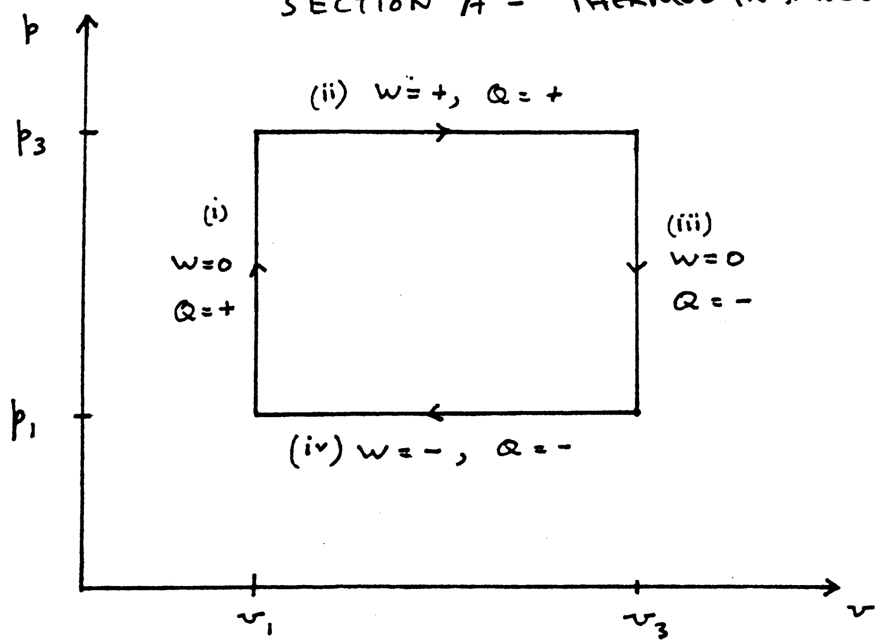
SECTION A - THERMODYNAMICS

1.

(a)



Fully resisted : $W = \int p\,dv$

First Law : $Q = \Delta u + W = b\,\Delta(pv) + \int p\,dv$

$\begin{pmatrix} = b v \Delta p & \text{for const vol.} \\ = (b+1) p \Delta v & \text{for const pressure} \end{pmatrix}$

(b) $\therefore$ for process (i) $Q = b v_1 (p_3 - p_1)$

for process (ii) $Q = (b+1)\, p_3 (v_3 - v_1)$

Net work for the whole cycle $= (p_3 - p_1)(v_3 - v_1)$

$\therefore$ Thermal efficiency $= \dfrac{\text{net } W}{Q \text{ in}} = \underline{\underline{\left((b+1)\dfrac{p_3}{p_3 - p_1} + b\dfrac{v_1}{v_3 - v_1} \right)^{-1}}}$

1 contd

(c) For steady flow control volumes:

shaft work (reversible) $w_x = -\int v\,dp$

SFEE $Q = \Delta h + w_x = (b\Delta pv + \Delta pv) - \int v\,dp$

$\left(\begin{array}{ll} = b\,v\,\Delta p & \text{for const volume} \\ = (b+1)\,p\,\Delta v & \text{for const pressure} \end{array} \right)$

