IA 1998 answers

Engineering Tripos Part IA 1998

Paper 1: Mechanical Engineering

ANSWERS

Section A

1. (c) 2 or more; $\dfrac{\dot{V}}{ND^3}$ (d) 78%; 275 MW

2. (b) $c_v T_1(\tau - 1)$ (d) $\sqrt[\gamma]{\tau}$

3. (c) $W \le AT_H \ln\left(\dfrac{T_2}{T_1}\right) + BT_H(T_2 - T_1) - A(T_2 - T_1) - \dfrac{1}{2}B(T_2^2 - T_1^2)$

   (d) $1 - \dfrac{T_C}{T_H}$

4. (c) 0.2604; 0.0263 (d) 119.0 kJ/kg; 3.563

Section B

5. (b) $\dot{\underline{r}} = -\dfrac{1}{2}v\underline{e}_r + \dfrac{\sqrt{3}}{2}v\underline{e}_\theta$; $\ddot{\underline{r}} = -\dfrac{\sqrt{3}}{4}\dfrac{v^2}{r}\underline{e}_\theta - \dfrac{3}{4}\dfrac{v^2}{r}\underline{e}_r$

   (c) $\ddot{s} = 0$; $R = \dfrac{2}{\sqrt{3}}r$

6. (a) 0.026; 0.13 rad/s anticlockwise

   (b) 0.045 rad/s anticlockwise; 3.05 mm/s right; 10.6 mm/s down

   (c) (i) $P = 70.8$ N (ii) $P = 72.9$ N (answers approximate)

7. (a) $\dfrac{H}{30}$ (c) $\dfrac{91}{60}mgH$ (d) $0.379H$

8. (c) $\dfrac{1}{2\pi}\sqrt{\dfrac{2k}{m}(r_0 - l_0) + 3\dot{\theta}_1^2}$

Section C

9. (b) 42.9 μH (c) (ii) 218; $218v_i$; 5.2×10^{-4}%

10. (c) $0.39A$; $0.32A$; $0.22A$

11. (c) $0.85\sqrt{k/m}$; $1.17\sqrt{k/m}$

Part 1A Engineering Tripos 1998

Paper 2 Section A, Structures

Answers

1 (a) (i) $v = 0,\ h = \dfrac{wL^2}{2H}$

(b) (i) Max. magnitude of bending moment = $\dfrac{wL^2}{16}$

(ii) Max. magnitude of bending moment = $\dfrac{wL^2}{16}$

2 (a)

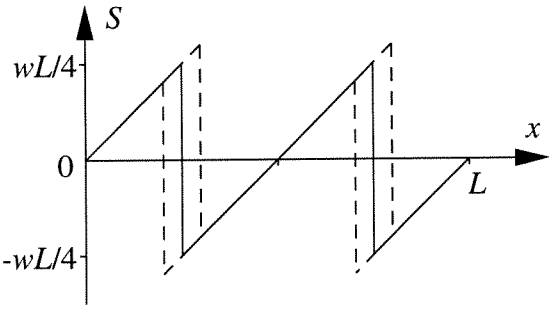
Bar	Tension (all $\times W$)	Extension (all $\times \dfrac{WL}{AE}$)
AB	-0.5	-0.5
BC	-0.5	-0.5
CD	-0.5	-0.5
AE	0	0
BE	$1/\sqrt{2}$	2
CE	$1/\sqrt{2}$	2
DE	0	0

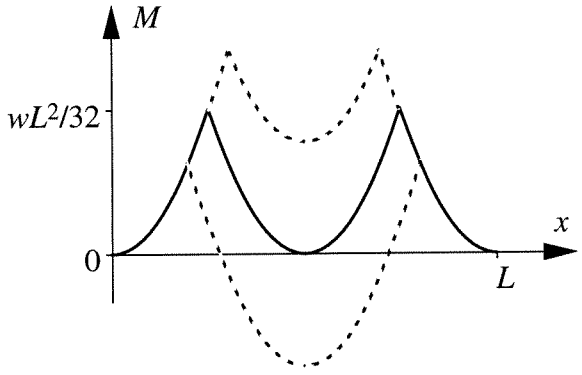
(b) $\delta = 7.16\dfrac{Wl}{AE}$

(c) $\delta_{\text{additional}} = 0.01L$

3 (a) $R_A = R_B = \dfrac{wL}{2}$

(b), (c)





