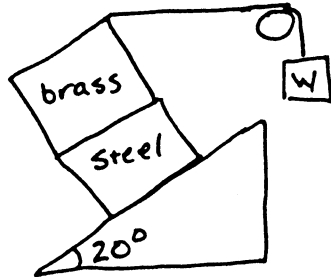


SECTION A: STRUCTURES

1)



FROM DATA BOOK

$$\rho_b = 8410 \text{ kg/m}^3$$

$$\rho_s = 7840 \text{ kg/m}^3$$

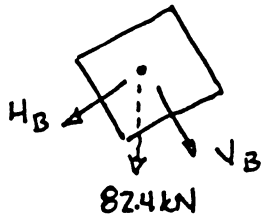
GIVEN

$$\mu_{b/s} = 0.3$$

$$\mu_{s/s} = 0.6$$

a)  $W=0$

i) FOR BRASS BLOCK : weight of block =  $8410 \times 1^3 \times 9.8$   
 $= 82418 \text{ N}$   
 $= 82.4 \text{ kN}$



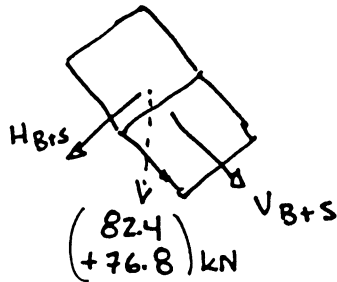
$$H_B = 82.4 \sin 20^\circ = 28.2 \text{ kN}$$

$$V_B = 82.4 \cos 20^\circ = 77.4 \text{ kN}$$

SLIDING OCCURS IF  $\frac{H_B}{V_B} > \mu_{b/s}$

$$\frac{H_B}{V_B} = \frac{28.2 \text{ kN}}{77.4 \text{ kN}} = 0.36 > 0.3 \quad \therefore \text{SLIDING OCCURS}$$

FOR STEEL BLOCK: WEIGHT OF block =  $7840 \times 1^3 \times 9.8$   
 $= 76832 \text{ N}$   
 $= 76.8 \text{ kN}$

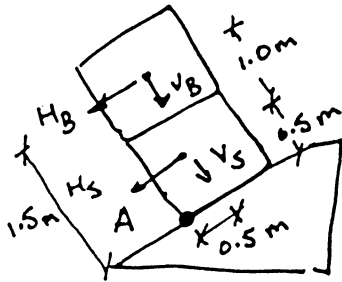


$$H_{B+s} = 159.2 \sin 20^\circ = 54.4 \text{ kN}$$

$$V_{B+s} = 159.2 \cos 20^\circ = 149.6 \text{ kN}$$

$$\frac{H_{B+s}}{V_{B+s}} = \frac{54.4 \text{ kN}}{149.6 \text{ kN}} = 0.36 < \mu_{s/s} = 0.6 \quad \therefore \text{NO SLIDING}$$

1 a) (i) OVERTURNING ABOUT A:



- FROM PART i)

$$H_B = 28.2 \text{ kN}, V_B = 77.4 \text{ kN}$$

$$- H_S = 76.8 \text{ kN} \sin 20^\circ = 26.3 \text{ kN}$$

$$V_S = 76.8 \text{ kN} \cos 20^\circ = 72.2 \text{ kN}$$

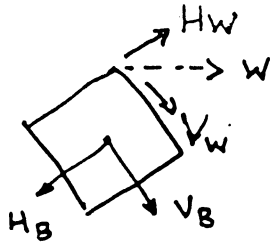
MOMENTS ABOUT A

$$H_B (1.5 \text{ m}) + H_S (0.5 \text{ m}) - V_B (0.5 \text{ m}) - V_S (0.5 \text{ m})$$

$$= 28.2 \text{ kN} \times 1.5 \text{ m} + 26.3 \text{ kN} \times 0.5 \text{ m} - 77.4 \text{ kN} \times 0.5 \text{ m} - 72.2 \text{ kN} \times 0.5 \text{ m}$$

$$m_A = -19.4 \text{ kN.m} \quad \therefore \underline{\underline{\text{NO OVERTURNING}}}$$

b)



MAXIMUM + MINIMUM VALUES OF W FOR SLIDING NOT TO OCCUR

FOR SLIDING DOWNWARDS  $\Rightarrow W_{\text{MIN}}$

$$H_W = W \cos 20^\circ$$

$$V_W = W \sin 20^\circ$$

AT POINT AT WHICH SLIDING OCCURS  $\mu_{B/S} N = \sum H$

$$\mu_{B/S} (V_B + V_W) = (H_B - H_W)$$

$$0.3 (77.4 \text{ kN} + W \sin 20^\circ) = (28.2 \text{ kN} - W \cos 20^\circ)$$

$$0.3 W \sin 20^\circ + W \cos 20^\circ = 28.2 \text{ kN} - 0.3 \times 77.4 \text{ kN}$$

$$W = \frac{4.98 \text{ kN}}{0.3 \sin 20^\circ + \cos 20^\circ} = \underline{\underline{4.78 \text{ kN}}} = \underline{\underline{W_{\text{MIN}}}}$$

FOR SLIDING UPWARDS  $\Rightarrow W_{\text{MAX}}$

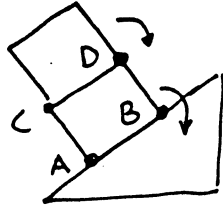
$$\mu_{B/S} (V_B + V_W) = (H_W - H_B)$$

$$0.3 (77.4 \text{ kN} + W \sin 20^\circ) = (W \cos 20^\circ - 28.2 \text{ kN})$$

$$W = \frac{0.3 \times 77.4 \text{ kN} + 28.2 \text{ kN}}{\cos 20^\circ - 0.3 \sin 20^\circ} = \underline{\underline{61.4 \text{ kN}}} = \underline{\underline{W_{\text{MAX}}}}$$