$\uparrow$
ie same as in non-flow

For process (i), $w_x = -v_1(p_3 - p_1)$

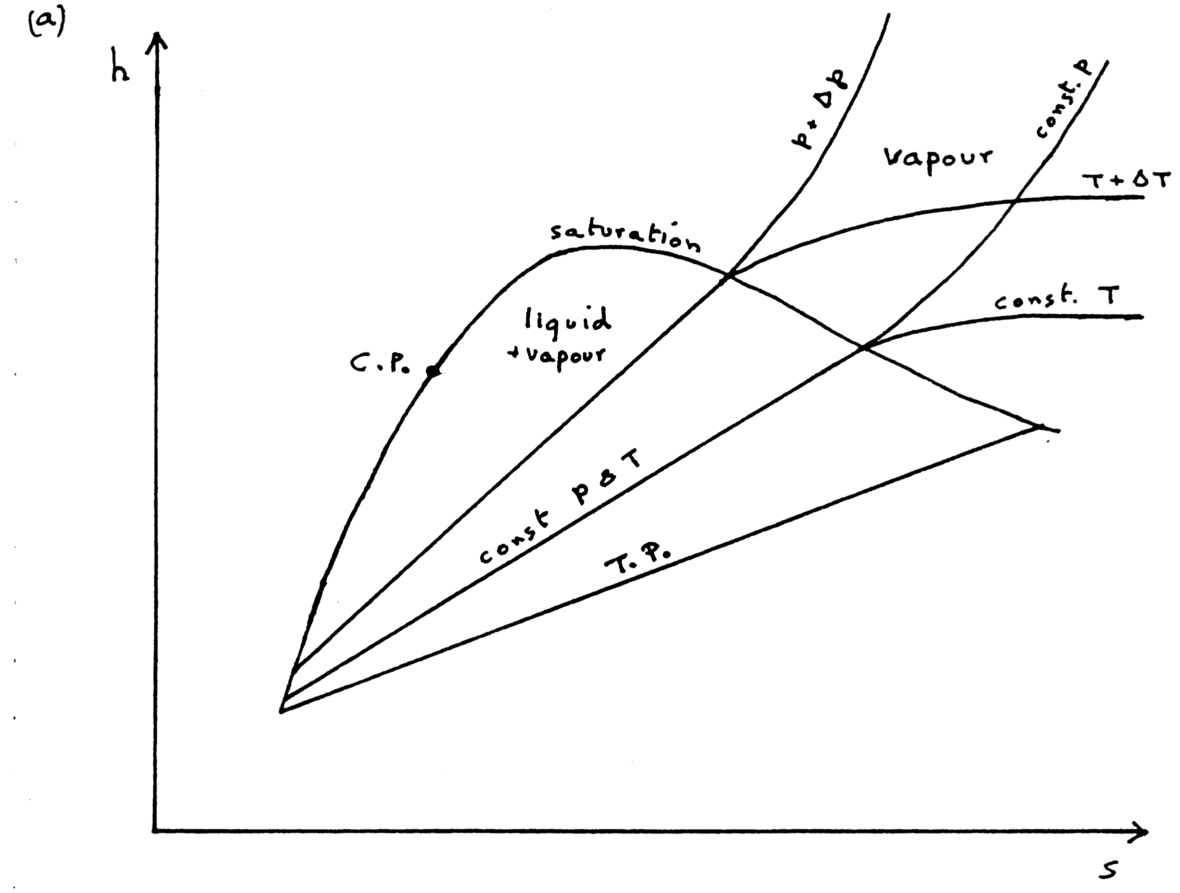
$Q = b\,v_1(p_3 - p_1)$

For process (ii), $w_x = 0$

$Q = (b+1)\,p_3(v_3 - v_1)$

2.

(a)



From $T\,ds = dh - v\,dp$

$$\left(\frac{\partial h}{\partial s}\right)_p = T$$

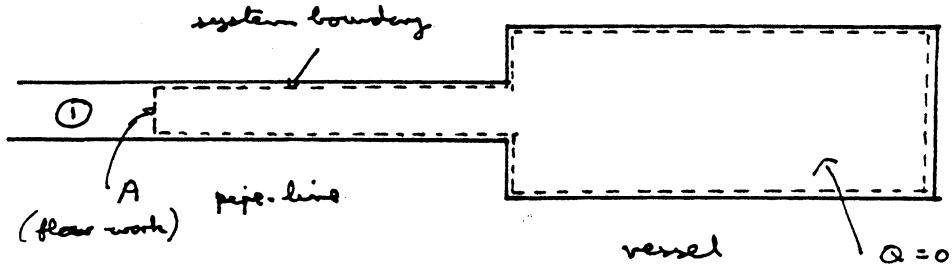
In the two phase region, $p = p(T \text{ only})$, so that lines of constant p are lines of constant T and have constant slope.

In the superheated vapour region, T increases with h at const. p and the slope of the isobars becomes steeper.

(Also in the vapour region, behaviour becomes closer to that of an ideal gas on moving away from the saturation line. Hence isotherms flatten.)

2 contd

(b) (i)



Consider as system all the H_2O contained in the vessel at the time of interest in the charging process, when the state is denoted ②. Initially the system (mass m) was in the pipe line at state ①.

Work done by system at A $= - m\, p_1 v_1$

First law $\quad 0 = m(u_2 - u_1) - m\, p_1 v_1 \quad \therefore \underline{\underline{u_2 = h_1}}$

(ii) At 20 MN/m², $h_f = 1826.5$

$u_2 = 1306(1-x_2) + 2571.7\, x_2$ where x is dryness

(values of u_f and u_g show that there is wet steam in vessel at 8 MN/m²)

