IA 1999 Answers

Engineering Tripos Part 1A 1999

Paper 1: Mechanical Engineering

ANSWERS

Section A

1. (a) w0, Q+; w+, Q+; w0, Q-; w-, Q-

(b) $Q = bv_1(p_3 - p_1)$; $Q = (b+1)p_3(v_3 - v_1)$; $\left((b+1)\frac{p_3}{p_3 - p_1} + b\frac{v_1}{v_3 - v_1}\right)^{-1}$

(c) $w_x = -v_1(p_3 - p_1), Q = bv_1(p_3 - p_1)$; $w_x = 0$, $Q = (b+1)p_3(v_3 - v_1)$

2. (b) 9.54 kg

3. (a) 13.86 kg

4. (b) 270.5K; 1229.5K; 3.76 bar; 348.8K; 37.5%

Section B

5. Answers approximate

(a) (ii) 1.1×10^{-3} rad/s anti-clockwise, 1.0×10^{-3} rad/s clockwise, 0.45×10^{-3} rad/s anti-clockwise, 0.058 mm/s,

(iii) 94 N

6. (a) (i) $\Omega_1 d\underline{k}$, $\omega R\underline{k}$, $\Omega_2 d\underline{k}$

(ii) $2R\cos\alpha\ \underline{i}$, $-2R\omega\cos^2\alpha\ \underline{k}$

(b) $\Omega_1 d - \omega R$, $\omega R - 2R\omega\cos^2\alpha - \Omega_2 d$

(c) $1 - 2\cos^2\alpha$, -1

7. (b) $\underline{v} = \alpha\dot{\lambda}S\underline{e}_t$ where $S = \sqrt{1 + \sin^2\lambda}$

$\underline{a} = \left(\alpha\ddot{\lambda}S + \alpha\dot{\lambda}^2\sin\lambda\cos\lambda / S\right)\underline{e}_t - \alpha\dot{\lambda}^2\cos\lambda / S\ \ \underline{e}_n$

(c) (i) $F = m\underline{a}\cdot\underline{e}_t$, $N = m\underline{a}\cdot\underline{e}_n$

(ii) $F = -N\sin\lambda$

8. (b) (i) 5747 m/s

(c) 9.52×10^5 Ns, 3367 m/s

Section C

9. (a) $B = T = \dfrac{\rho c R D}{2h(D+R)}$

(b) 17.7 mins

(c) Working in minutes:
For t ≤ 12, $\theta = (15T - 25)\exp(-t/T) + 20 - 15T + 15t$
For t > 12, $\theta = 200 - 282.4\exp(-t/T)$
56.5°C, 30.7 mins.

10. (a) $\theta = (y_2 - y_1)/L$, $y_g = (y_2 + y_1)/2$

(b) $m\ddot{y}_g = -k(y_1 - x_1) - k(y_2 - x_2)$

(d) 5μm

11. (b) $c = \dfrac{1}{\sqrt{8}}\left(\dfrac{1}{R}\sqrt{\dfrac{L}{C}} + R\sqrt{\dfrac{C}{L}}\right)$

(c) $\sqrt{2} - 1 < \dfrac{1}{R}\sqrt{\dfrac{L}{C}} < \sqrt{2} + 1$

Part 1A Engineering Tripos 1999

Paper 2 Section A, Structures

Answers

1 (a) (i) Sliding will occur.

(b) $W_{max} = 61.4$ kN; $W_{min} = 4.78$ kN

2 (a) (i) $2W$

(ii)

Bar	Tension
AC, EG	$-2W/\sqrt{3}$
BC, EF	$2W/\sqrt{3}$
CD, DE	$2W/\sqrt{3}$

(b) (i) vertical = $10WL/3EA$, horizontal = 0

(ii) reduced by $2WL/\sqrt{3}EA$

3 (a) $T_{CD} = \sqrt{2}wa/4$; reactions at A: $\uparrow 3wa/4$, $\leftarrow wa/4$;

reactions at B: $\uparrow wa/4$, $\rightarrow wa/4$

(b) $M_B = wa^2/4$

(c) $M_B = wa^2/4$, $M_{min} = -wa^2/32$, $M_C = 0$

4 (a) $M_C = 2wa^2$

(b) Using clockwise +ve, $\theta_B = 3wa^3/24EI$, $\theta_C = 21wa^3/24EI$

(c) $\delta_A = 3wa^4/24EI \uparrow$, $\delta_D = 90wa^4/24EI \downarrow$

5 (b) $S_{max} = 1$ kN, $S_{min} = -1$ kN; $M_{max} = 0$, $M_{min} = -300$ Nm

(c) $\sigma_{max} = 1.64 \times 10^6$ N/m^2

(d) 182 N

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Paper 2 Section B MATERIALS

Answers

6 (a) $\varepsilon_t = \ln_e(A_o/A)$

$\sigma_{TS} = \sigma_t$ (at necking) x (A / A_o).