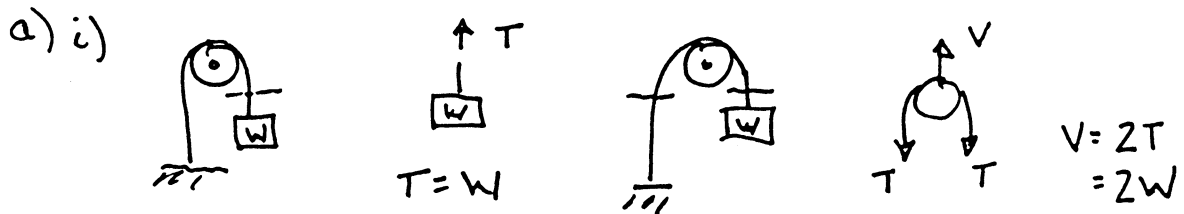
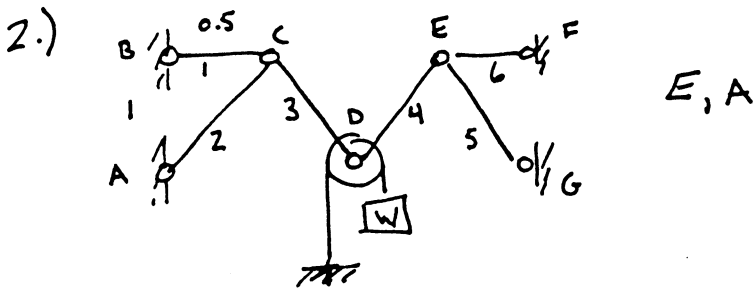
Engineering Tripos Part IA 1999 Paper 2: Structures and Materials - Cribs (3)

1) c) CHECK FOR OVERTURNING AT THE FOLLOWING LOCATIONS B, D!



SINCE BLOCKS DO NOT OVERTURN WHEN  $W=0$  DO NOT NEED TO CHECK OVERTURNING ABOUT A + C.

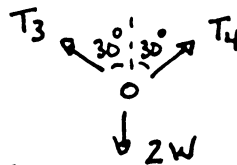
CRUSHING OF CORNERS MAY ALSO OCCUR.



FORCE ON D =  $2W$

(ii) SYMMETRIC - CONSIDER  $\frac{1}{2}$  THE STRUCTURE

- AT JOINT D

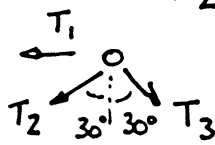


$$T_3 = T_4$$

$$(T_3 + T_4) \cos 30^\circ = 2W$$

$$T_3 = \frac{W}{\cos 30^\circ} = \frac{W \cdot 2}{\sqrt{3}}$$

- AT JOINT C



$$T_3 \cos 30^\circ = -T_2 \cos 30^\circ$$

$$T_2 = -T_3 = -\frac{W \cdot 2}{\sqrt{3}}$$

$$T_1 = T_3 \sin 30^\circ - T_2 \sin 30^\circ$$

$$= 2T_3 \cdot \frac{1}{2} = \frac{2W}{\sqrt{3}}$$

Q. 2

2 a) ii) cont'd

FROM SYMMETRY

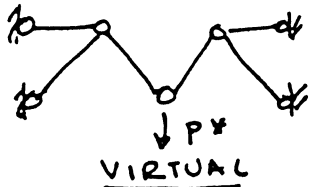
$$T_1 = T_6 = 2W/\sqrt{3}$$

$$T_2 = T_5 = -2W/\sqrt{3}$$

$$T_3 = T_4 = 2W/\sqrt{3}$$

b) i) DISPLACEMENT AT D  
USE VIRTUAL WORK

$$\underbrace{\sum P^* \cdot \delta}_{\text{virtual}} = \underbrace{\sum T^* \cdot e}_{\text{real}}$$



CHOOSE  $P^* = 2W$

BAR	LENGTH	$T_x^*$ (1)	$e \times (\frac{2W}{\sqrt{3}})$ (2)	$\frac{(1) \times (2)}{T^* e}$
BC	$L/2$	1	$1/2$	$4W^2L/6EA$
AC	$L$	-1	-1	$4W^2L/3EA$
CD	$L$	1	1	$4W^2L/3EA$

CONSIDER 1/2 THE STRUCTURE

$$\sum P^* \cdot \delta = \sum T^* \cdot e$$

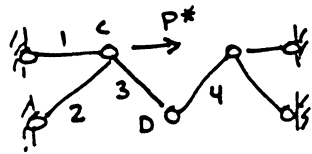
$$\delta_{VD} = 2 \cdot \frac{20W^2L}{6EA} \cdot \frac{1}{2W} = \frac{10}{3} \frac{WL}{EA}$$

FOR ENTIRE STRUCTURE

$$\delta_{HD} = 0$$

$$\frac{4W^2L/6EA + 4W^2L/3EA + 4W^2L/3EA}{1/2 \text{ THE STRUCTURE}} = \frac{20W^2L}{6EA}$$

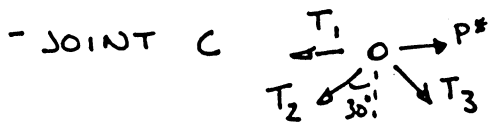
ii) DISPLACEMENT AT C HORIZONTALLY



LOAD AT D = 0

$T_3$  &  $T_4$  CANNOT BALANCE UNLESS

$$T_3 = T_4 = 0 \quad T_3 \swarrow \quad T_4 \nearrow$$



$$T_3 = 0, \quad T_2 \cos 30^\circ + T_3 \cos 30^\circ = 0 \quad \therefore T_2 = 0$$

