ENGINEERING TRIPOS PART IA

Monday 7 June 1999 9 to 12

Paper 1

MECHANICAL ENGINEERING

Answer not more than **eight** *questions, of which not more than* **three** *may be taken from Section A, not more than* **three** *from Section B and not more than* **two** *from Section C.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer not more than **three** *questions from this section.*

1 The specific internal energy of a gas, *which is not perfect*, is given by

$$u = a + bpv,$$

where p is the pressure and v the specific volume and a and b are positive constants. The gas executes the following fully resisted cycle in a cylinder:

(i) volume held constant while the pressure rises, the initial state having pressure p_1 and specific volume v_1,

(ii) pressure held constant while the volume rises, the final state having pressure p_3 and specific volume v_3,

(iii) volume held constant while the pressure falls,

(iv) pressure held constant while the volume falls and the state returns to p_1, v_1.

(a) Sketch the cycle on a p-v diagram and indicate against each of the four processes whether work is being done by the gas ($W+$), on the gas ($W-$) or there is no work ($W0$). Indicate also against each process the heat transfer and its direction (Q+ for transfer to the gas, Q- from the gas or Q0). [8]

(b) Find expressions in terms of p_1, v_1, p_3, v_3 and b for the heat transfer per kg of gas in those processes where the transfer is *to* the gas. Determine the thermal efficiency of the cycle. [6]

(c) If the cycle were to be executed reversibly in a steady flow plant with the following components

(i) a compressor with heat transfer such that the specific volume is held constant,

(ii) a constant-pressure heater,

(iii) a turbine with heat transfer such that the specific volume is held constant,

(iv) a constant-pressure cooler,

determine the shaft-work and heat exchanges per kg of gas for the components where heat transfer is *to* the gas. [6]

2 (a) Sketch the enthalpy-entropy diagram for superheated and wet steam, showing the saturation and triple-point lines and the position of the critical point. Draw typical isobars and isotherms on the sketch, ensuring that the representation of their slopes is qualitatively correct. Explain why the isobars have the shape which you have shown. [8]

(b) Saturated hot water at $20\ \text{MN/m}^2$ is delivered from a pipeline via a throttle to an initially evacuated vessel of volume $0.1\,\text{m}^3$. The process is adiabatic and the state in the vessel at any instant is uniform.

(i) Show that during this charging process, the specific internal energy of the contents of the vessel is constant and equal to the specific enthalpy in the pipeline. [6]

(ii) Determine how much water will have been delivered to the vessel when the pressure in it reaches $8\ \text{MN/m}^2$. [6]

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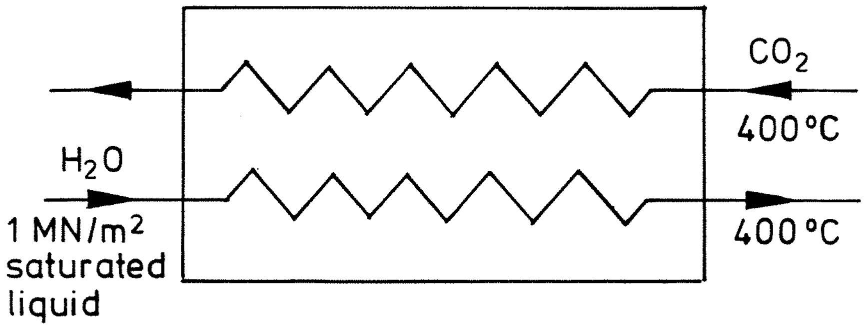

Fig. 1

Figure 1 shows diagramatically a counter-flow heat exchanger which is a component in a power plant where carbon dioxide is used as a heat-transfer medium between a heat source and the working substance of a steam power plant. Steam is raised from the saturated liquid state at a constant pressure of 1 MN/m^2 and superheated to 400°C. The carbon dioxide, which may be treated as a perfect gas, is cooled at constant pressure from 400°C to the temperature of the saturated liquid water. Overall heat loss from the exchanger may be neglected.

