ENGINEERING TRIPOS PART IA

Monday 8 June 1998 9 to 12

Paper 1

MECHANICAL ENGINEERING

Answer not more than **eight** *questions, of which not more than* **three** *may be taken from Section A, not more than* **three** *from Section B and not more than* **two** *from Section C.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer not more than **three** *questions from this section.*

1 Hydroelectric power stations utilise the pressure difference created by a change of height of water to drive a turbine to generate electrical power.

(a) Use the equation $Tds = dh - vdp$ to show that, when an *incompressible* fluid undergoes an *isentropic* process, the change in its specific enthalpy is given by

$$\Delta h = v\Delta p$$ [3]

(b) Show that the power output from a reversible, adiabatic turbine receiving a steady volumetric flow rate of water of $\dot{V}$ in a hydroelectric power station is

$$\dot{W}_{x\,\mathrm{isen}} = -\dot{V}\Delta p$$

where Δp is the pressure change across the turbine. (You may neglect all kinetic and potential energy terms.) [5]

(c) A manufacturer has built a series of geometrically similar water turbines of diameter D, rotational speed (in radians per second) N and measured isentropic efficiencies η.

The manufacturer wants to establish a relationship which will enable the isentropic efficiency of turbines to be estimated under a variety of operating conditions. Assuming that η, N, D and $\dot{V}$ are the only relevant variables, how many independent dimensionless groups can be formed using them?

Taking the isentropic efficiency η to be one of the dimensionless groups, obtain a second dimensionless group ϕ (the non-dimensional flow rate) in the form

$$\phi = \frac{\dot{V}}{\mathrm{function}(N, D)}$$ [5]

(cont.

(d) After conducting a series of experiments the manufacturer is able to plot the graph shown in Fig. 1 relating efficiency η and non-dimensional flow rate ϕ, calculated using the basic SI units for N, D and $\dot{V}$. Use this graph to find the efficiency of a water turbine with diameter 3.42 m rotating at 50 radians per second while receiving a volumetric flow rate of water of 70 m^3s^{-1} in a hydroelectric power station.

Given that the pressure difference Δp created by a height difference Δz of fluid of density ρ is given by $\Delta p = -\rho g \Delta z$, calculate the power output from this turbine if the change in height of water is 514 m. Take the density of water to be 1000 kgm^{-3}. [7]