LET  $P^* = 1 \Rightarrow T_1 = 1$

$$T_1 = P^*$$

BAR	LENGTH	$T^*$
BC	$L/2$	1

$$e \times (\frac{2W}{\sqrt{3}} \frac{L}{EA}) \quad 1/2$$

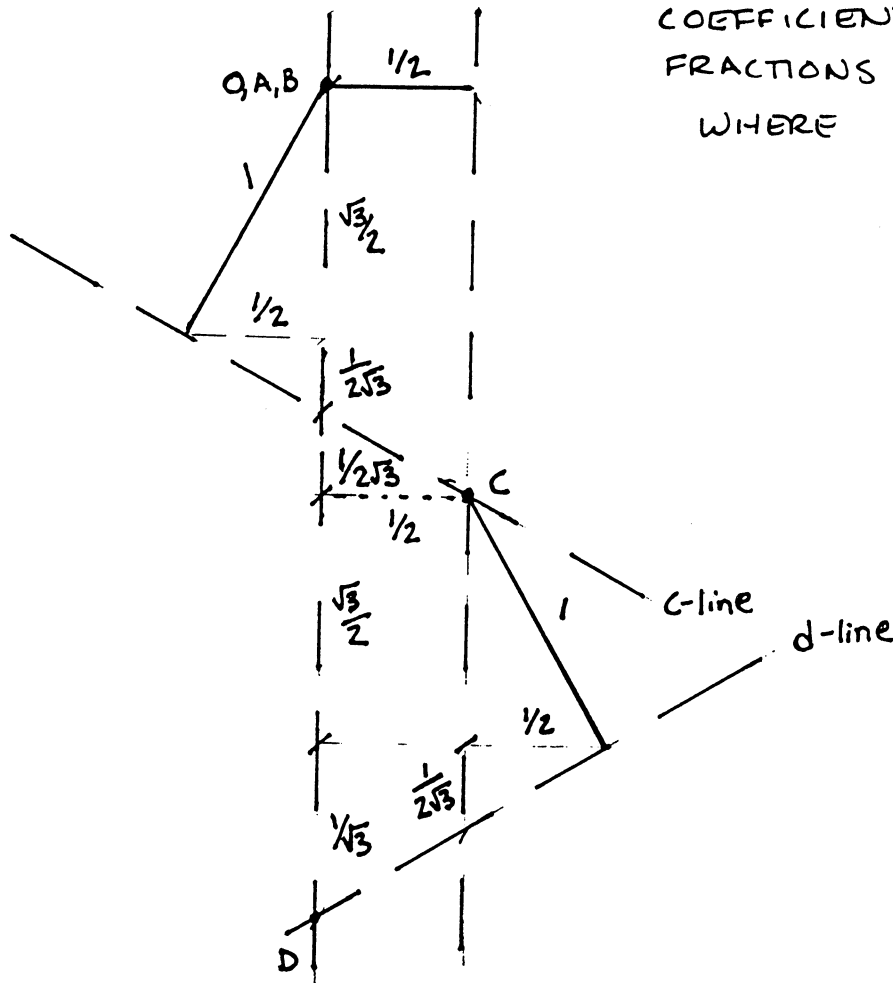
$$\frac{T^* e}{WL/\sqrt{3}EA}$$

$$\sum P^* \cdot \delta = \sum T^* \cdot e \quad 1 \cdot \delta_{HC} = WL/\sqrt{3}EA$$

THE CHANGE IN HORIZONTAL DISTANCE =  $2\delta_{HC} = \frac{2WL}{\sqrt{3}EA}$

2b) cont'd

- ALTERNATE SOLUTION USING A DISPLACEMENT DIAGRAM - DISPLACEMENT AT D + C



COEFFICIENTS ARE FRACTIONS OF  $\delta$  WHERE  $\delta = \frac{2WL}{\sqrt{3}EA}$

$$\delta_{VD} = \left[ 2 \left( \frac{\sqrt{3}}{2} \right) + \frac{1}{\sqrt{3}} + 2 \left( \frac{1}{2\sqrt{3}} \right) \right] \delta$$

$$= \left[ \sqrt{3} + \frac{2}{\sqrt{3}} \right] \delta = \left( \frac{5}{\sqrt{3}} \right) \left( \frac{2WL}{\sqrt{3}EA} \right) = \underline{\underline{\frac{10WL}{3EA}}}$$

$$\delta_{HD} = 0$$

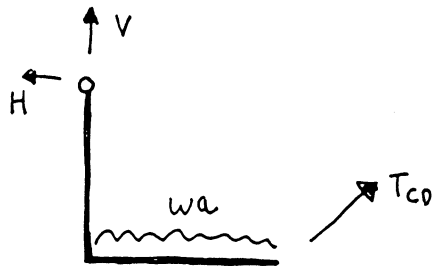
$$\delta_{HC} = \frac{1}{2} \delta = \left( \frac{1}{2} \right) \left( \frac{2WL}{\sqrt{3}EA} \right) = \frac{WL}{\sqrt{3}EA}$$

CHANGE IN HORIZONTAL DISTANCE BETWEEN C & E =  $2\delta_{HC} = \underline{\underline{\frac{2WL}{\sqrt{3}EA}}}$

- C) - STRUCTURE BECOMES STATICALLY INDETERMINATE  
 - CAN PROCEED BY CONSIDERING COMPATIBILITY CONDITIONS + MATERIAL PROPERTIES

Engineering Tripos Part IA 1999 Paper 2: Structures and Materials - Cribs (6)

3(a) FBD for ABC



Moments about A :

$$wa \cdot \frac{a}{2} = T_{cd} \cdot a\sqrt{2}$$

$$T_{cd} = \frac{\sqrt{2} wa}{4}$$

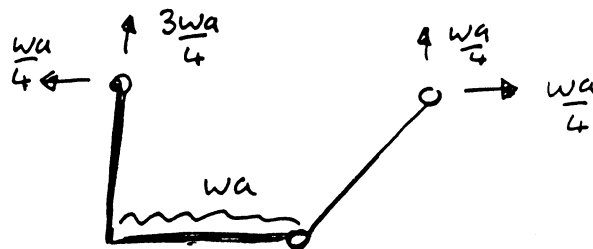
Horiz. equil :

$$H = \frac{T_{cd}}{\sqrt{2}} = \frac{wa}{4}$$

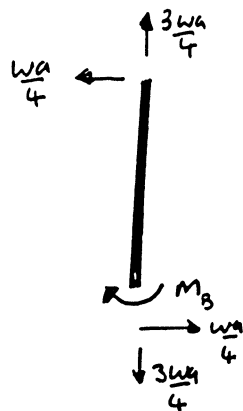
Vertical equil :

$$V = wa - \frac{T_{cd}}{\sqrt{2}} = \frac{3wa}{4}$$

Reactions



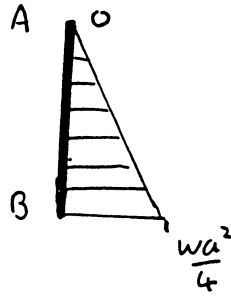
(b) FBD for AB



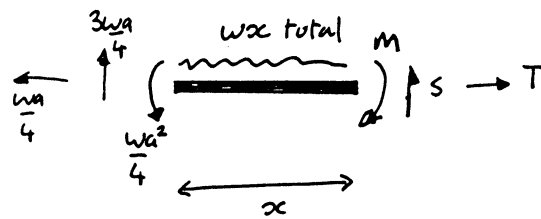
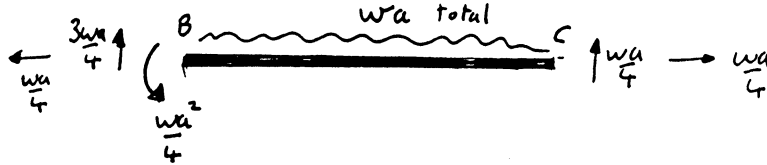
$$M_B = \frac{wa}{4} \cdot a = \frac{wa^2}{4}$$

3(b) cont.