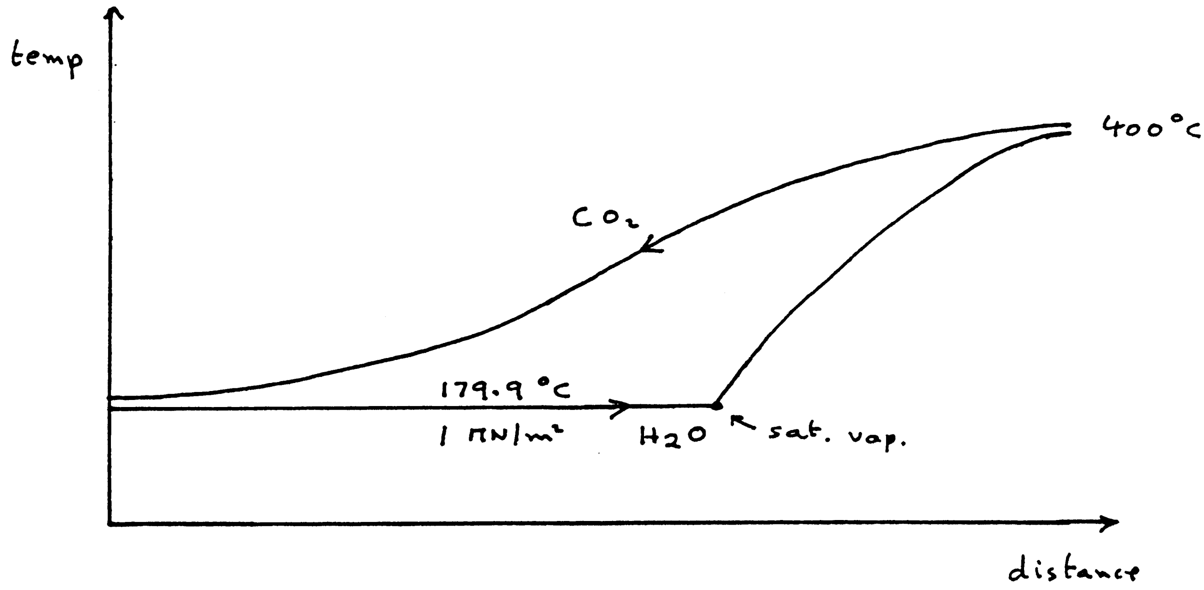
$\therefore x_2 = 0.4112$

Specific volume $= 0.001384(1-x_2) + 0.0235 x_2$

$= 0.01048$ m³/kg

$\therefore$ Mass in vessel $= \dfrac{0.1}{0.01048} = \underline{\underline{9.54 \text{ kg}}}$

3 (a)



Since no overall heat transfer between exchanger and surroundings and no shaft work, SFEE gives enthalpy drop of CO_2 = enthalpy rise of H_2O. With sp. ht. capacity of CO_2 = 0.82 (table 2) and outlet temp of $CO_2 \simeq 179.9\ °C$, mass of CO_2 per kg H_2O given by

$$m_{CO_2} \times 0.82\,(400 - 179.9) = 1\,(3264 - 762.6)$$

ie $\underline{\underline{m_{CO_2} = 13.86 \text{ kg}}}$

The heat-exchange process is irreversible because heat is transferred between the streams across a finite temperature difference (except near ends of exchanger). The difference is unavoidable because h varies linearly with T for CO_2 but not for evaporating steam.

3 cont

(b) From second law, if reversible and adiabatic overall, the entropy fluxes must balance (as well as the enthalpy fluxes. If T is the outlet absolute temperature of CO_2

$$m_{CO_2} \times 0.82 \ln\left(\frac{400+273.15}{T}\right) = 1.\,(7.467 - 2.138)$$

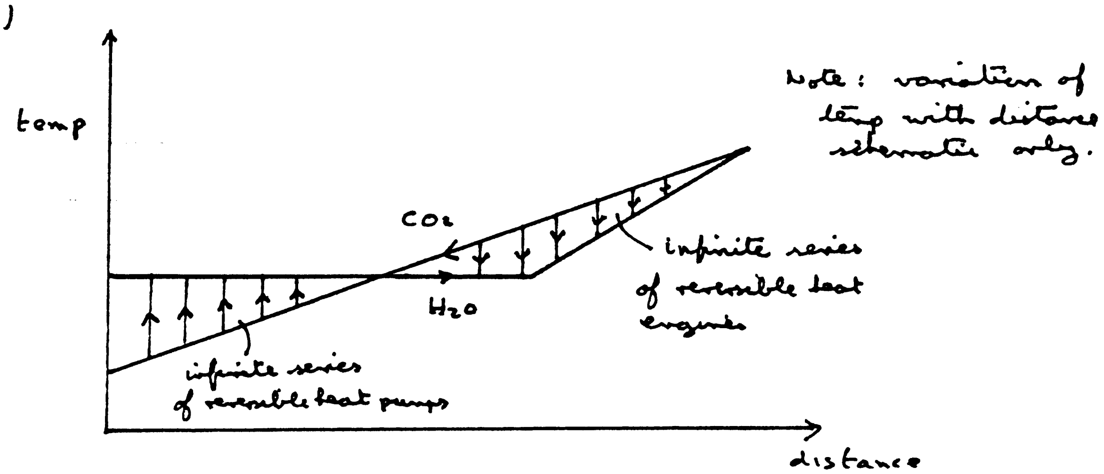
$$\& \quad m_{CO_2} \times 0.82\,(673.15 - T) = 1\,(3264 - 762.6)$$

making use of data-card formula $s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