6 (b) $\varepsilon_t = n - B$.............at necking.

$\sigma_{TS} = \sigma_o(n)^n \exp(n - B)$

$B = 1.86$.

7 $\varepsilon = t/2R$ and $\sigma = E(t/2R)$.

R must satisfy $R \geq (t/2)\ (E/\sigma_y)$.

The merit index M (for a given t) is $M = \sigma_y/E$.

Best choices include PE, PP, Nylon, elastomers.

8 (a) The elastic stress concentration factor $K_t = \sigma_{local}/\sigma_{nominal}$

For a hole, $K_t = 1 + 2\,(a/\rho)^{1/2} = 3$ (a is notch length, ρ is notch tip radius)

The stress intensity factor $K = \sigma_{nominal}\,(\pi a)^{1/2}$

($K = K_c$ when $\sigma_{nominal} = \sigma_f$).

The local tensile stress $\sigma_{local} = K/(2\pi r)^{1/2}$ (r is distance)

8 (b) The steel's endurance fatigue limit is roughly half its yield strength, i.e., 150 MPa. Since the cyclic stress is ± 25MPa, the likelihood of a crack nucleating elsewhere away from any pre-existing flaw is negligible.

$\Delta K_{(threshold)}$ is 6 $MPam^{1/2}$; K_{IC} is 50 $MPam^{1/2}$, approximately.

$\Delta\sigma = 50$ MPa and $\Delta K_{(threshold)} = 6$ $MPam^{1/2}$; acceptable "a" is 3.65mm.

9 (b) $\sigma_y = \sigma_o + 2(2Gb/l)$ ($2\tau_y = \sigma_y$, approximately)

$\sigma_y = 421$ MPa.

9 (c) The new T_D is 148 K.

10 (b) $T_1 = 1308$ K = 1035 C, an increase in 35 C.

Paper 3: Electrical and Information Engineering

Answers

1. (a) $R_2 = 3\ \text{k}\Omega$, $V_{DD} = 23$ V, $R_1 = 100\ \text{k}\Omega$ to $10\ \text{M}\Omega$ typically.
(b) $R_{in} = R_1$ as in (a), $R_{out} = 8.33\ \text{k}\Omega$, Gain = - 41.65 (i.e. inverting).
(c) $R_{load} = R_{out} = 8.33\ \text{k}\Omega$.
(d) C = 95.5 nF.

2. (b) $R_{IN} = 1099\ \Omega$, G = - 9.01
(c) $v_o = -4.72\ v_s$.
(d) $f_{3dB} = 1.67$ MHz (*not* 1.59 MHz).

3. (b) $V_{th} = -5$ V, $R_{th} = 20\ \Omega$.
(c) $R_A = 20\ \Omega$, $R_B = 40\ \Omega$.
(d) chain 3 circuits as in (c).

4. (a) $P_{loss} = 590.8$ W, $V_{in} = 247.8$ V.
(b) C = 902 μF, ($X_c = 3.53\ \Omega$).
(c) $V_{load} = 242.6$ V.

Section B

5. (b) Sum-of-products:

$$
\begin{aligned}
O_0 &= I_0.\overline{I_1} + \overline{I_0}.I_1 = I_0 \oplus I_1 \\
O_1 &= \overline{I_1}.I_2 + \overline{I_0}.I_2 + I_0.I_1.\overline{I_2} = I_2.(\overline{I_0} + \overline{I_1}) + I_0.I_1.\overline{I_2} \\
O_2 &= I_0.I_1.I_2
\end{aligned}
$$

Product-of-sums:

$$
\begin{aligned}
O_0 &= (I_0 + I_1).(\overline{I_0} + \overline{I_1}) \\
O_1 &= (I_1 + I_2).(I_0 + I_2).(\overline{I_0} + \overline{I_1} + \overline{I_2}) \\
O_2 &= I_0.I_1.I_2
\end{aligned}
$$