(a) Sketch on one diagram the distribution of temperature along the flow passages for the two streams in the heat exchanger. Determine the mass of CO_2 cooled for every kg of steam raised. Explain why a process of pure heat transfer between CO_2 and evaporating water cannot be reversible. [8]

(b) An inventor claims that he can replace the heat exchanger with a device which is reversible, as well as having no overall heat loss and no overall work exchange. Of the inlet and outlet states only the temperature of the CO_2 at outlet would be changed. Show that this temperature would be approximately 311.6 K and the mass of CO_2 cooled for every kg of steam raised under these circumstances would be approximately 8.44 kg. [8]

(c) Devise a scheme for implementing the inventor's device using ideal heat engines and pumps. [4]

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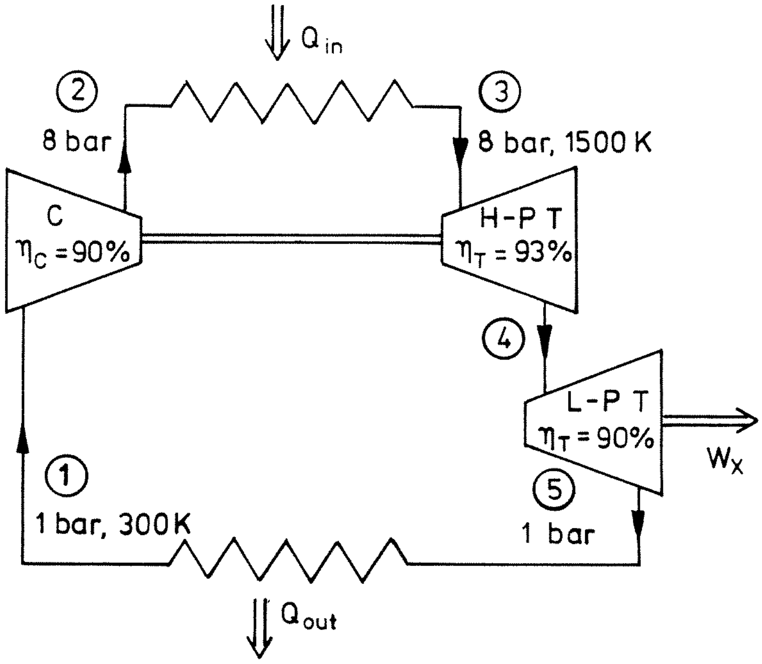

Fig. 2

Figure 2 is a block diagram for a closed-cycle gas-turbine plant, having separate high-pressure and low-pressure turbines. The overall work output from the plant is provided by the low-pressure turbine of isentropic efficiency 90%, while the high-pressure turbine, of isentropic efficiency 93%, drives the compressor. The single compressor has an isentropic efficiency 90%. The working substance of the plant is air, which may be treated as a perfect gas, and its state at various points is as shown on the figure. The compressor and the turbines are adiabatic.

(a) Sketch the cycle on a T-S diagram. [4]

(b) Determine

(i) the temperature rise in the compressor, [4]

(ii) the temperature and pressure at state 4 between the high-pressure and low-pressure turbines, [6]

(iii) the temperature drop in the low-pressure turbine, [3]

(iv) the thermal efficiency of the plant. [3]

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SECTION B

Answer not more than **three** *questions from this section.*

5 Figure 3 shows the mechanism of a mole wrench used to grip objects between its jaws. The upper handle and jaw assembly ABC is held fixed. The lower handle EFG is attached to the upper handle by a link AF, which is freely hinged at F. AF is connected to the upper handle at A by a frictionless pivot. The location of A can be adjusted in the horizontal direction along AB using the screw S. The lower jaw is attached to the upper and lower handles at B and E respectively by frictionless hinges. Two equal and collinear forces of 45 N are applied to the handles at A and G in a vertical direction as shown. The jaws grip a circular bar at points C and D with a force R acting in the direction shown. All links are rigid and light. Figure 3 is drawn full-size and sufficiently accurate answers can be obtained by measuring from this figure and any constructions. **A separate copy of Fig. 3 is provided and should be handed in with your answers.**

(a) In the first instance A is held fixed while the handles are forced together, deforming the circular bar such that D on the lower jaw closes with a speed of 0.03 mms^{-1}.