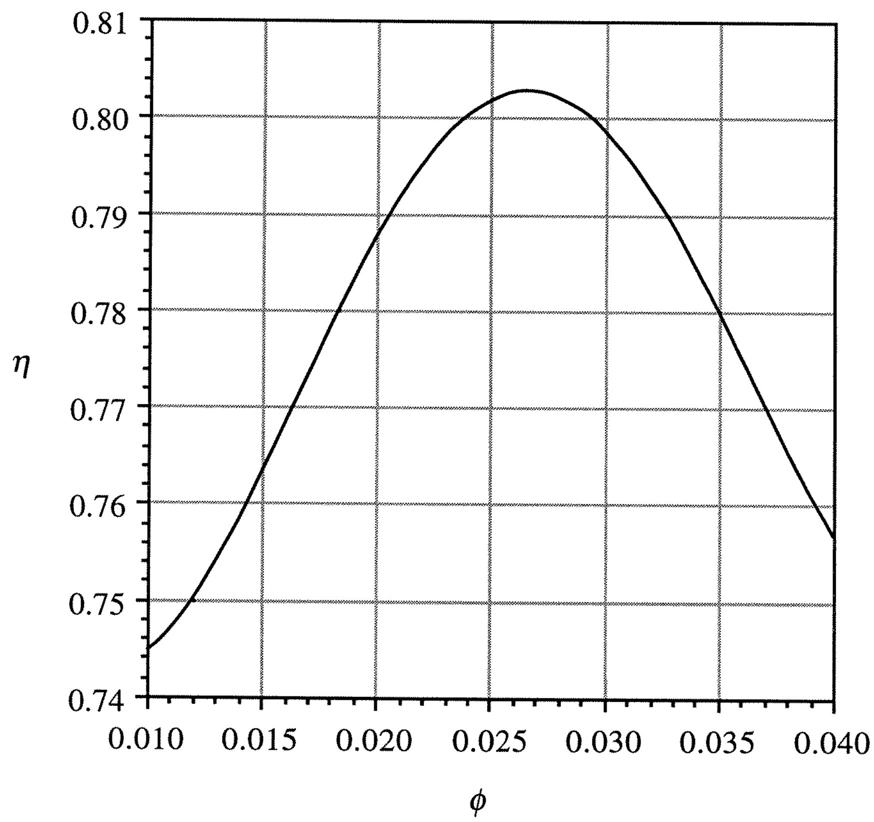

Fig. 1

2 The Lenoir Cycle consists of the following four reversible processes:

A: Constant volume heating from state T_1, p_1, V_1 to state T_2, p_2, V_1;
B: Isentropic expansion from state T_2, p_2, V_1 to state T_3, p_3, V_3;
C: Constant volume cooling from state T_3, p_3, V_3 to state T_4, p_1, V_3;
D: Constant pressure cooling from state T_4, p_1, V_3 to state T_1, p_1, V_1.

(a) Sketch the cycle on p-V and T-s diagrams. [5]

(b) If the working fluid is a perfect gas, find the heat input to the cycle per kg of gas in terms of T_1 and τ, where τ is the temperature ratio $\frac{T_2}{T_1}$. [3]

(c) Show that the net work output from the cycle per kg of gas is given by

$$W_{\text{net}} = c_v T_1 \tau \left(1 - r^{1-\gamma}\right) + R T_1 \left(1 - r\right)$$

where r is the volume ratio $\frac{V_3}{V_1}$. [7]

(d) With reference to your p-V diagram, explain why, if T_1 and τ are fixed, while the volume ratio r is increased, the net work output reduces beyond a certain value of r.

Hence, or otherwise, find the value of r which maximises the net work output from the cycle for fixed values of T_1 and τ. [5]

3 An inventor has produced a device in which a steady flow of a certain gas (<u>not</u> a perfect gas) enters at a temperature T_1 and leaves at a temperature $T_2 > T_1$. The pressure at entry to and exit from the device is equal to atmospheric pressure.

The specific heat capacity at constant pressure of the gas varies with temperature as

$$c_p = A + BT$$

where A and B are constants and temperature T is in Kelvin.

(a) Starting with $c_p = \left(\frac{\partial h}{\partial T}\right)_p$, show that the enthalpy change of the gas between the entry and exit of the device is given by

$$h_2 - h_1 = A(T_2 - T_1) + \tfrac{1}{2}B\left(T_2^{\,2} - T_1^{\,2}\right)$$

Explain carefully why this equation is valid even if the pressure is not constant throughout the device. [5]

(b) Show that the entropy change of the gas between the entry and exit of the device is given by

$$s_2 - s_1 = A\ln\left(\frac{T_2}{T_1}\right) + B\left(T_2 - T_1\right)$$ [4]

(c) The only heat flow to or from the device is from a single source at constant temperature T_H, and the device produces a steady work output W per kg of gas. Find an expression for the maximum possible value of W. You may assume that changes in the kinetic and potential energy of the gas entering and leaving the device are negligible. [6]

(d) An engineer observes that the work output of the device could be increased by constant pressure cooling of the exit gas, returning it to T_1 and using the heat extracted to run a series of heat engines. If all the heat rejected from these engines is at the environmental temperature T_C, what now is the maximum possible ratio of the work output from the device to the heat input to it? [5]

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4 (a) Explain, giving an example, what information is required in order to determine completely the thermodynamic state of a pure substance in a two-phase region. [2]

A vapour compression refrigerator, which operates in steady flow, has Refrigerant 12 as the working fluid. The fluid leaves the condenser as saturated liquid at 25 °C and is throttled adiabatically to the evaporator pressure of 1.508 bar. The mixture is then allowed to evaporate at constant pressure while receiving heat from the cold box of the refrigerator. It leaves the evaporator as saturated vapour and is compressed adiabatically to the condenser pressure, leaving the compressor with a temperature 20 K above the saturation temperature in the condenser.

(b) Sketch the cycle on *p-h* and *T-s* diagrams. [5]

(c) Using Table 13 of the Thermodynamics Data Book determine the dryness fraction of the mixture leaving the throttle valve and the ratio of the volumetric flow rates before and after the valve. [5]

(d) Show that the work input to the compressor per kg of fluid is 33.4 kJ and find the heat transferred in the evaporator. Hence calculate the coefficient of performance of the refrigerator. [4]

(e) What are the practical advantages and disadvantages of replacing the throttle by a turbine? [4]

SECTION B

Answer not more than **three** *questions from this section.*

5 Figure 2 shows a particle P at a distance r from the origin O. $\underline{e}_r$ and $\underline{e}_\theta$ are unit vectors along and normal to the radius vector OP. $\underline{i}$ and $\underline{j}$ are fixed unit vectors. The angle between the vectors $\underline{i}$ and $\underline{e}_r$ is θ. The particle moves with constant velocities $v/2$ towards the origin O and $\sqrt{3}\,v/2$ in the $\underline{e}_\theta$ direction.

