ENGINEERING TRIPOS PART IA

Monday 8 June 1998 1.30 to 4.30

Paper 2

STRUCTURES AND MATERIALS

Answer not more than **eight** *questions, of which not more than* **four** *may be taken from Section A and not more than* **four** *from Section B.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer not more than **four** *questions from this section.*

1 A railway bridge of span length $2L$ is constructed as a symmetrical three-pin arch AOB, and is shown in Fig. 1. The shape of the arch is defined by the equation

$$y = H(x/L)^2$$

relative to the axes shown. Due to the weight of a train passing, a uniform vertical load of w per unit horizontal length is transmitted to a length αL of the arch, as shown in Fig. 1.

(a) For a uniformly distributed load over the full span of the bridge $(\alpha = 2)$:

(i) calculate the horizontal and vertical force transmitted by the pin at O; [6]

(ii) show that the bending moment in the arch is everywhere zero. [4]

(b) When the train is only loading half the span of the bridge $(\alpha = 1)$:

(i) calculate the maximum magnitude of the bending moment in the *unloaded* span AO; [6]

(ii) calculate the maximum magnitude of the bending moment in the *loaded* span OB. [4]

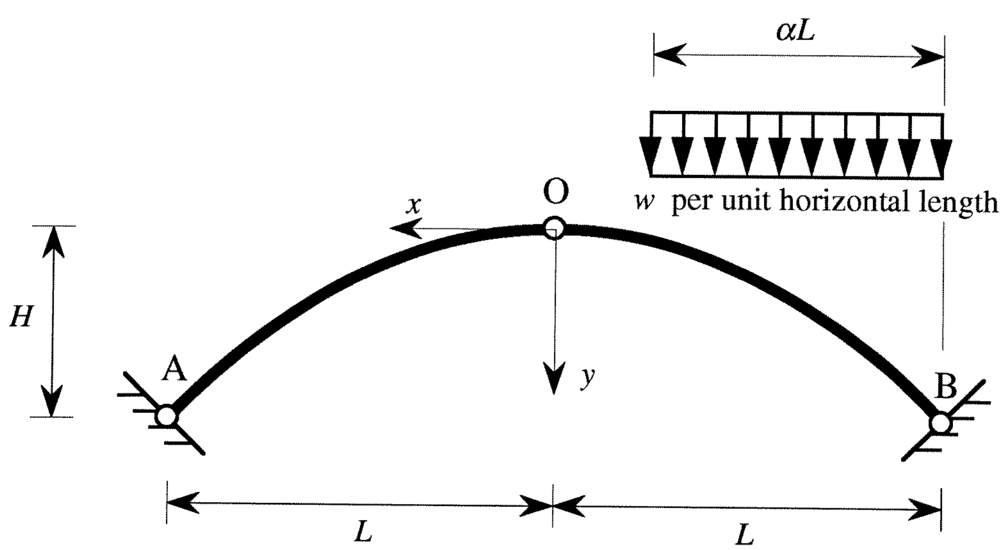

Fig. 1

2 Figure 2 shows a two-dimensional pin-jointed structure in which all the bars are made of material with a Young's modulus E. The outer bars AB, BC and CD have a cross-sectional area A, and length L, while the cross members AE, BE, CE, and DE have a cross-sectional area $A/4$, and length $L/\sqrt{2}$. A vertical load W is applied at joint E.