(d) (i) $M = \dfrac{wx^2}{2}$

(ii) $M = \dfrac{w}{2}\left(x^2 - Lx + \alpha L^2\right)$

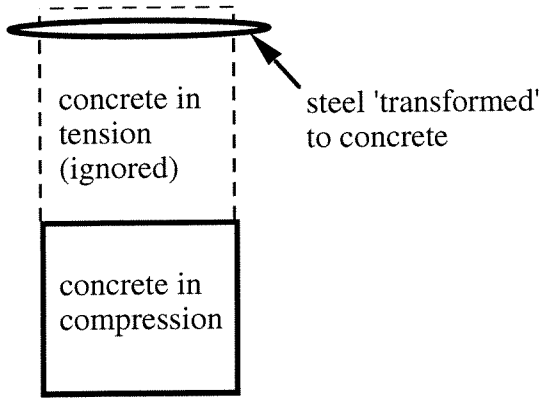
4 (a) $M = 125$ Nm

(b) $\phi_E = 0.0104$ rad

(c) $u'_C = 0.0078$ m, $v'_C = 0.0091$ m, $\phi'_C = 0.0391$ rad

(d) $v_A = 0.0241$ m, $h_A = 0.0378$ m

5 (a)



(b) 159.6 mm

(c) $EI = 12.37 \times 10^6$ Nm^2

(d) Steel will yield first at a curvature of 7.92×10^{-3} m^{-1}

Engineering Tripos Part IA 1998

Paper 2 Section B, Materials

Answers

6 (b) (ii) $M_1 = \dfrac{\sigma_f^{2/3}}{\rho}$; CFRP or wood.

(iii) $\sigma_f \geq 122$ MPa; CFRP or Al alloy.

7 (a) $\sigma_t = \sigma_n(1+\varepsilon_n)$; $\varepsilon_t = \ln(1+\varepsilon_n)$

$\varepsilon_n \leq 0.01$

(b) (i) 0.5% proof stress = 280MPa; Tensile strength = 362 MPa

(iii) True strain = 0.18; $H \approx 1320$ MPa (by extrapolation)

9 (b) (i) 9 mm

(ii) $n \approx 5$; $Q \approx 195$ kJ / mol

(c) (i) 6.51 kN

(ii) new strain-rate = 0.4 x old strain-rate, so failure in 25, 000 hrs (insufficient)

10 (b) (i) $\Delta\sigma_o$ = 840 MPa

(ii) and (iii)

Test	$\Delta\sigma$	$\Delta\sigma_o$	N_f
1	630	840	10^4
2	460	614	10^6
3	510	680	2.4×10^5
4	560	746	5.9×10^4

(iv) α = 0.065

Engineering Tripos Part IA 1998
Paper 3: Electrical and Information Engineering
Numerical Answers

Section A

3. Power supply voltage must be greater than 4 V

4. Amplifier gain = 0.93
   Output resistance = 232 Ω
   Noise source voltage = 0.86 mV

Section B

5. (a) $\overline{\overline{D}\overline{C} \,.\, D\overline{B} \,.\, D\overline{A} \,.\, \overline{D}CB}$ (b) $\overline{\overline{D+C} + \overline{B+D} + \overline{A+B+C+D}}$

6. $J_B = A\overline{U} + \overline{A}UC$ $K_B = \overline{A}\overline{U} + AU = \overline{A \oplus U}$
   Clocked synchronous system - should not be susceptible to static hazards.

7. (b) +127, -128
   $+47_{10}$ = 0010 1111
   -89_{10} = 1010 0111
   $+68_{10}$ = 0100 0100
   Sum = 0001 1010 $= 26_{10}$. 1 is carried out of the MSB.