(a) Write down expressions for $\dot{\underline{e}}_r$ and $\dot{\underline{e}}_\theta$. [4]

(b) Find vector expressions for the position, the velocity and the acceleration of P in polar coordinates using $\underline{e}_r$ and $\underline{e}_\theta$. [9]

(c) Find expressions for the radius of curvature of the path of P and the magnitude of the acceleration of P along the path, in terms of v and r. [7]

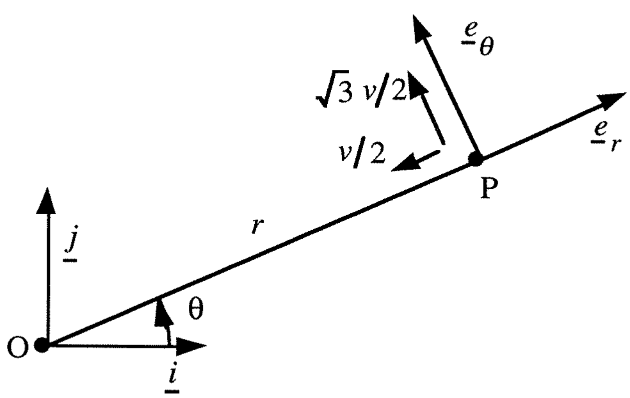

Fig. 2

6 Figure 3 illustrates a simple can-crushing mechanism. The can is placed on a stationary platform, and is crushed by a moving platform attached to point A, which is restrained to move vertically. Bar BCD is pinned to a stationary point at B and hinged to the link AC at C. Bar DEF is attached to BCD via a hinge and is free to slide through a swivel at E. Member lengths are as shown on Fig. 3.

(a) The can is being compressed such that A moves down at a velocity of 3 mms^{-1}. Draw a velocity diagram for the members AC and BCD when the mechanism is in the position shown and find the angular velocities of these members. A suitable scale for the velocity diagram is for 10 mm to represent a velocity of 1 mms^{-1}. Figure 3 is drawn to scale and sufficiently accurate answers may be obtained by tracing from this figure and measuring distances from the velocity diagram. [8]

(b) Complete your diagram to include bar DEF. Find the angular velocity of DEF and the components of the velocity of point F in the horizontal and vertical directions. [6]

(c) Find the vertical force P which has to be applied at F to compress the can against a resistive force R of 250 N:

(i) assuming that all the joints are frictionless;

(ii) including frictional torques at joints C and D of 120 Nmm, with the other joints frictionless. [6]

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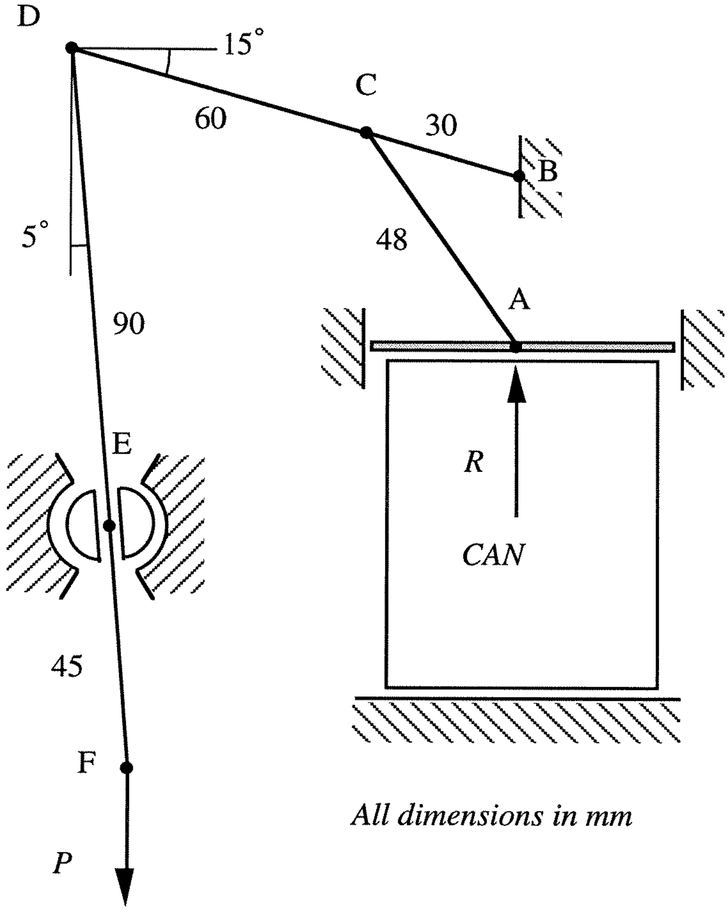