Assuming $\underline{\underline{T = 311.6\text{ K}}}$ entropy equation gives $\underline{\underline{m_{CO_2} = 8.437}}$

enthalpy equation gives $\underline{\underline{m_{CO_2} = 8.437}}$

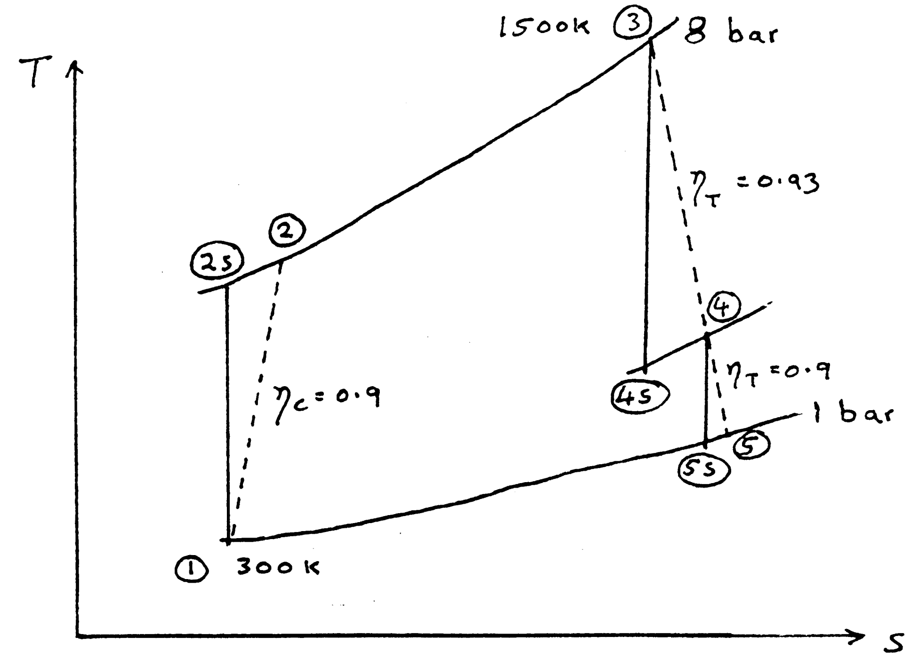
(c)



Temperature differences between streams used to drive engines (or overcome by heat pumps - as appropriate). Since there is overall enthalpy balance, no net shaft work and engines just drive pumps. BUT result may be only transfer of irreversibility to heat source.

4.

(a)



(b) (i) For the compressor: $T_{2s} = 300 \times 8^{0.4/1.4} = 543.43$ K

$T_2 - T_1 = \frac{1}{0.9}(543.43 - 300) = \underline{\underline{270.48 \text{ K}}}$

(ii) Since work output of h.p. turbine drives the compressor,

Temperature drop $T_3 - T_4 = 270.48$ $\therefore$ $\underline{\underline{T_4 = 1229.52 \text{ K}}}$

also $T_3 - T_{4s} = \frac{270.48}{0.93} = 290.84$

& $T_{4s} = 1209.16$

So $\frac{8}{p_4} = \left(\frac{1500}{1209.16}\right)^{1.4/0.4}$ giving $\underline{\underline{p_4 = 3.762 \text{ bar}}}$

(iii) For the l.p. turbine $T_{5s} = 1229.52 \div \left(\frac{3.762}{1}\right)^{0.4/1.4}$

$= 842.01$ K

$\therefore$ $T_4 - T_5 = 0.9(1229.52 - 842.01)$

$= \underline{\underline{348.76 \text{ K}}}$

4 cont)

(iv) Temperature rise in heater = $T_3 - T_2$

$$= 1500 - (270.48 + 300) = 929.52 \text{ K}$$

$$\text{Thermal efficiency} = \frac{348.76}{929.52} = \underline{\underline{37.5\%}}$$