BMD for AB, moments plotted on tension side of frame



(c) FBD for BC, cut at  $x$  from B



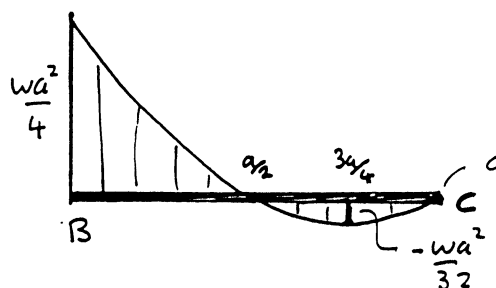
$$S = -\frac{3wa}{4} + wx \quad \left( = \frac{wa}{4} \text{ at } x = L \checkmark \right)$$

$$M = \frac{wx^2}{2} - \frac{3wax}{4} + \frac{wa^2}{4} \quad \left( = 0 \text{ at } x = L \checkmark \right)$$

$$S = 0 \text{ at } x = \frac{3a}{4}, \quad M = -\frac{wa^2}{32}$$

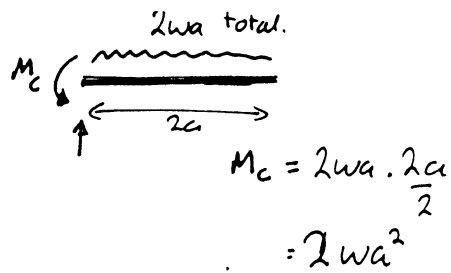
$$M = 0 \text{ at } x = \frac{a}{2}$$

BMD for BC, moments plotted on tension side of frame.

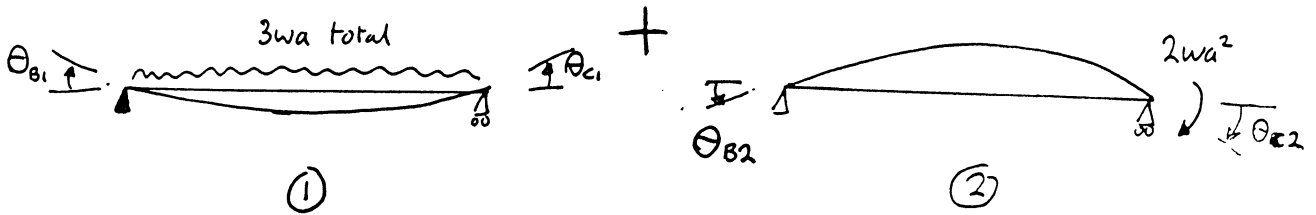


Engineering Tripos Part IA 1999 Paper 2: Structures and Materials - Cribs (8)

4(a) FBD for CD



(b) Consider BC with 2 loading cases applied



From Data Book:

$$\theta_{B1} = \theta_{C1} = \frac{(3wa)(3a)^2}{24EI} = \frac{27}{24} \frac{wa^3}{EI}$$

in directions shown.

$$\theta_{C2} = \frac{(2wa^2)(3a)}{3EI} = \frac{48}{24} \frac{wa^3}{EI}$$

$$\theta_{B2} = \frac{\theta_{C2}}{2} = \frac{24}{24} \frac{wa^3}{EI}$$

Combine rotations, using clockwise +ve

$$\theta_B = \theta_{B1} - \theta_{B2} = \underline{\underline{\frac{3}{24} \frac{wa^3}{EI}}}$$

$$\theta_C = \theta_{C2} - \theta_{C1} = \underline{\underline{\frac{21}{24} \frac{wa^3}{EI}}}$$

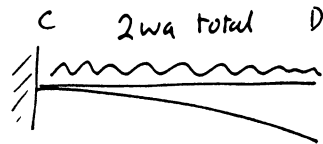


Engineering Tripos Part IA 1999 Paper 2: Structures and Materials - Cribs (9)

4 (cont.) (c)

$$\text{Defln. at A} = \theta_B \times a = \frac{3}{24} \frac{wa^4}{EI} \quad \uparrow$$

Defln at D =  $\theta_C \times a$  + Defln of cantilever



$$\begin{aligned} \delta_{D3} &= \frac{(2wa)(2a)^3}{8EI} \\ &= \frac{48}{24} \frac{wa^4}{EI} \end{aligned}$$

$$\begin{aligned} \therefore \text{Defln at D} &= \frac{21}{24} \frac{wa^3}{EI} \times 2a + \frac{48}{24} \frac{wa^4}{EI} \\ &= \frac{90}{24} \frac{wa^4}{EI} \quad \downarrow \end{aligned}$$

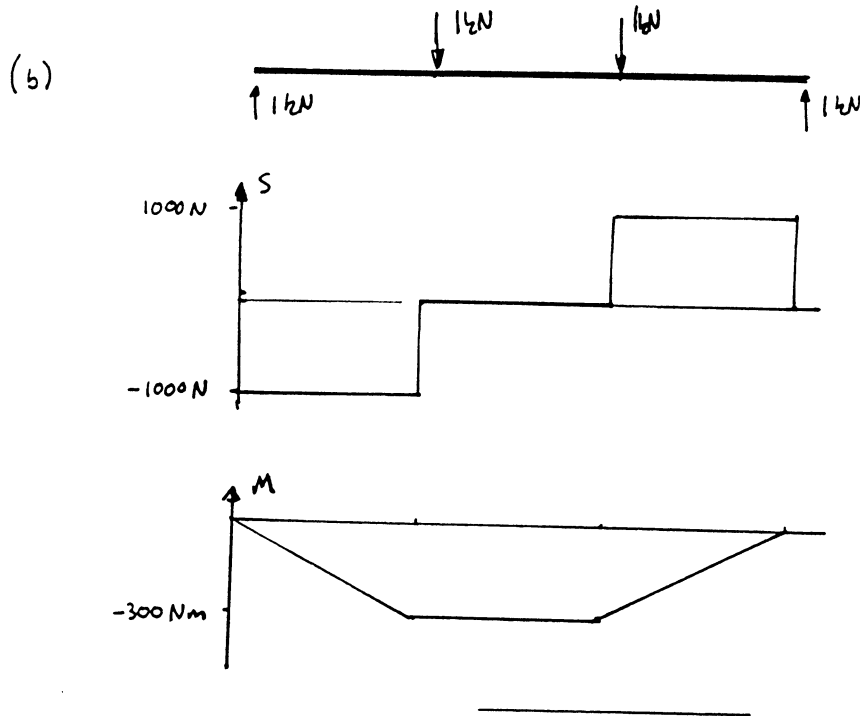
Engineering Tripos Part IA 1999 Paper 2: Structures and Materials - Cribs (10)