Fig. 3

7 A ball of mass m is dropped from a height $4H$ above the ground onto a platform of mass m which is supported by a spring, as illustrated in Fig. 4. On impact the ball sticks to the platform so that there is no subsequent relative motion between the two bodies. The spring has an **unstretched** length H. The stiffness of the spring equals $30mg/H$, so that the restoring force F exerted by the spring is related to its change in length x by $F = 30mgx/H$, where g is the gravitational acceleration. Air resistance and the weight of the spring may be neglected.

(a) The platform is at rest before the ball is dropped. By how much is the spring compressed at this point? [1]

(b) Show that the velocity of the ball immediately before the impact is equal to $\sqrt{91gH/15}$. [3]

(c) Find an expression for the energy dissipated in the impact. [5]

(d) Find the maximum compression of the spring in the subsequent motion. [8]

(e) The movement of the platform is now resisted by a constant frictional force. What can you say, using energy arguments but without further detailed calculations, about the qualitative change in the maximum compression in the spring, as compared with your answer for part (d) above? [3]

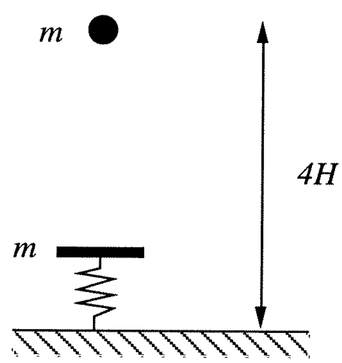

Fig. 4

8 In an apparatus for practising tennis shots, a ball of mass m is attached to a fixed point O by a light spring. The spring has an unstretched length ℓ_0, and exerts a restoring force F which varies parabolically with the spring extension x, i.e. $F = kx^2$, where k is a constant. It is assumed that the ball and spring remain in a horizontal plane and that gravitational forces can be neglected. The ball is initially in a steady circular trajectory around O at a radius r_0. An impulsive force is now applied to the ball along the direction of its motion, so that its tangential speed is increased by an amount Δv.

(a) Show that the angular velocity $\dot{\theta}_0$ of the ball immediately before the impulse is given by

$$\dot{\theta}_0 = \left(r_0 - \ell_0\right)\sqrt{\frac{k}{mr_0}}$$

and write down an expression for the angular velocity $\dot{\theta}_1$ immediately after the impulse. [4]

(b) Show that the radius r of the ball from O in the subsequent motion is governed by the differential equation

$$\ddot{r} + \frac{k}{m}\left(r - \ell_0\right)^2 - \dot{\theta}_1^2 \frac{r_0^4}{r^3} = 0$$ [7]

(c) By writing the radius r as $r = y + r_0$ show that, for small perturbations in the orbit, the above differential equation can be simplified to the form

$$\ddot{y} + ay = b$$

and hence show that the length of the spring varies sinusoidally with time, finding an expression for the frequency of oscillation. [9]

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SECTION C

Answer not more than **two** *questions from this section.*

9 Part of a radio circuit is illustrated in Fig. 5. A resistance R, a capacitance C and an inductance L are connected in series, driven by a harmonic input voltage $v_i = V_i \cos \omega t$. The output voltage v_o is measured across the inductor. No current is drawn at the output.

(a) Derive the governing differential equation relating v_i and v_o, and show that it corresponds to Case (b) of the Mechanics Data Book. [7]

(b) The capacitance C is fixed at 0.4 nF. To choose an appropriate inductor, it is assumed that the resistance R is zero. Find the value of the inductance L such that the undamped natural frequency of the circuit is 1215 kHz. [2]

(c) In practice the resistance R in the circuit has a value of 1.5 Ω.