(i) In what direction does point D on the lower jaw move? [1]

(ii) Locate the instantaneous centres of BED and EFG on the separate copy of Fig. 3. Hence determine the angular velocities of BED, EFG and AF, and the magnitude and direction of the velocity of G. [9]

(iii) Find the magnitude of the gripping force R. [4]

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(b) Now joint A is moved horizontally by the screw S, while point G moves such that bar EFG does not rotate. D on the lower jaw continues to close at a speed of 0.03 mms^{-1}. Sketch the corresponding velocity diagram, including an approximate scale. An accurate diagram drawn to scale is **not** required. [6]

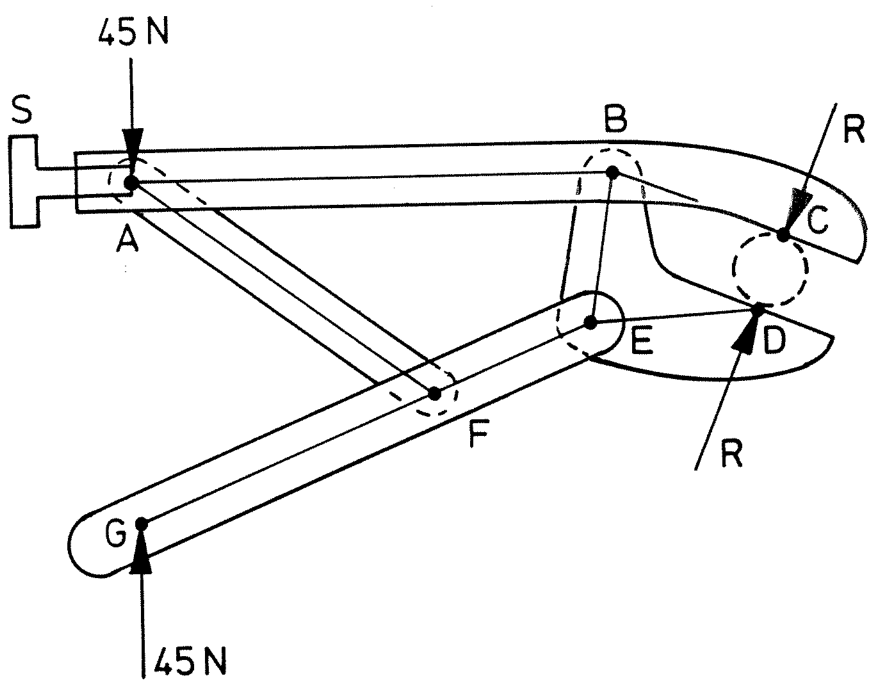

Fig. 3

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6 Figure 4 shows the cross section through a traction drive. The input and output discs 1 and 2 are solids of revolution, which rotate about the fixed axis of symmetry OO' with angular velocities $\underline{\Omega}_1$ and $\underline{\Omega}_2$ respectively. Relevant dimensions of the drive and unit vectors $\underline{i}$, $\underline{j}$, and $\underline{k}$ are given in Fig. 4. The conical faces of the discs are held in contact with a spherical ball of radius R. All the contact points A, B, D and E between the ball and the discs are at a distance d from the axis of the discs. The centre of the ball C is held fixed, but the ball is free to rotate with angular velocity $\underline{\omega}$. The axis of rotation of the ball lies in the $\underline{i}-\underline{j}$ plane and passes through C. This axis is inclined at angle α to the unit vector $\underline{j}$, so that it lies parallel to OA, as illustrated in Fig. 4.