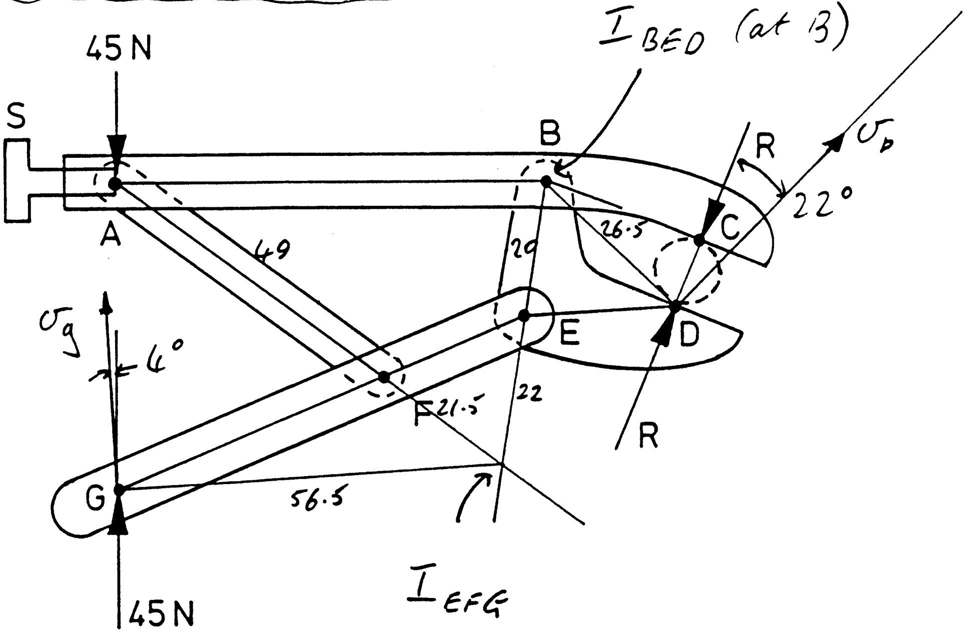
Mechanical Engineering

SECTION B

Q.5

a(i) ⊥ BD as shown

(ii) See Fig. for instantaneous centres



a(ii) $\omega_{BED} = v_D/BD = 0.03\,\text{mm/s}/26.5\,\text{mm} = 1.13\times10^{-3}\,\text{rad/s}$ ↻ (2 s.f. accuracy)

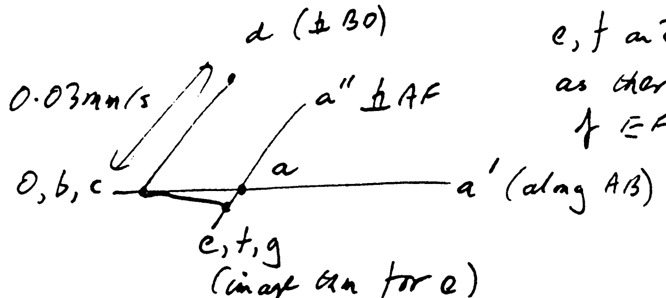
$\omega_{EFG} = \omega_{BED}\times BE/EI = 1.03\times10^{-3}\,\text{rad/s}$ ↺

$\omega_{AF} = \omega_{EFG}\times FI/FA = 0.451\times10^{-3}\,\text{rad/s}$ ↻

$v_g = \omega_{EFG}\times GI = 0.058\,\text{mm/s}$. Direction as shown.

(iii) From work $\Sigma \underline{F}.\underline{v} = 0$ $\quad R = 45\times0.058\times\cos4/0.03\cos22 = 94\,N$

(b)



e, f and g are coincident as there is no relative motion of EFG.

A popular question, but the first part was answered incorrectly by many, as were the locations of the instantaneous centres.

Q6. (a) (i) $\underline{r}_{A/O} = d\underline{j} + d\tan\alpha\,\underline{i}$

$$\underline{v}_A = \underline{\omega}\times\underline{r} = \Omega_1\underline{i}\times(d\underline{j} + d\tan\alpha\,\underline{i}) = \Omega_1 d\,\underline{k}$$

$$\underline{r}_{B/C} = R(-\underline{i}\cos\alpha + \underline{j}\sin\alpha)$$

$$\underline{v}_{B/C} = \underline{v}_B = \omega(\underline{j}\cos\alpha + \underline{i}\sin\alpha)\times R(-\underline{i}\cos\alpha + \underline{j}\sin\alpha)$$

$$= \omega R\,\underline{k}$$

$$\underline{v}_E = \Omega_2 d\,\underline{k}$$

(ii) $\underline{r}_{D/B} = 2R\cos\alpha\,\underline{i}$

$$\underline{v}_{D/B} = \underline{\omega}\times\underline{r}_{D/B} = \omega(\underline{j}\cos\alpha + \underline{i}\sin\alpha)\times 2R\cos\alpha\,\underline{i}$$

$$= -2R\omega\cos^2\alpha\,\underline{k}$$

[N.B. Note all velocities in $\underline{k}$ direction as expected.]

(b) Sliding velocity $= |\underline{v}_1 - \underline{v}_2|$

At AB $\Delta v_1 = |\underline{v}_A - \underline{v}_B| = |\Omega_1 d - \omega R|$

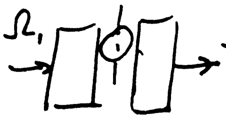
At DE $\Delta v_2 = |\underline{v}_D - \underline{v}_E| = |\underline{v}_{D/B} + \underline{v}_B - \underline{v}_E| = |-2R\omega\cos^2\alpha + \omega R - \Omega_2 d|$

(c) $\Delta v = 0 \Rightarrow \Omega_1 d = \omega R$

$$\Omega_2 d = \omega R - 2R\omega\cos^2\alpha$$

Eliminate ω: $\dfrac{\Omega_2}{\Omega_1} = 1 - 2\cos^2\alpha$

when $\alpha = 0$ $\dfrac{\Omega_2}{\Omega_1} = -1$:-



Straight through - Direction of rotation changes.