(a) Calculate the bar forces in the structure, and hence determine the change in length of each bar caused by the load W. [8]

(b) By virtual work or otherwise, determine the horizontal displacement of joint D. [6]

(c) Due to local heating, bar AE extends by 1% of its original length. Calculate the *additional* horizontal displacement of joint D due to this extension. [6]

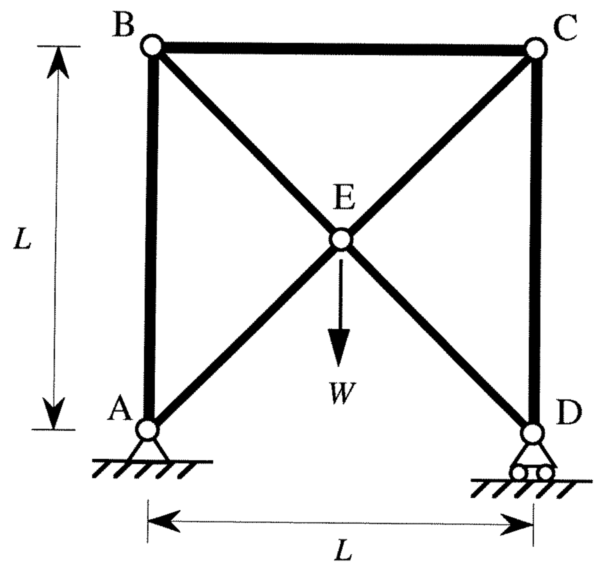

Fig. 2

3 Figure 3 shows a beam ABCD of length L, with supports at B and C, positioned at a distance αL from the ends of the beam. The beam is subjected to a uniformly distributed load of w per unit length.

(a) Calculate the reactions at B and C due to this load. [2]

(b) Sketch a shear force and a bending moment diagram for the case $\alpha = 0.25$, indicating salient values. [4]

(c) Show by adding dashed lines to your plots from (b) the effect of decreasing α to 0.2, and the effect of increasing α to 0.3. (Salient values are not required). [4]

(d) Derive expressions for the bending moment in the beam as a function of x and α:

(i) for AB, $0 \le x \le \alpha L,$ $0 < \alpha < 0.5;$

(ii) for BC, $\alpha L \le x \le (1-\alpha)L,$ $0 \le \alpha < 0.5.$ [6]

(e) Show from your answers to (d) that the value of α that minimises the maximum *magnitude* of the bending moment carried by the beam is $\alpha = \left(\sqrt{2}-1\right)/2$. [4]

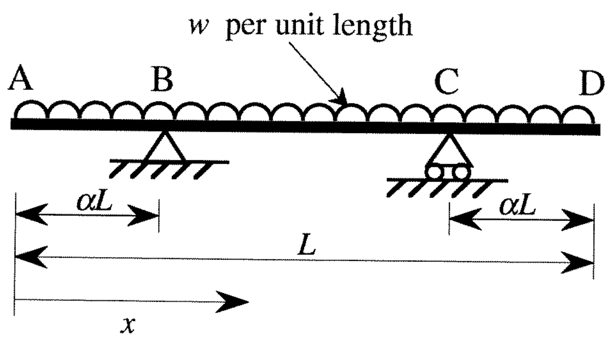

Fig. 3

4 Figure 4 shows a simple 2D model of the frame of a chair. The frame has rigid joints, and a uniform bending stiffness EI of 2000 Nm^2. When someone is sitting on the chair, a vertical load of 500 N is applied to the frame at C, half-way along the seat.

(a) Calculate the bending moment in the frame at E due to the applied load. [2]

(b) By considering the beam EF, and using Data Book coefficients, or otherwise, find the rotation at E. [4]

(c) Initially neglecting the rotation at E calculated in (b), i.e. regarding beam EF as temporarily rigid, find the horizontal and vertical deflection of C, and also the rotation at C. [8]

(d) Using your answers to (b) and (c), find the horizontal and vertical deflection of point A. [6]

