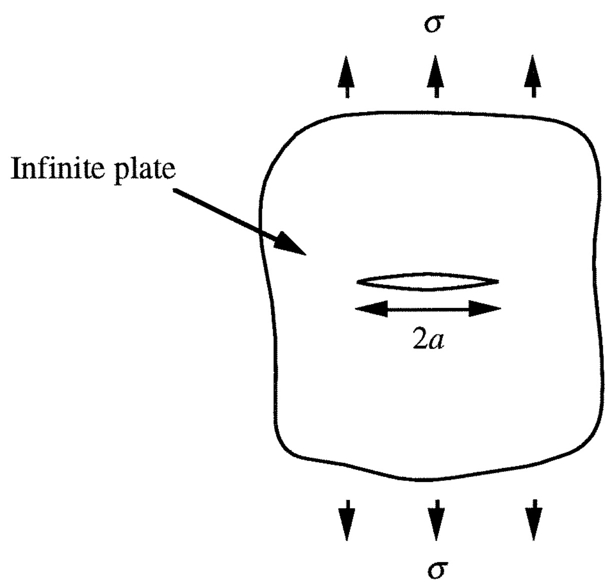

Fig. 7

(TURN OVER

10 Describe briefly the factors which determine the rate at which a metal corrodes when exposed to water-based environments containing oxygen. [4]

Account in detail for the following:

(i) Zinc-plated steel does not corrode in an aqueous environment, even when the surface is scratched to expose the underlying steel; [4]

(ii) Car exhausts made of mild steel corrode much more quickly than those made of stainless steel; [4]

(iii) Mild steel radiators in domestic central heating systems undergo little corrosion even after several years of service; [4]

(iv) Titanium has a much lower electrode potential than lead but appears well above it in the galvanic series. [4]

END OF PAPER