(d) Gears, two disc machine or pulleys.

An unpopular question, with a fair number of good answers, but too many floundering with the simple $\underline{v} = \underline{\omega}\times\underline{r}$ expressions in the first part.

Q7 (a) $\dot{x} = \alpha \dot{\lambda}$ $\quad \dot{y} = -\alpha \dot{\lambda} \sin \lambda$

$\underline{v} = \dot{x}\underline{i} + \dot{y}\underline{j} = \alpha \dot{\lambda} \underline{i} - \alpha \dot{\lambda} \sin \lambda \underline{j}$

(b) $\underline{v} = \dot{s}\underline{e}_t$ where $\dot{s} = |\underline{v}| = \alpha \dot{\lambda} \sqrt{1 + \sin^2 \lambda}$

and $\underline{e}_t = (\underline{i} - \sin\lambda \underline{j})/\sqrt{1+\sin^2\lambda}$ is a unit vector in the direction of $\underline{v}$.

$\underline{a} = \alpha \ddot{\lambda} \underline{i} - \alpha \ddot{\lambda} \sin\lambda \underline{j} - \alpha \dot{\lambda}^2 \cos\lambda \underline{j}$

$\underline{a}\cdot\underline{e}_t = (\alpha\ddot{\lambda} + \alpha\ddot{\lambda}\sin^2\lambda + \alpha\dot{\lambda}^2 \sin\lambda\cos\lambda)/\sqrt{1+\sin^2\lambda}$

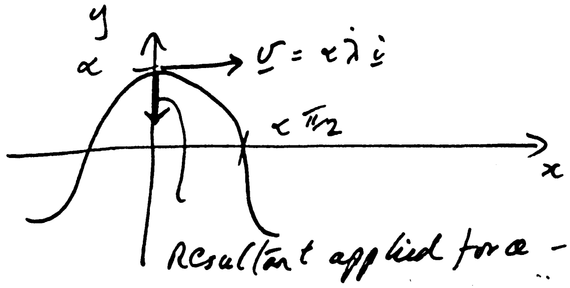
$\underline{e}_n = (\sin\lambda \underline{i} + \underline{j})/\sqrt{1+\sin^2\lambda}$

$\underline{a}\cdot\underline{e}_n = (\alpha\ddot{\lambda}\cancel{\sin\lambda} - \alpha\ddot{\lambda}\cancel{\sin\lambda} - \alpha\dot{\lambda}^2\cos\lambda)/\sqrt{1+\sin^2\lambda}$

$$\underline{a} = (\alpha\ddot{\lambda}(1+\sin^2\lambda) + \alpha\dot{\lambda}^2\sin\lambda\cos\lambda) \times \frac{1}{\sqrt{1+\sin^2\lambda}}\underline{e}_t - \frac{\alpha\dot{\lambda}^2\cos\lambda}{\sqrt{1+\sin^2\lambda}}\underline{e}_n$$

(c)(i) $\underline{F} = m\underline{a} \Rightarrow F = m\underline{a}\cdot\underline{e}_t$, $N = m\underline{a}\cdot\underline{e}_n$

where $\underline{a}\cdot\underline{e}_t$ and $\underline{a}\cdot\underline{e}_n$ are given above.



$\ddot{\lambda} = 0$, $F = m\alpha\dot{\lambda}^2\sin\lambda\cos\lambda/\sqrt{\ }$

$N = -m\alpha\dot{\lambda}^2\cos\lambda/\sqrt{\ }$

$F = -N\sin\lambda$

when $\lambda = 0$

$\underline{v} = \alpha\dot{\lambda}\underline{i}$

$F = 0$

$N = -\alpha\dot{\lambda}^2\underline{j}$

The first part of section (b) (ie using $\underline{v} = \dot{s}\underline{e}_t$) was not completed well. Those who understood this concept went on to score good marks.

Q 8

(a)

Tangentially, as the force only acts in the radial direction the momentum of momentum is conserved $\Rightarrow$

$r^2\dot{\theta} = \text{const} = h$

Radially 'F = ma' $\Rightarrow -GMm/r^2 = m(\ddot{r} - r\dot{\theta}^2)$

Combining $\ddot{r} - \dfrac{h^2}{r^3} + \dfrac{GM}{r^2} = 0$

(b)(i) On surface of Earth $GM/R^2 = g = 10$, where R is Earth's radius.