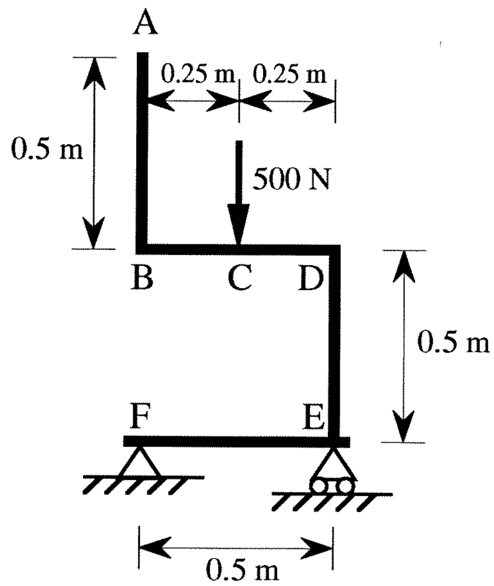

Fig. 4

5 Figure 5 shows the cross-section of a reinforced concrete beam. The concrete has a Young's modulus of 14×10^3 N/mm^2 in compression, but can be considered to have zero Young's modulus in tension due to the formation of cracks. It is reinforced by a steel bar of radius 15 mm, which is centred 50 mm below the top surface of the beam. The steel has a Young's modulus of 210×10^3 N/mm^2. The beam is initially straight, and is subjected to pure bending moments that cause tension at the top of the beam.

(a) Sketch a cross-section for the beam, 'transformed' to a single material. [4]

(b) Calculate the distance of the neutral axis from the bottom of the beam. [4]

(c) Find the bending stiffness EI of the transformed composite section. [6]

(d) The steel has a yield stress of 400 N/mm^2, and the concrete fails in compression at a stress of 40 N/mm^2. Calculate which will occur first: failure of the concrete in compression, or yielding of the steel. What is the curvature of the beam when this occurs? [6]

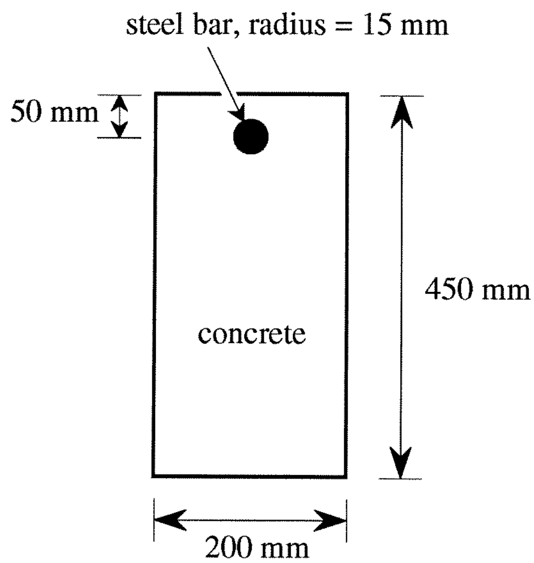

Fig. 5

SECTION B

Answer not more than **four** *questions from this section.*

6 (a) Define Young's modulus E for an engineering material. Briefly describe three methods for measuring Young's modulus, and comment on their accuracy. [6]

(b) The Engineering Department requires a new set of rising bollards in the entrance roadway for security purposes. The bollards have a specified height l, and may be assumed to be built-in at the ground. A solid cylindrical cross-section has been chosen, but the radius R may be varied. In service the bollards are vertical and must support a given maximum load W, applied horizontally at mid-height, without failure. To minimise the power requirements for lifting the bollards, they must be as light as possible.

(i) Show that the maximum stress in the bollard is given by

$$\sigma = \frac{2Wl}{\pi R^3}$$

(ii) Derive a *performance index*, including only material properties, which should be maximised for the given design specification. Hence select the best two materials for the bollards from those given in Table 1.