5(a) Cross-sectional area =  $0.12 \times 0.1 - 0.08 \times 0.08$   
=  $5.6 \times 10^{-3} \text{ m}^2$

Volume of wood used =  $5.6 \times 10^{-3} \text{ m}^2 \times 1 \text{ m}$   
=  $5.6 \times 10^{-3} \text{ m}^3$

$\rho_{\text{softwood}} \approx 800 \frac{\text{kg}}{\text{m}^3} \Rightarrow \text{Mass} \approx 5.6 \times 10^3 \times 800 = 4.5 \text{ kg}$

$\therefore$  Weight  $\approx 45 \text{ N}$  total, negligible compared with  $2000 \text{ N}$  applied load.



(c) Bending stress, use  $\sigma = \frac{My}{I}$

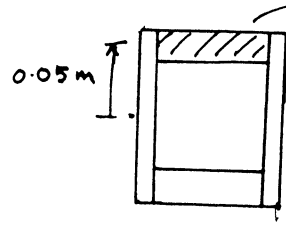
$$I = \frac{0.1 \times 0.12^3}{12} - \frac{0.08 \times 0.08^3}{12} = 10.98 \times 10^{-6} \text{ m}^4$$

$$y_{\text{max}} = 0.06 \text{ m}, \quad M_{\text{max}} = 300 \text{ Nm}$$

$$\sigma_{\text{max}} = \frac{300 \times 0.06}{10.98 \times 10^{-6}} = \underline{\underline{1.64 \times 10^6 \text{ N/m}^2}}$$

Engineering Tripos Part IA 1999 Paper 2: Structures and Materials - Cribs (11)

(d) 'Cut out' section connected by nails.



$$\text{Area} = 0.08 \times 0.02 = 1.6 \times 10^{-3} \text{ m}^2$$

$$\bar{y} = 0.05 \text{ m}$$

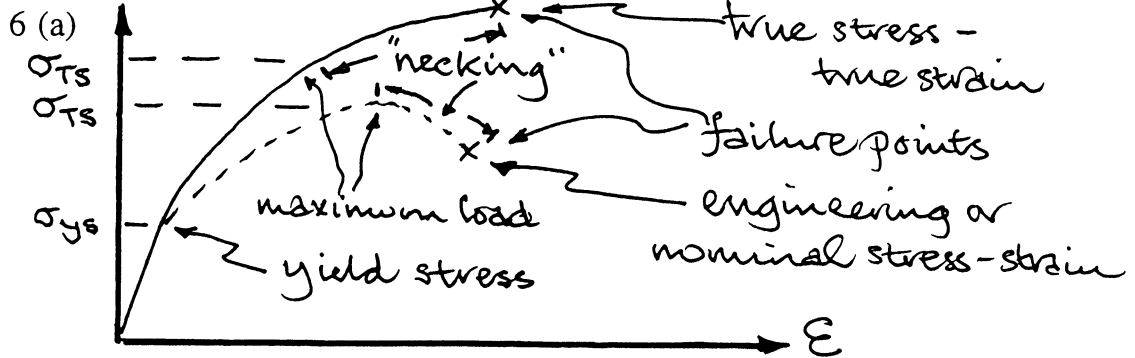
$$A\bar{y} = 80 \times 10^{-6} \text{ m}^3$$

$$\begin{aligned} \text{Force carried by nails / unit length} &= \frac{F A \bar{y}}{I} \quad (F_{\text{max}} = 1000 \text{ N}) \\ &= 7286 \text{ N} \end{aligned}$$

In 1m, this is carried by  $\frac{1000 \times 2}{50} = 40$  nails

$$\text{force}_{\text{nail}} = \underline{\underline{182 \text{ N}}}$$

SECTION B MATERIALS



By definition:

$$\epsilon_t = \ln_e(A_0/A)$$

By definition:

$$\sigma_{TS} = \sigma_t \text{ (at necking)} \times (A / A_0).$$

6 (b) Given:

$$\sigma_t = \sigma_0(B + \epsilon_t)^n$$

$$\begin{aligned} \text{Therefore: } d\sigma_t/d\epsilon_t &= n\sigma_0(B + \epsilon_t)^{n-1} = n\sigma_0(B + \epsilon_t)^n / (B + \epsilon_t) \\ &= n\sigma_t / (B + \epsilon_t) \end{aligned}$$

$$\text{Now: } d\sigma_t/d\epsilon_t = \sigma$$

$$\text{Hence: } n\sigma_t / (B + \epsilon_t) = \sigma_t$$

$$\text{Therefore: } \epsilon_t = n - B \dots \dots \dots \text{at necking.}$$

$$\text{By definition: } \sigma_{TS} = \sigma_t \text{ (at necking)} \times (A / A_0).$$

$$\text{Now: } \sigma_t = \sigma_0(n)^n \quad (\text{where } n = (B + \epsilon_t))$$

$$\text{Therefore: } \sigma_{TS} = \sigma_0(n)^n (A/A_0) \quad (\text{at necking})$$

$$\text{By definition: } \epsilon_t = \ln_e(A_0/A)$$

$$\text{Therefore: } (A_0/A) = \exp(\epsilon_t) = \exp(n - B)$$

$$\text{Therefore: } \sigma_{TS} = \sigma_0(n)^n \exp(n - B)$$

$$\text{Therefore: } \sigma_{TS} = \sigma_0(n/e)^n e^B$$

$$\text{At yielding: } \epsilon_t = 0.$$

$$\text{Therefore: } \sigma_{ys} = \sigma_0(B)^n$$

$$\text{Recall: } \sigma_{TS} = \sigma_0(n/e)^n e^B$$

$$\text{Taking: } \sigma_{ys} = \sigma_{TS} / 2 \quad (\text{given in qu.})$$

$$\text{Therefore: } \sigma_0(B)^n = 1/2 (\sigma_0(n/e)^n e^B)$$

$$\text{For: } n = 0.5 \text{ (given): } B^{1/2} = 1/2 (1/2e)^{1/2} e^B$$

$$\text{Therefore: } 1/2 (\ln B) = \ln(1/2) - 1/2 (\ln 2e) + B$$

$$\text{This gives: } B = 1.86.$$

7 A ligament of thickness  $t$  bent elastically to a radius  $R$  has a surface strain  $\epsilon = t/2R$  and a corresponding maximum stress  $\sigma = E(t/2R)$  which must not exceed the yield stress  $\sigma_y$  of material. The radius  $R$  to which the ligament can be bent without yielding must satisfy  $R \geq (t/2) (E/\sigma_y)$ . The best material is the one that can be bent to the *smallest* radius without yielding; therefore, the best choice is the one with greatest value of the merit index  $M$  (for a given  $t$ ):  $M = \sigma_y/E$ .