(a) Derive **vector** expressions, in terms of d, R, α, ω, Ω_1, Ω_2 and the unit vectors, for:

(i) the absolute velocities of A, B and E,

(ii) the position and velocity of D relative to B. [8]

(b) Find expressions for the sliding velocity at the contacts between the ball and **each** of the discs. [5]

(c) Find an expression for the ratio of output to input angular speeds Ω_2/Ω_1 when there is no slip at either contact between the ball and discs. What value does Ω_2/Ω_1 take when $\alpha = 0$? [5]

(d) Suggest another mechanism that could be used to give different input and output shaft speeds. [2]

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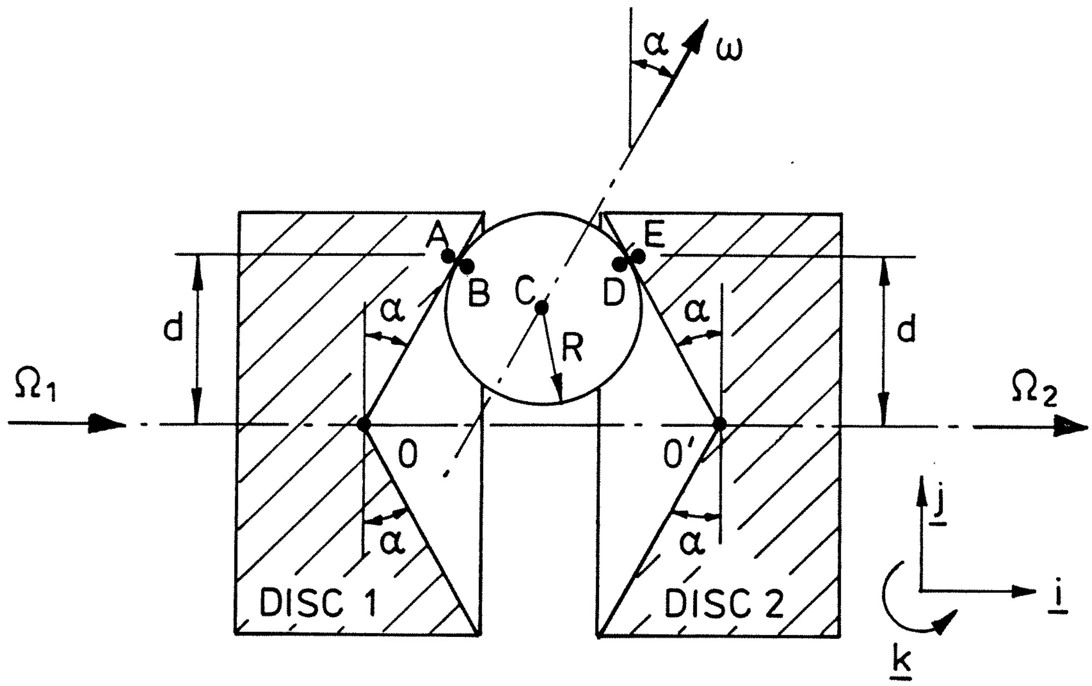

Fig. 4

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7 A particle of mass m moves in a plane such that its Cartesian co-ordinates of position x and y are given by the equations

$$x = \alpha\lambda ,$$
$$y = \alpha \cos\lambda ,$$

where α is a constant and λ varies with time.

(a) Show that the velocity $\underline{v}$ of the particle in cartesian co-ordinates is given by

$$\underline{v} = \alpha\dot{\lambda}\underline{i} - \alpha\dot{\lambda}\sin\lambda\ \underline{j} ,$$

where $\underline{i}$ and $\underline{j}$ are unit vectors in the x and y directions respectively. [3]

(b) Derive vector expressions in intrinsic co-ordinates for the velocity and acceleration of the particle, in terms of α, λ and time derivatives of λ. [10]

(c) As a particle follows the path described above, it is acted on by forces F and N applied in directions along and normal to the direction of travel, respectively.

(i) Derive expressions for the forces F and N which must be applied to maintain the particle on its path.