(iii) A bollard of height 0.6 m is required to resist a load of 40 kN at mid-height without failure. An upper limit of 50 mm is imposed on the radius of the bollard. Show that this sets a lower limit on the failure stress of the material, and if necessary revise your material selection from Table 1. [12]

(c) Briefly outline two factors other than strength which might eliminate either of your chosen materials. [2]

Material	Density (Mg/m^3)	Failure stress (MPa)
CFRP	1.6	300
Wood	0.6	100
Mild steel	7.6	350
Al alloy	2.7	350
Concrete	2.1	80

Table 1

7 (a) Define what is meant by nominal stress and strain, and true stress and strain. Using the formulae in the Materials Databook, derive expressions for true stress and true strain in terms of nominal values, stating any assumptions made. Find the maximum nominal strain for which the true and nominal values of stress differ by less than 1%. [6]

(b) A tensile nominal stress - strain curve for an aluminium alloy tested at room temperature is shown in Fig. 6.

A separate copy of Fig. 6 is provided, and should be handed in with your answer.

(i) Evaluate the 0.5% proof stress and the tensile strength for this alloy, and indicate these on Fig. 6.

(ii) Convert the curve to a true stress-strain curve as far as the tensile strength of the alloy, and plot this on Fig. 6.

(iii) A strip of the alloy of thickness 3 mm is cold rolled to a final thickness of 2.5 mm. The width of the strip was found to be unchanged. Explain whether the hardness of the rolled strip can be estimated from the data given. [12]

(c) A specimen of the aluminium alloy was tensile tested in liquid nitrogen, and gave a very similar stress-strain curve to that obtained at room temperature. In what ways might the tensile behaviour differ for specimens of medium carbon steel tested at room temperature and in liquid nitrogen? [2]

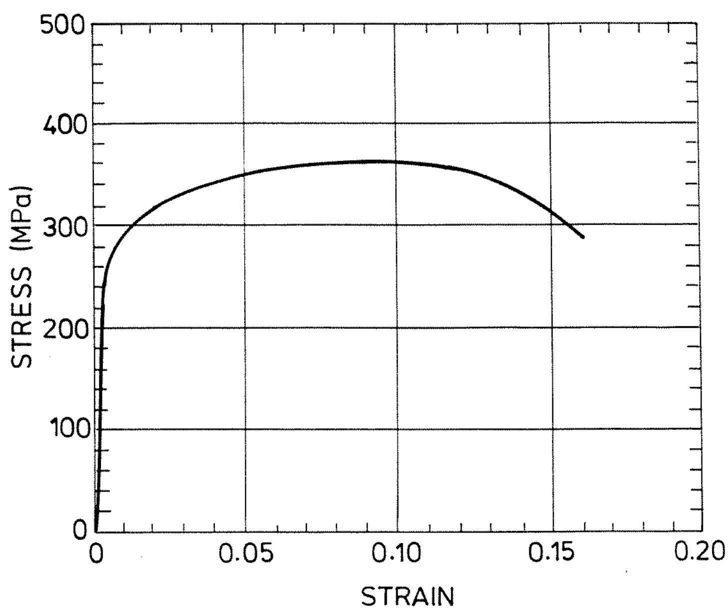

Fig. 6

8 Account for the following:

(a) The presence of welds in carbon steels can be detrimental to fatigue life, particularly in the presence of water. [4]

(b) The mild steel radiators in a house did not rust significantly for 10 years, then rusted rapidly when a continuous leak was left unrepaired. [4]

(c) Aluminium and gold do not visibly oxidise at room temperature while iron does oxidise, though this does not appear consistent with the values in the Materials Databook for the free energy of formation of these oxides. [4]

(d) When two machined steel surfaces are placed in contact, the true contact area is only a small fraction of the nominal contact area. [4]

(e) The wear of a steel bearing was greatly reduced by maintaining a good supply of oil to the bearing, but this method could not be used to protect the dies which were used for hot forging of the bearing during manufacture. [4]

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9 (a) What is meant by a thermally activated process? Explain the role of diffusion in creep of metallic alloys at low stresses. [4]

(b) Steady-state creep curves for a stainless steel at two constant temperatures are shown in Fig. 7. The steel obeys the steady-state creep equation

$$\dot{\varepsilon} = A\sigma^n \exp(-Q/RT)$$

where $R = 8.314$ J/mol K.