In orbit $\ddot{r} = 0 \Rightarrow \dfrac{h^2}{r^3} = r_0\dot{\theta}^2 = GM/r_0^2$ where r_0 is radius of orbit

$\Rightarrow v_0 = r_0\dot{\theta} = \sqrt{\dfrac{GM}{r_0}} = \sqrt{\dfrac{gR^2}{r_0}} = \underline{5747 \text{ m/s}}$

(ii) Faster half just escapes $\Rightarrow v = 0$ far away.

Using energy, Gain in PE = Loss in KE

$\displaystyle\int_{r_0}^{\infty} \frac{m}{2}\frac{GM}{r^2}\,dr = \frac{1}{2}\left(\frac{m}{2}\right)v_1^2 + 0$:- v_1 is velocity after separation.

$\Rightarrow \dfrac{GM}{r_0} = \dfrac{1}{2}v_1^2$; $v_1 = \sqrt{\dfrac{2GM}{r_0}} = \sqrt{2}\,v_0 = \underline{8128} \text{ m/s}$

Momentum is conserved in separation (no external forces)

Impulse $I = \dfrac{m}{2}(v_1 - v_0) = 9.52 \times 10^5$ kg m/s or Ns

Speed of slower half v_2 : $400(8127 + v_2) = 800 \cdot 5747$

$\underline{v_2 = 3367 \text{ m/s}}$

Standard question well answered by most. Putting numbers in too early, then losing factors of 1000, tended to make answers inaccurate.

SECTION C

Q9. (a) Balancing heat flow

$$\frac{d\theta}{dt} = \frac{\dot{Q}}{\rho c V} = \frac{hA}{V}\frac{1}{\rho c}(\theta_0 - \theta) \quad \text{where} \quad A = 2\pi R D + 2\pi R^2, \quad V = \pi R^2 D$$

ie $B = T = \frac{\rho c V}{hA} = \frac{\rho c}{h}\frac{RD}{2D+2R}$

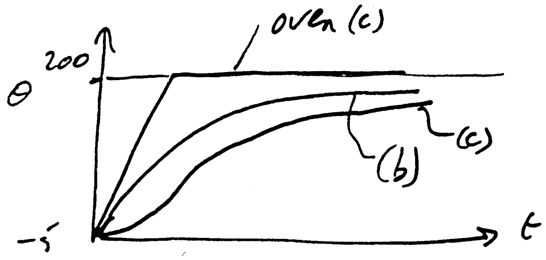
(b) C.F. $\theta = Ae^{-t/T}$, P.I. $\theta = \text{constant} = C$ } Work in °C and minutes

B.C. $t=0,\ \theta=-5 \Rightarrow C + A = -5$

$t = 25,\ \theta = 150$

$t \to \infty\ \theta \to 200 \Rightarrow C = 200,\ A = -205$

$e^{-25/T} = (150-200)/-205 \Rightarrow T = 17.7$ mins.



(c) $T\dot{\theta} + \theta = 20 + 15t \quad t < 12$

$= 200 \quad t \geq 12$

For $t<12$ C.F. $\theta = Ae^{-t/T}$

P.I. Try $\theta = \alpha + \beta t$

$\Rightarrow T\beta + \alpha + \beta t = 20 + 15t$

$\beta = 15,\ \alpha = 20 - 15T$

B.C. $t = 0,\ \theta = -5 \Rightarrow -5 = 20 - 15T + A,\ A = -25 + 15T$

$\theta = (15T - 25)e^{-t/T} + 20 - 15T + 15t$

At $t = 12$, $\theta = 56.5$°C

For $t > 12$ $\theta = Ae^{-t/T} + 200$

B.C. $t = 12$ $\theta = 56.5 \Rightarrow \theta = -282.6e^{-t/T} + 200$

when $\theta = 150$°C, $t = 30.7$ mins

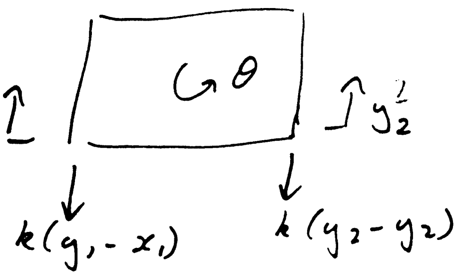
Quite a long question, but well answered. Sketches were surprising patchy. ~~Atria~~ The initial slope was often quoted as a way of deriving T, but as this slope is not given this is inappropriate.

Q 10.