The selection chart (see next page) is used by superimposing this guideline of slope = 1. Best choices include **PE, PP, Nylon, elastomers**; the latter appear superior but may be too flexible and, therefore, a minimum modulus requirement is defined by the horizontal line on the chart. Spring steel is possible which can be used when high stiffness is required. Polymers give more design freedom than metals and are cheap and easy to mould in one operation. Their spring-like properties offer opportunities that can be exploited; eg, parts easily snap together.

*Fatigue* can be a problem and lead to premature failure. An additional failure criterion based on *endurance limit* can be used and appropriate index chosen.

8 (a) The concentration of elastic tensile stress at a notch is expressed analytically in terms of an elastic stress concentration factor

$$K_t = \sigma_{\text{local}}/\sigma_{\text{nominal}}$$

which is the ratio of the maximum local stress near the discontinuity to the nominal stress acting across the gross cross section of the component. The elastic stress distribution depends on the shape (and size) of the notch. The simplest case is that of a hole where at the tip of the hole

$$K_t = 1 + 2 (a/\rho)^{1/2} = 3.$$

$a$  is notch length and  $\rho$  is notch tip radius so that  $K_t$  increases as a notch becomes longer and sharper.  $K_t$  decreases with increasing distance from the notch tip.

The stress intensity factor  $K$  can be used to predict the critical values of stress and crack length at which a crack will propagate in an unstable manner. In this context

$$K = \sigma_{\text{nominal}} (\pi a)^{1/2} \quad (K = K_c \text{ when } \sigma_{\text{nominal}} = \sigma_f).$$

We then consider fracture occurs when  $K$  is equal to or greater than the critical stress intensity factor  $K_c$  (or  $K_{Ic}$  usually called the fracture toughness of the material). The local tensile stresses near the crack tip can be described by

$$\sigma_{\text{local}} = K/(2\pi r)^{1/2}$$

where  $r$  is distance ahead of the crack tip in the plane of the crack. A comparison between the equations for  $K_t$  and  $K$  indicates that  $K$  is proportional to the limiting value of  $K_t$  as the root radius  $\rho$  of the notch tip approaches zero.