(i) A specimen of the steel of length 750 mm was subjected to a tensile stress $\sigma = 40$ MPa at a temperature $T = 538$ °C. Calculate the extension after 5 000 hours.

(ii) Use the creep curves to evaluate the constants n and Q. [7]

(c) A solid cylinder of the same steel was designed for operation as a support for a turbine housing under a uniform tensile load $P = 6$ kN and a temperature $T = 450$ °C. A routine check after 10 000 hours in service revealed that the extension of the cylinder was 50% higher than expected. The extension which had occurred was half of the maximum allowable before the component would need to be replaced.

(i) Evaluate the load which had actually been applied, given that the operating temperature had been correctly maintained.

(ii) The loading on the component could not be corrected, but a drop in operating temperature of 20 °C was possible. Check if this modification was sufficient to ensure a further 40 000 hours of operation before replacement. [7]

(d) Outline briefly one metallurgical and one processing technique that are employed to enhance the creep resistance of metallic alloys. [2]

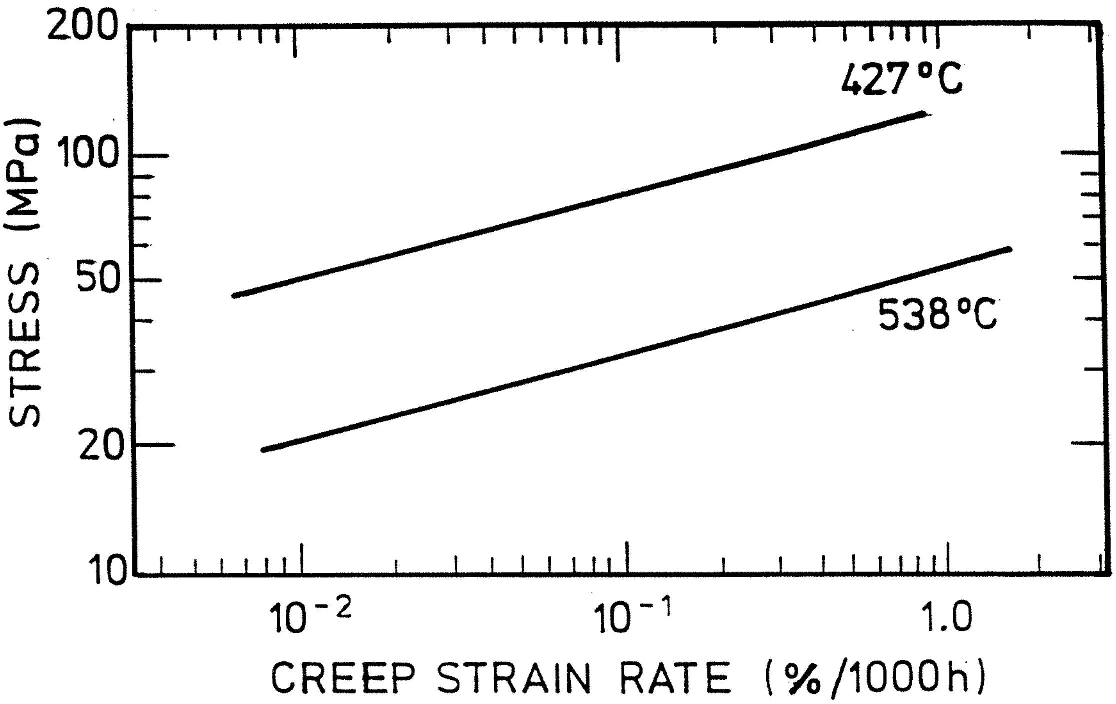

Fig. 7

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10 (a) Describe briefly what is meant by fatigue failure in metals. Distinguish between low cycle fatigue and high cycle fatigue, and explain the underlying physical mechanisms which control the fatigue life in each case. [6]

(b) A medium carbon steel was tested to obtain high cycle fatigue data for number of cycles to failure N_f in terms of the applied stress range $\Delta\sigma$ (peak-to-peak). As the test equipment was only available for a limited time, the tests had to be accelerated. This was achieved by testing the specimens using the loading history shown schematically in Fig. 8. A first set of N_1 cycles was applied with a stress range $\Delta\sigma_1$, followed by a second set of N_2 cycles at a lower stress range $\Delta\sigma_2$.