8. (b) -0.64 to +0.635 V Samples/second = $8 \times 10^6 / n$,
where n is the number of clock cycles required to execute the sequence of instructions that acquire, process and output a single sample. One algorithm gave n=31, implying 258,000 samples per second. Several other possible algorithms exist.

Section C

9. (a) 357 V (b) 94.7 nF m^{-2} (c) 506 N m^{-2}

10. (a) $B = \dfrac{\mu_0 NI}{\dfrac{\ell}{\mu_r} + g + x}$ T (b) 2.04 mH (c) $F = \dfrac{B^2 A}{2\mu_0} = 1.19$ N

    (d) F is about 36 times greater (not 100 times, owing to saturation).

11. (a) $\dfrac{\pi \varepsilon_0}{\ln\left[\dfrac{d-r}{r}\right]}$ Fm^{-1} (b) $\dfrac{\mu_0 I}{2\pi r}$ T (c) $\dfrac{\mu_0}{\pi}\ln\left[\dfrac{d-r}{r}\right]$ Hm^{-1}

    (d) $\dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ ms^{-1} (= c)

Numerical answers to 1998 Part Ia, Paper 4 Mathematical Methods

Section A

Question 1.

a) Perpendicular distance $3/\sqrt{5}$, independent of α. When $\alpha = 26$ lines are parallel.
b) Point of intersection is

$$\left(\frac{2}{\beta+1}, \frac{3-\beta}{\beta+1}, \frac{6-4\beta}{\beta+1}\right) .$$

When $\beta = -1$ the line and plane are parallel, and no intersection.

Question 2.

a) $3/2$
b) $2n\pi + \mathrm{i}1.32$, for $n = 0, 1, 2, \ldots$
c) $\frac{x^2}{2} - x^3 + \frac{29x^4}{24} + \ldots$

Question 3.

a) $y = x + \cot x + \left(1 - \frac{\pi}{2}\right)\mathrm{cosec}x$
b) $y = A\exp(6t) + B\exp(-4t) + \frac{t}{10}\exp(6t)$

Question 4.

a) $y_n = 2^{-n}$
b)

$$R = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$$

$$A' = \begin{pmatrix} -3/2 & -5/\sqrt{2} & 3/2 \\ -3/\sqrt{2} & 0 & 9/\sqrt{2} \\ 3/2 & 7/\sqrt{2} & 5/2 \end{pmatrix}$$

Question 5.

a) $\lambda_1^n, \lambda_2^n, \lambda_3^n$, $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$.
b) $\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1}$, $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$.
Eigenvalues $10, 5, 1$, eigenvectors $(1,0,0)^{\mathrm{T}}$, $(0,1,-1)^{\mathrm{T}}$, $(0,1,1)^{\mathrm{T}}$.
$A^{10}\mathbf{x} \approx 10^{10}(1,0,0)^{\mathrm{T}}$
$A^{-1^{10}} \approx \frac{3}{2}(0,1,1)^{\mathrm{T}}$

Engineering Tripos Part IA 998

Paper 4: Mathematical Methods

ANSWERS

Section B

6. (a) 1 (b) $t^3/3$ (c) $1-e^{-t}$

7. (a) $y(t) = 3 + \sum_{n=1}^{\infty}\left[\frac{6}{n^2+1}\cos nt - \frac{2n}{n^2+1}\sin nt\right]$ (b) $y(t) = \sum_{n=-\infty}^{\infty} \frac{2(-1)^n \sinh \pi}{\pi(1+n^2)} e^{\text{int}}$

8. (a) 0.0952 (b) 0.092 (c) 0.0025 (but see crib)

9. (a) $y = \frac{1}{2} + \frac{1}{2}e^{-2t} - e^{-t}$ (b) $y(t) = \frac{5}{2}te^{-t} + \frac{3}{2}e^{-t} - \frac{1}{2}\cos t$

10. (b) $\begin{pmatrix} -2x \\ -2y \\ 1 \end{pmatrix}$

So,

If you want this in electronic form email me at ar417@eng, Adrian