(ii) For the case where $\dot{\lambda}$ is constant and λ increases with time, find an expression for F in terms of N and λ. Sketch the path of the particle, indicating the magnitude and direction of the velocity of the particle and the resultant applied force when $\lambda = 0$. [7]

8 (a) A satellite of mass m moves in an orbit around the Earth under the action of a central force $F = \frac{GMm}{r^2}$, where r is the distance from the centre of the Earth to the satellite. Show carefully that the orbit satisfies the equation

$$\ddot{r} - \frac{h^2}{r^3} + \frac{GM}{r^2} = 0 \ ,$$

where h is a constant. [6]

(b) (i) A satellite of mass 800 kg is initially put into a circular orbit at an altitude of 6000 km above the Earth's surface. Find the velocity of the satellite. Take the Earth's radius as 6400 km and g at the Earth's surface as 10 ms^{-2}. [4]

(ii) The satellite is then separated into two halves of equal mass by an internal mechanism in the satellite which imparts to each half of the satellite an equal and opposite impulsive force acting along the direction of travel. The faster half is just able to escape from the Earth's gravitational field. Calculate the magnitude of the impulsive force of separation and the speed of the slower half just after separation. [10]

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SECTION C

Answer not more than **two** *questions from this section.*

9 Heating of a pizza in an oven can be modelled by assuming that the pizza is a circular disc of radius R and depth D, made of a material with density ρ and specific heat capacity c. The temperature θ is assumed to be uniform throughout the pizza and the heat flow $\dot{Q}$ into the pizza is given by

$$\dot{Q} = hA(\theta_0 - \theta) ,$$

where h is a constant, A is the surface area of the pizza and θ_0 is the temperature of the oven.

(a) Show that the variation of pizza temperature with time t is governed by an equation of the form

$$B\frac{\mathrm{d}\theta}{\mathrm{d}t} + \theta = \theta_0$$

and find the constant B in terms of the given geometric, heat transfer and material constants. Write down an expression for the time constant T of the system. [6]

(b) A pizza at a temperature of –5°C is placed in a pre-heated oven which is at a temperature of 200°C. It is observed that the pizza reaches its required temperature of 150°C after 25 minutes. Sketch the variation of pizza temperature with time and estimate the time constant T. [5]

(c) On the next day a similar pizza at the same initial temperature of –5°C is warmed up. However the oven is now at 20°C when the pizza is placed in it. At this point the oven begins to heat up at a constant rate of 15°C per minute until it

(cont.

reaches 200°C, at which temperature it then stays. Derive equations describing the variation of pizza temperature with time. Calculate the temperature of the pizza when the oven reaches 200°C and the time needed to heat the pizza up to 150°C. Add the variation with time of oven and pizza temperatures to your sketch for part (b) above. [9]

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10 Excessive vibration is found to be a problem with an optical instrument to measure surface roughness. The instrument is rigid, and is mounted on a flat table using flexible rubber pads. Figure 5 illustrates a simple model of the instrument, which considers small displacements in a plane. The mounting is modelled by two linear springs of stiffness k, placed a distance L apart. The instrument has a mass m, with the centre of mass equidistant between the mounting points, and a moment of inertia $J = mL^2/6$. Damping should be neglected. The position of the instrument is measured using the displacements y_1 and y_2 at the mounting points. The input vibration from the table is harmonic and defined by displacements $x_1 = X_1 \cos\omega t$ and $x_2 = X_2 \cos\omega t$, again measured at the mounting points.

(a) Write down expressions relating the displacements y_1 and y_2 of the instrument mounting points to the displacement y_G and the rotation θ of the centre of mass of the instrument. [3]

(b) Show that the rotation θ of the centre of mass of the instrument is governed by the differential equation

$$J\ddot{\theta} = \frac{kL}{2}(y_1 - x_1) - \frac{kL}{2}(y_2 - x_2)$$

and derive the corresponding differential equation for the displacement y_G of the centre of mass. [5]

(cont.

(c) Show that the equation of motion for the displacements $\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ of the system can be expressed in the form

$$[\mathbf{m}]\ddot{\underline{y}} + [\mathbf{k}]\underline{y} = \underline{f},$$

where $[\mathbf{m}] = \begin{bmatrix} m/2 & m/2 \\ -m/3 & m/3 \end{bmatrix}$, $[\mathbf{k}] = \begin{bmatrix} k & k \\ -k & k \end{bmatrix}$ and $\underline{f} = \begin{bmatrix} kx_1 + kx_2 \\ -kx_1 + kx_2 \end{bmatrix}$. [6]