Table 2 shows the test program. In every test the mean stress σ_m was held constant at 150 MPa, and each test was continued until specimen failure. A separate tensile test gave a tensile strength for the steel of 600 MPa.

(i) Using Goodman's rule, show that the stress range which would give failure in 10^4 cycles with *zero* mean stress is 840 MPa.

(ii) Use Miner's rule to find the expected number of cycles to failure for each of the stress ranges $\Delta\sigma_2$ listed in Table 2, with the given mean stress of 150 MPa.

(iii) Convert the $\Delta\sigma$ - N_f data obtained to the equivalent data for zero mean stress.

(iv) Draw a suitable graph to show that all the fatigue life data for zero mean stress are consistent with Basquin's law for high cycle fatigue

$$\Delta\sigma\, N_f^{\alpha} = C_1$$

and find the constant α. [12]

(c) Give two examples of engineering components which commonly fail by fatigue. [2]

(cont.

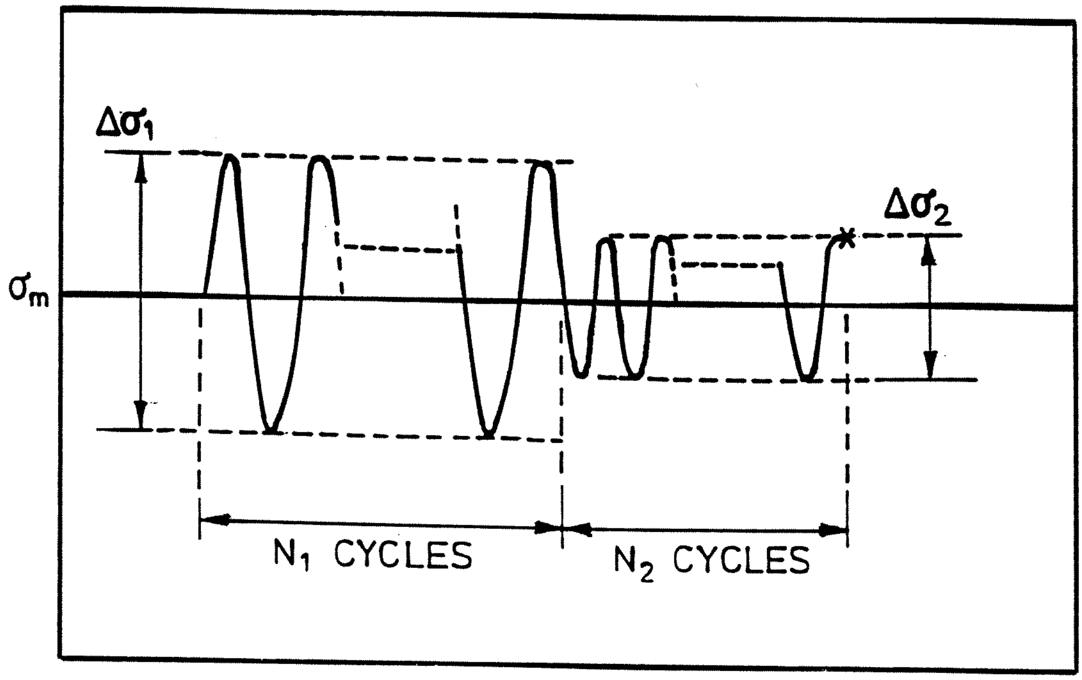

Fig. 8

Test	$\Delta\sigma_1$ (MPa)	N_1	$\Delta\sigma_2$ (MPa)	N_2
1	630	10^4	---	---
2	630	5×10^3	460	5×10^5
3	630	5×10^3	510	1.2×10^5
4	630	2.5×10^3	560	4.4×10^4