Question 7 Fig. 7 To be included as part of your answer

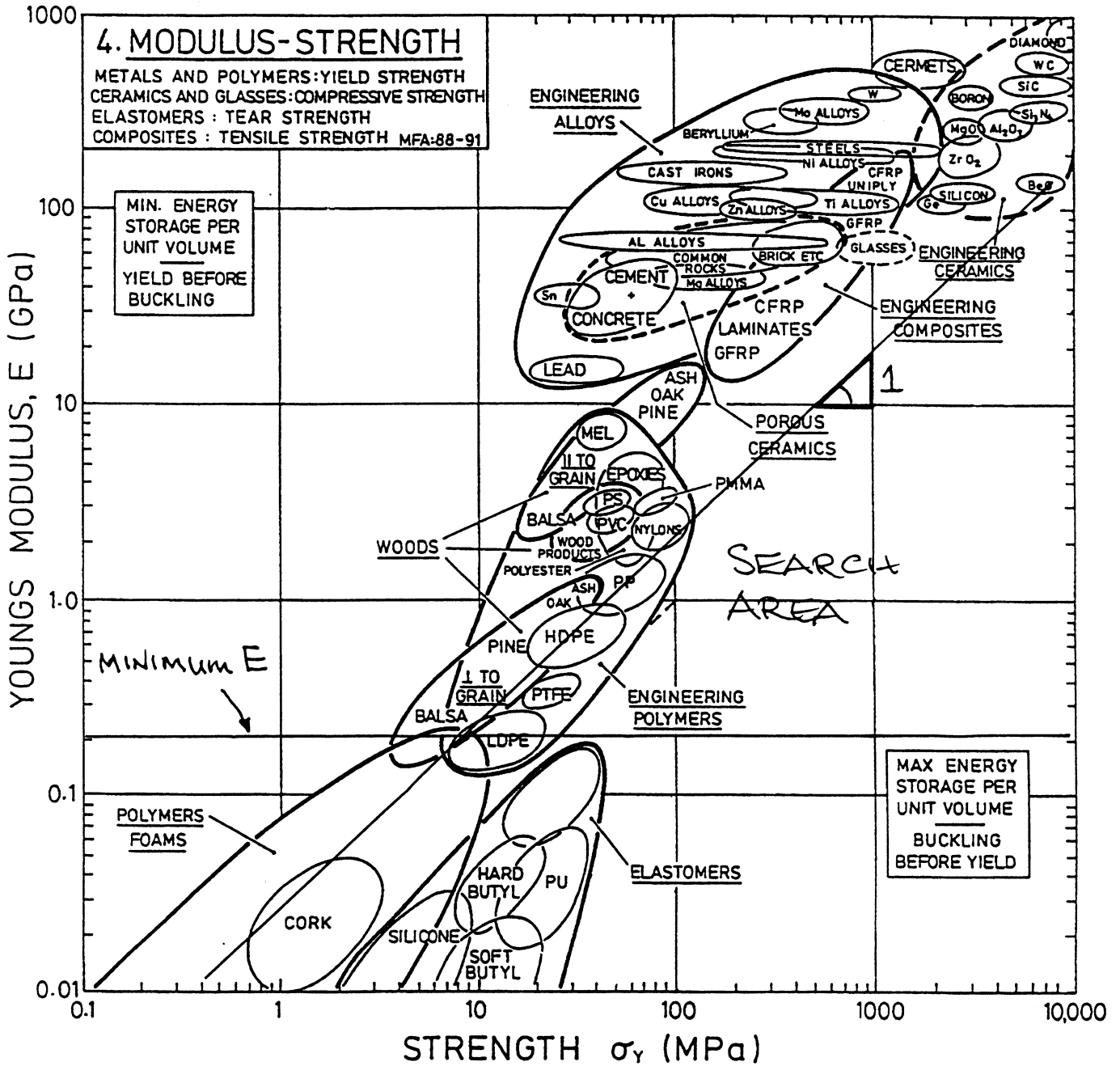


Fig. 7 A Materials Selection Chart

8 (b) **Step 1:**

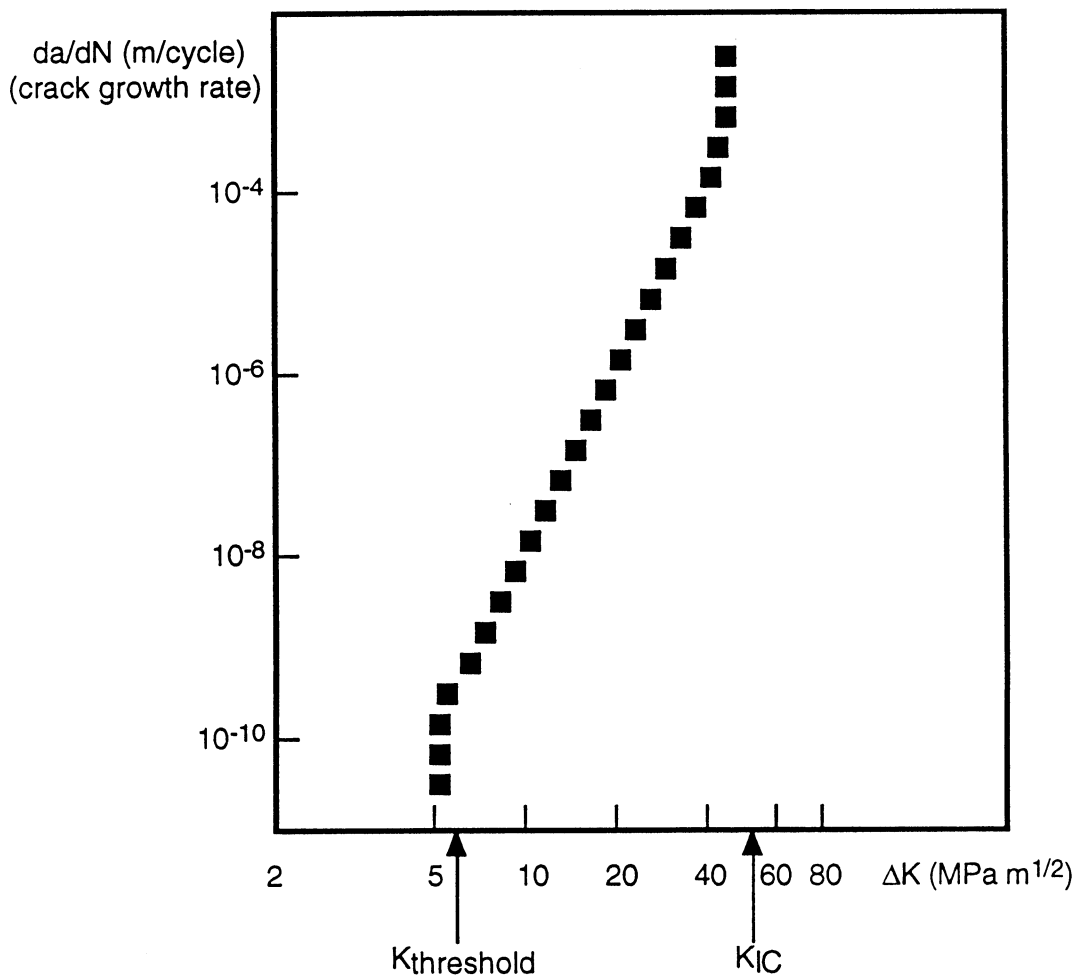
a starting point is to find out if there have been any service failures during the past 20 years (you are told there have been none reported).

**Step 2:**

recall the steel's endurance fatigue limit is roughly half its yield strength, i.e., 150 MPa. Since the cyclic stress is  $\pm 25$ MPa, **the likelihood of a crack nucleating elsewhere away from any pre-existing flaw is negligible.**

**Step 3:**

proceed with fracture mechanics calculation to establish a safe allowable flaw size, then base a new inspection criterion on this calculation. Since the fatigue life is 30 years (more than 10 billion cycles), pre-existing cracks must have a near zero crack growth rate and not be allowed to grow. This implies that  **$\Delta K$  level must be less than the threshold value  $\Delta K_{(threshold)}$ .** An estimate of the  $K$  threshold for this material is obtained by experiment. The figure below defines  $\Delta K_{(threshold)}$  and  $K_{IC}$ .



The value of  $\Delta K_{(\text{threshold})}$  is  $6 \text{ MPam}^{1/2}$ ; that of  $K_{IC}$   $50 \text{ MPam}^{1/2}$ , approximately; the former denotes the limit level below which crack growth is effectively zero (it is undetectable) and the latter is the value of  $K$  at which fast (unstable) crack growth occurs.

**Step 4:**

insert  $\Delta\sigma = 50 \text{ MPa}$  and  $\Delta K_{(\text{threshold})} = 6 \text{ MPam}^{1/2}$  into the FM equation and solve for "a" which turns out to be  $3.65 \text{ mm}$ . Hence **flaws that are 3.5 times deeper than used in the current inspection criterion (1mm) will not lead to failures of the shaft.** Since defects have never been known to exceed  $2 \text{ mm}$  (given), the calculation shows not only that all rejected shafts would be acceptable but also that there is no need to inspect all the shafts. **The recommendation should be to inspect only one shaft out of several to ensure that some checks are maintained on the process and to accept all shafts that have cracks up to 3mm deep.** The machining allowance of  $1 \text{ mm}$  ensures that an initial  $3 \text{ mm}$  crack will never be larger than  $2 \text{ mm}$  deep after machining. This provides additional safety against service failures.

9 (a) When a moving dislocation encounters precipitates it will not in general cut through them because precipitates are generally stronger than the matrix. Consequently, the dislocation bows between neighbouring precipitates and around them. This creates dislocation loops around precipitates and effectively shortens the distance between them with the passage of successive dislocations. This is a source of work hardening. The stress for dislocation bowing is inversely dependent on the spacing between precipitates. High stresses can be involved in dislocation bowing and the material exhibits a high yield strength.