(a) $\theta = (y_2 - y_1)/L$, $y_G = (y_1 + y_2)/2$

(b) Taking moments about G
('F = ma')

$$\underline{\frac{kL}{2}(y_1 - x_1) - \frac{kL}{2}(y_2 - x_2) = J\ddot{\theta}} \quad (1)$$



Resolving ↑ 'F = ma'

$$-k(y_1 - x_1) - k(y_2 - x_2) = m\ddot{y}_G \quad (2)$$

(c) Need to get things in terms of y_1 and y_2

(1) Becomes $\frac{kL}{2}(y_1 - x_1) - \frac{kL}{2}(y_2 - x_2) = \frac{mL^2}{6}(\ddot{y}_2 - \ddot{y}_1)/L$

rearranging $-m/3\ \ddot{y}_1 + m/3\ \ddot{y}_2 - ky_1 + ky_2 = -kx_1 + kx_2$

(2) Becomes $\frac{m}{2}\ddot{y}_1 + \frac{m}{2}\ddot{y}_2 + ky_1 + ky_2 = kx_1 + kx_2$

which are the equations given in matrix form in the question.

(d) $\underline{Y} = \left[[k] - \omega^2 m\right]^{-1} \underline{F}$

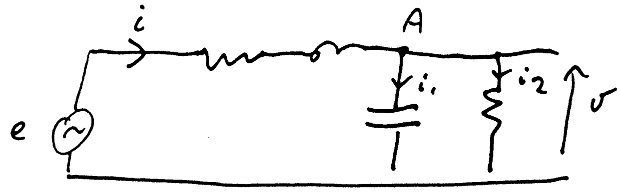
$$= \left[1000\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} - (20)^2\begin{pmatrix} 15 & 15 \\ -10 & 10 \end{pmatrix}\right]^{-1}\begin{pmatrix} 1000 \cdot 50 \\ -1000 \cdot 50 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -5 \\ 3 & -3 \end{pmatrix}^{-1}\begin{pmatrix} 50 \\ -50 \end{pmatrix} = \begin{pmatrix} -40/3 \\ 10/3 \end{pmatrix} \mu m$$

$|Y_G| = |(Y_1 + Y_2)|/2 = 5\,\mu m$

Add damping.
(Alternatively change mass and/or stiffness, though it is not obvious how. In this case the natural frequencies are below 20 rad/s, so increase mass or reduce stiffness).
Many people were not able to do the first two parts of the question. This seemed to hinder them in part (d) although this was relatively simple and stand-alone.
The lowest scoring question of Sections B and C.

Q11



(a) Summing voltages around circuit

$$e = iR + L\frac{di}{dt} + v$$

Summing currents at A

$$i = i_1 + i_2 = \frac{v}{R} + C\frac{dv}{dt} \Rightarrow \frac{di}{dt} = \frac{\dot{v}}{R} + C\ddot{v}$$

Combining:

$$e = \left(\frac{v}{R} + C\dot{v}\right)R + L\left(\frac{\dot{v}}{R} + C\ddot{v}\right) + v$$

$$\Rightarrow \underline{\frac{LC}{2}\ddot{v} + \left(\frac{L}{2R} + \frac{CR}{2}\right)\dot{v} + v = \frac{e}{2}}$$

(b) Need to transform to D.B $\quad \frac{\ddot{y}}{\omega_n^2} + \frac{2c\dot{y}}{\omega_n} + y = \begin{cases} 0 & t<0 \\ x & t>0 \end{cases}$

$y = v + D \Rightarrow \frac{LC}{2}\ddot{y} + \left(\frac{L}{2R} + \frac{CR}{2}\right)\dot{y} + y = \frac{e}{2} + D$

This is right where $D = -\frac{e_0}{2}$ giving $x = \frac{e}{2} - \frac{e_0}{2}$

where e_0 is 10 V, $e = 4$ V $\Rightarrow D = -5$ V, $x = -3$ V.

$\omega_n = \sqrt{\frac{2}{LC}}$, $\quad \frac{2c}{\omega_n} = \left(\frac{L}{2R} + \frac{CR}{2}\right) \Rightarrow c = \frac{1}{\sqrt{2LC}}\left(\frac{L}{R} + CR\right)\frac{1}{2} = \frac{1}{\sqrt{8}}\left(\frac{1}{R}\sqrt{\frac{L}{C}} + R\sqrt{\frac{C}{L}}\right)$

(c) For v to fall below 2V in subsequent response with $y = v - 5$, $x = -3 \Rightarrow y/x$ rises above 1

ie need $c < 1$.

Put $c = 1 \Rightarrow \sqrt{8} = x + \frac{1}{x}$ where $x = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow x^2 - \sqrt{8}x + 1 = 0$

$$\Rightarrow x = \frac{\sqrt{8} \pm \sqrt{8-4}}{2} = \sqrt{2} \pm 1$$

For large or small x, $c \to \infty$

So for $c < 1$ we need $\underline{\sqrt{2} - 1 < \frac{1}{R}\sqrt{\frac{L}{C}} < \sqrt{2} + 1}$

The first part was done well, but relating the given inputs to the data book (part b) was not. This tended to give problems in part (c).

MPFS
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