(d) Due to passing lorries, the table vibrates with an amplitude $X_1 = 50$ μm, $X_2 = 0$ at a frequency $\omega = 20$ rads^{-1}. For an instrument of mass $m = 30$ kg and mounting springs of stiffness $k = 1000$ N/m, find the amplitude of the displacement y_G of the centre of mass of the instrument. What could be done to reduce the amplitude of vibration of the instrument? [6]

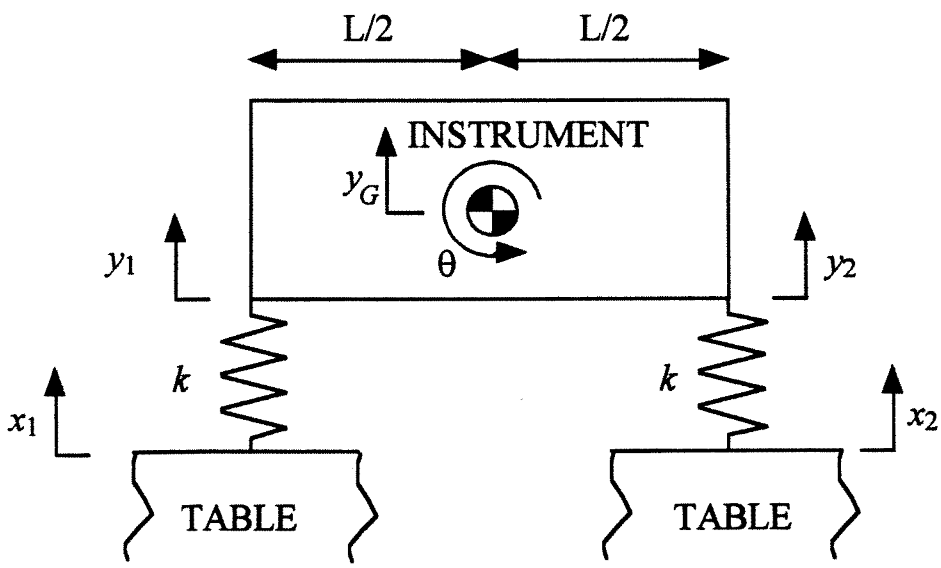

Fig. 5

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11 The circuit shown in Fig. 6 contains two equal resistances R, a capacitance C and an inductance L. It is driven by a voltage e. A voltage v is measured across the output terminals. No current is drawn at the output terminals.

(a) Derive the following governing differential equation relating the change of output voltage v with time t

$$\frac{LC}{2}\frac{\mathrm{d}^2v}{\mathrm{d}t^2}+\left(\frac{L}{2R}+\frac{CR}{2}\right)\frac{\mathrm{d}v}{\mathrm{d}t}+v=\frac{e}{2}.$$ [8]

(b) The circuit is initially in a steady state with the input voltage $e = 10\text{V}$. The voltage e is then altered by a step change to a value of 4 V.

(i) Show, by making a substitution of the form $y = v + D$, where D is a suitable constant, that the change in output voltage v can be modelled using section 4.4 of the Mechanics Data Book. Find an expression in terms of L, C and R for the damping factor c. [6]

(ii) Find an expression in terms of L and C for the range of values of R when the output voltage v would fall below 2 V in the subsequent response. [6]

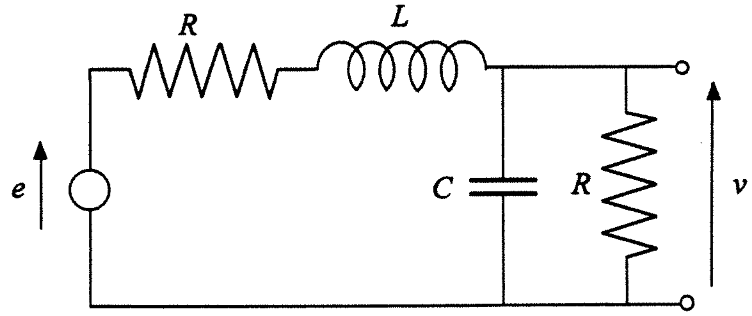

Fig. 6

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