The growth of a precipitate at elevated temperature is a diffusion-controlled process and hence is time and temperature sensitive. (An example is the age hardening of Al alloys). Coarsening of precipitates is accompanied by an increase in spacing between neighbouring precipitates and the yield strength can fall with time and increasing temperature. Rogue (much larger) spacings can result in parts of the microstructure allowing a dislocation to "escape" more easily and the yield strength is therefore less than might have been expected assuming a uniform distribution of precipitate spacings. A sketch showing dislocation bowing and the formation of loops around precipitates would be useful.

9 (b)  $\sigma_y = \sigma_0 + 2(2Gb/l)$  ( $2\tau_y = \sigma_y$ , approximately)

$\langle l \rangle = l + d$       or  $l = \langle l \rangle - d$ ;      therefore  $l = 0.5 \times 10^{-6} \text{ m}$ .

$$\sigma_y = (175 \times 10^6) + \frac{2(2 \times 105 \times 10^9 \times 3 \times 10^{-10})}{(0.5 \times 10^{-6})}$$

$$= 175 + 2(123)$$

$$= (175 + 246) \text{ MPa.}$$

$$\sigma_y = \underline{421 \text{ MPa.}}$$



9 (c) (i) Service temperature is an important variable for many steels and bcc materials. For example, a particular steel at moderate temperature where the fracture load is greater than the yield load, it exhibits some ductility. At low temperature, the fracture load can be much less than the yield load because the steel has become notch-sensitive and any small discontinuity (and associated stress concentration) can trigger catastrophic (elastic) fracture with little or no ductility. The steel can be extremely brittle. **Below a ductile-brittle transition temperature (in practice, a narrow band), the steel can be unsafe and above that temperature they may be safely used.**

(ii) The stress at which the low C steel fractures in a brittle manner increases linearly with temperature and at 200 K is given by:

$$1190 = A + 200B.$$

Now by definition,  $\sigma_f = \sigma_y$  @  $T_D$  and  $\sigma_y$  varies with temperature according to:

$$\sigma_y = (1400 - 3.6 T) \text{ (given).}$$

For this particular steel the  $T_D = 100$  K (which is also given).

$$\text{i.e., } \sigma_y = \sigma_f = 1400 - 3.6 (100) = 1040 \text{ MPa.}$$

Thus:  $1190 = A + 200B$  and  $1040 = A + 100B$ .

Solving for A and B gives  $A = 890$  and  $B = 1.5$ .

Therefore:  $\sigma_f = 890 + 1.5T$ .

By precipitation strengthening, if  $\sigma_y$  is raised by 245 MPa, then

$$\sigma_y = 1645 - 3.6T.$$

Therefore the new  $T_D$  is where  $1645 - 3.6T = 890 + 1.5T$  which is **148 K**. Alternatively, a graphical construction of  $\sigma_y$  and  $\sigma_f$  versus T and the intersection of these 2 straight lines at  $\sigma_y = \sigma_f$  gives the  $T_D$ .

10 (a) Basically, there are 2 types: low temperature creep (less than  $0.5T_{\text{melting point}}$ ) in which the plastic strain varies with time logarithmically and high temperature creep (above  $0.5T_{\text{melting point}}$ ) or steady state creep in which the plastic strain varies linearly with time after an initial transient stage. (A sketch could be shown). In high temperature creep, as plastic flow occurs, dislocations move and multiply (dislocation density goes up) and through interactions cause work or strain hardening. This causes the creep rate to diminish. Recovery processes such as dislocation annihilation and rearrangements counteract the effects of strain hardening, i.e., recovery mechanisms cause the creep rate to accelerate. In steady state creep, strain hardening effects and softening effects of recovery are exactly compensated.

Dislocation rearrangements and a decrease in dislocation density involves mechanisms like dislocation climb around, for example, precipitates which is diffusion controlled and involves an activation energy  $Q$  for the steady state creep process. Because steady state creep is proportional to the diffusion coefficient  $D$ , it follows that the variation in creep rate with temperature is dictated by the temperature dependence of  $D$  by an Arrhenius type of relation involving  $Q$ .

Creep resistant alloys are those materials for which the coefficient of diffusion  $D$  is small. Examples include: high melting point materials (oxides, refractory metals, Ni and Co based superalloys). Precipitation and dispersion strengthened alloys are creep resistant. All dislocation strengthening mechanisms can contribute to high temperature strength; solid solution strengthening, grain boundary strengthening. Specifically this means that when the material is strengthened by, for example, a precipitation process, the spacing between the dislocation defects must remain small if the process is to remain effective. Dispersed second phase particles of high melting point that are insoluble in the matrix do not coarsen and are most effective in reducing creep.

In addition, grain boundary (sliding) creep and diffusional creep (at  $T$  greater than  $0.9T_m$  and at stresses below the critical value for dislocation motion) contribute to high temperature creep. In the case of the former, deformation is inhomogeneous since it is all concentrated at the grain boundaries and can lead to cracking and rupture. The contribution of grain boundary sliding increases as the grain size decreases. Dislocation motion within the grains also can occur which prevents microvoids opening up. At very high temperature, the directional diffusion of vacancies (net flow or migration in one direction) can cause a macroscopic shape change. Sketches showing microstructural changes during creep would be useful.

$$10 \text{ (b)} \quad \text{Given:} \quad \sigma_0 = 70 \text{ MPa} \quad T_0 = 1000 \text{ C (1273 K)}$$

$$\sigma_1 = 80 \text{ MPa.} \quad T_1 = ?$$

For new process:

$$\epsilon_1 = A\sigma_1^5 \exp(-\Delta Q/RT_0) = A\sigma_0^5 \exp(-\Delta Q/RT_1)$$

$$5 \ln (\sigma_1/\sigma_0) = (\Delta Q/R)[(1/T_0) - (1/T_1)]$$

Rearranging and solving for  $T_1$ :

$$\{(1/T_0) - (1/T_1)\} = (5R/\Delta Q) \ln (\sigma_1/\sigma_0)$$

$$(1/T_1) = (1/T_0) - (5R/\Delta Q) \ln (\sigma_1/\sigma_0)$$

Hence:  $T_1 = 1308 \text{ K} = 1035 \text{ C}$ , an increase in 35 C.

END