(i) What qualitative effect will this resistance have on the response of the circuit? [3]

(ii) Find the Q factor for the circuit, the maximum amplitude of the response and the percentage change in frequency from the undamped natural frequency of 1215 kHz at which this maximum response occurs. [8]

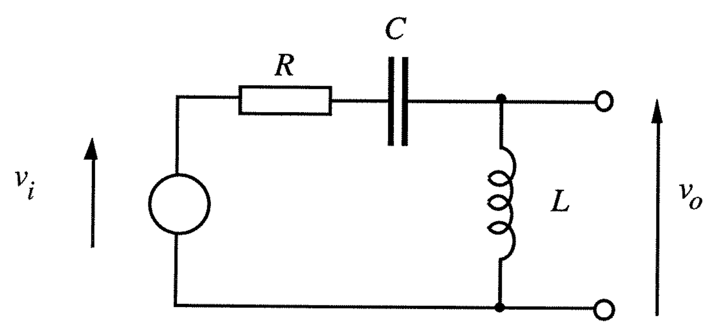

Fig. 5

10 Figure 6 shows an impact buffer consisting of two viscous dashpots and a spring. The dashpots both have rates λ and the spring has a stiffness k. Displacements x and y of the right hand end and the centre of the system are as shown in the figure. When the spring is in its unstretched state $y = 0$.

(a) Show that the displacements x and y are related by the differential equation

$$T\dot{y} + y = b\dot{x}$$

and express T and b in terms of λ and k. [5]

The system is initially at rest with $x = y = 0$. After time $t = 0$ the displacement x is increased according to the following variation with time t;

$$x = At/t_0 \qquad \text{for } 0 \le t \le t_0$$
$$x = A \qquad \text{for } t > t_0$$

where A and t_0 are constants.

(b) Derive an expression for the response y for $0 \le t \le t_0$. [5]

(c) For values of t_0/T of 0.5, 1 and 2, calculate the displacement y at time $t = t_0$ in terms of A, and sketch y as a function of t/T for $t \le t_0$. [6]

(d) For a value of t_0/T equal to 1, derive the response for $t > t_0$ and include this on your sketch in answer to part (c). [4]

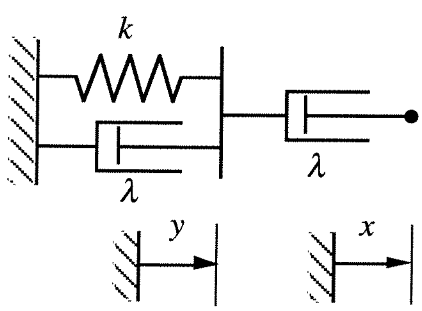

Fig. 6

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11 Figure 7 illustrates a tuned vibration absorber consisting of two masses A and B, constrained to move in a straight line. The masses of A and B are m and $m/10$ respectively. Mass A is connected to a fixed point by a spring of stiffness k and the two masses A and B are connected together by a spring of stiffness αk. A harmonic force $f = F\cos\omega t$ acts on mass A, and the displacements of the masses A and B from their equilibrium positions are y_1 and y_2 respectively.

For small displacements y_1 and y_2 of the masses, the equations of motion can be expressed as

$$[m]\underline{\ddot{y}} + [k]\underline{y} = \underline{f}$$

where $\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} Y_1 \cos\omega t \\ Y_2 \cos\omega t \end{bmatrix}$ and $\underline{f} = \begin{bmatrix} f \\ 0 \end{bmatrix} = \begin{bmatrix} F\cos\omega t \\ 0 \end{bmatrix}$.

(a) Find the mass and stiffness matrices $[m]$ and $[k]$. [6]

(b) Derive an expression for Y_1, the amplitude of vibration of mass A. [7]

(c) For a value of $\alpha = 0.1$, determine the values of the two natural frequencies of the system, and sketch $|Y_1|$ as a function of frequency ω. [7]

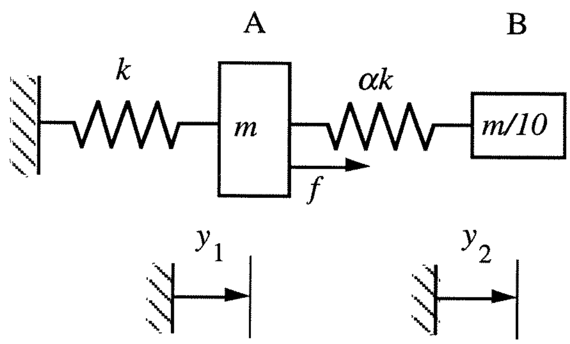

Fig. 7

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