(c) 2-input NAND implementation

$$
\begin{aligned}
O_0 &= \overline{\overline{(I_0.\overline{I_1})}.\overline{(\overline{I_0}.I_1)}} \\
O_1 &= \overline{\overline{I_2.(\overline{I_0.I_1})}.\overline{\overline{\overline{I_0.I_1}}.\overline{I_2}}} \\
O_2 &= \overline{\overline{\overline{\overline{I_0.I_1}}.I_2}}
\end{aligned}
$$

6. (b) i) Unused states: 0010, 0110, 1110, 1010
ii)

$$
\begin{aligned}
J_A &= Q_B.Q_C + Q_B.\overline{Q_D} \\
K_A &= \overline{Q_B}.\overline{Q_D} + Q_B.\overline{Q_C}.Q_D \\
J_B &= \overline{Q_A}.\overline{Q_D} + \overline{Q_A}.Q_C \\
K_B &= \overline{Q_A}.\overline{Q_C}.Q_D + Q_A.Q_C
\end{aligned}
$$

7. b) iii) 10.9 μs

8. b) ii) 65536 iii) 4

Section C

9. b) 0.53 T

   c) the inductance doubles

10. b) i) 8 pF
    ii) 16 pF

    c) 8 + 40d pF

    d) rises 0.043 mm up the tube

11. a)

$$\frac{N_A d}{\pi\,(1/\mu_A + 1/\mu_B)} \ln \frac{R_2}{R1}$$

    b) peak flux VT

    d) flux density is zero

Numerical answers to 1999 Part Ia, Paper 4 Mathematical Methods

Section A

Question 1.

b)

(i) $\sqrt{3}x + y + \frac{z}{\sqrt{2}} = \sqrt{3}$

(ii) $x = \sqrt{3}y = \sqrt{6}z$

(iii) $x = \frac{1}{2}, y = \frac{1}{2\sqrt{3}}$, z arbitrary.

c) $\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right)$.

Question 2.

a) $5^{\frac{1}{4}} \exp(i0.25\theta + 0.5n\pi i)$ for $n = 0, 1, 2, 3$, where $\theta = \tan^{-1} 4/3$. Solutions of polynomial are $(3 \pm 4i)^{\frac{1}{4}}$, so above roots together with their complex conjugates.

b) 1

c) $1 - \frac{x^2}{2} + \frac{5x^4}{24} + \ldots$

Question 3.

a) $y = \frac{\exp(-4x)}{5} - \frac{\exp(-2x)}{5} - \frac{x\exp(-4x)}{7}$.

b) $y_n = \frac{3}{5}2^n + \frac{2}{5}\left(-\frac{1}{2}\right)^n$.

Question 4.

a)

$$Q(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Question 5.

b) eigenvalues $2, 1, -1$, normalised eigenvectors $(1, 0, 0)^t$, $(0, 1, 1)^t/\sqrt{2}$, $(0, 1, -1)^t/\sqrt{2}$.

d)

$$A - 2I = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$

Section B

6. (b) $t - \frac{t^2}{2T}$ for $0 \le t \le T$, $\frac{T}{2}$ for $t \ge T$

(c) (i) $\frac{t^2}{2} - \frac{t^3}{6T}$ (ii) $\frac{5}{6}T^2$

7. (a) $\frac{1}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\theta}{2n-1}$

(b) $a_0 = \frac{1}{2}$, $a_n = \frac{1}{n^2\pi^2}\left[1-(-1)^n\right]$ $b_n = -\frac{1}{n\pi}$

(d) $\lambda = 1$ $\mu = \frac{1}{2}$

8. $\frac{q}{p+q}$

9. (a) $x(t) = \frac{1}{2}\sin t + e^{-t/2}\cos\frac{\sqrt{3}}{2}t$ $y(t) = \frac{1}{2}\sin t - \frac{1}{\sqrt{3}}e^{-t/2}\sin\frac{\sqrt{3}}{2}t$ for $t \ge 0$.

(b) 0 , 0.

10. $(-1, 2)$, $(-1, -\frac{4}{3})$ and $(1,0)$ are saddle points , $(-\frac{1}{3}, \frac{2}{9})$ is a maximum