Table 2

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ENGINEERING TRIPOS PART IA
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Paper 2 Question 7(b)

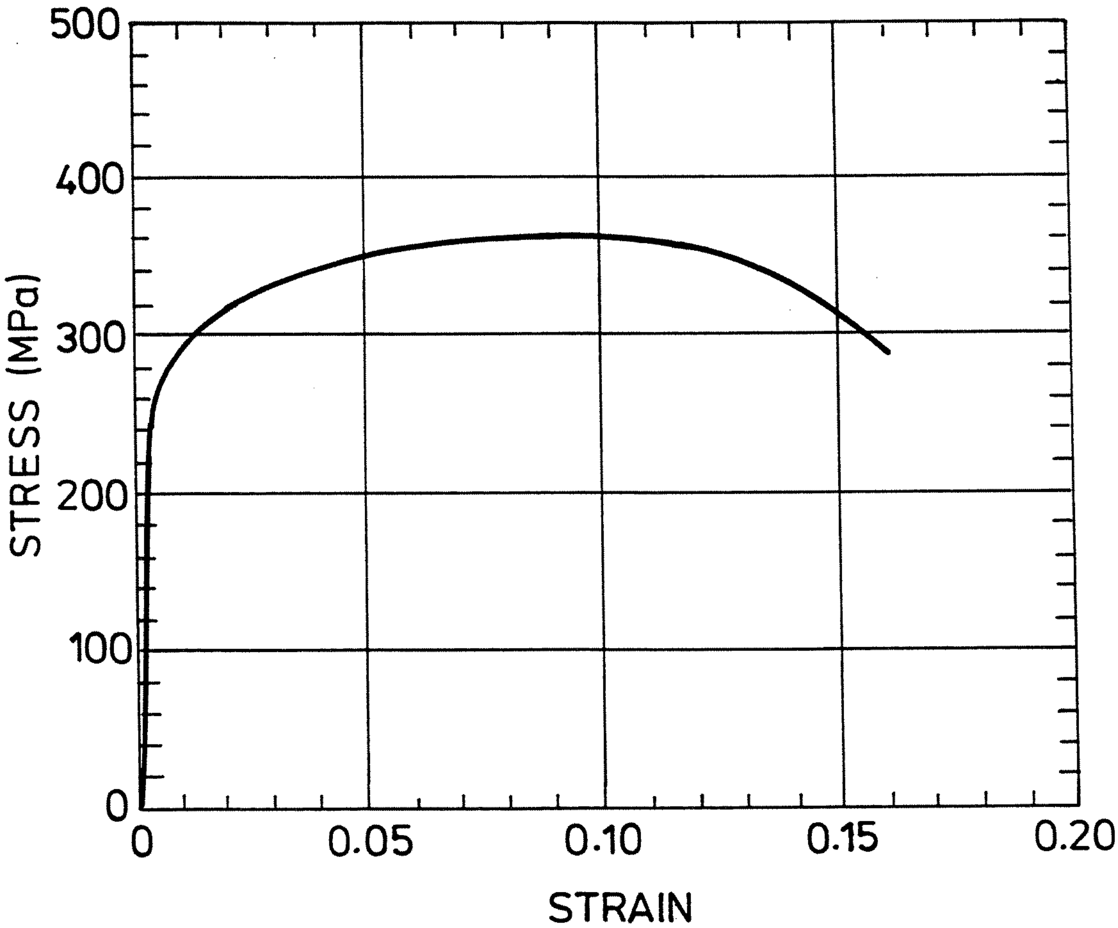

Fig. 6