ENGINEERING TRIPOS PART IA

Monday 7 June 1999 1.30 to 4.30

Paper 2

STRUCTURES AND MATERIALS

Answer not more than **eight** *questions, of which not more than* **four** *may be taken from Section A, and not more than* **four** *from Section B.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

All questions carry the same number of marks.

Answers to Sections A and B should be tied together and handed in separately.

(TURN OVER

SECTION A

Answer not more than **four** *questions from this section.*

1 A sculptor wishes to construct a statue using blocks made of steel and brass. The blocks are cubes of side length 1 m and are stacked on a wedge-shaped steel base (with a 20° slope) which is firmly anchored to the ground. A preliminary configuration is shown in Fig. 1. The top block is made of brass and the lower block is made of steel. The static coefficient of friction between steel and brass is 0.3 and between steel and steel is 0.6. The densities of these materials can be found in the Structural Data Book.

A cable is attached to the brass block at point B. The cable is horizontal until it passes over a light frictionless pulley. The other end of the cable is attached to a weight W.

(a) For the special case when $W = 0$:

(i) Determine if sliding will occur at one or more of the block interfaces. [4]

(ii) If the interfaces were roughened to prevent sliding, show that the blocks will not overturn about point A. [6]

(b) If the interfaces were not roughened, calculate the maximum and minimum weight W that could be attached to the pulley if sliding is not to occur at the brass/steel interface. [6]

(c) In addition to sliding between the interfaces, what other potential modes of failure should be considered? [4]

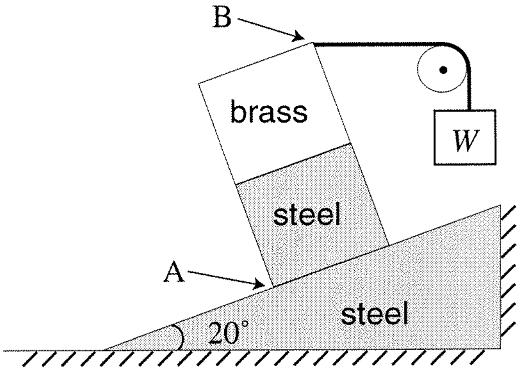


Fig. 1

2 Figure 2 shows a two-dimensional pin-jointed structure in which all the bars are made of material with a Young's modulus E. The cross-sectional area of the bars is A. The pin-jointed structure is to be used as a support for a frictionless pulley which will be attached at joint D.

(a) A rope passes over the pulley at D and is attached to a block of weight W.

(i) What is the vertical force applied to the structure at joint D (neglect the self-weight of the pulley)? [2]

(ii) Determine the forces in all the bars resulting from this loading. [6]

(b) Due to the force applied by the pulley, find:

(i) the displacement at D and [4]

(ii) the change in the horizontal distance between C and E. [4]

(c) An extra bar is now added from C to E. Explain why it is not possible to solve for the bar forces by equilibrium alone. How else is it possible to proceed? [4]

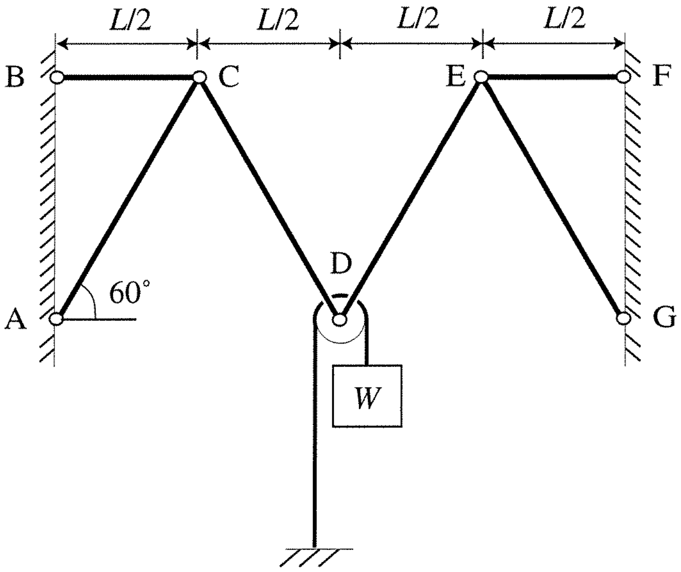


Fig. 2

3 Figure 3 shows a light structure ABCD. It consists of beams AB and BC, which are rigidly joined to each other at B, and a cable CD. A and D are pinned to supports which are at the same level. The beam BC is subjected to a uniformly distributed vertical load of w per unit length, as shown.

(a) Calculate the tension in the cable CD, and find the support reactions at A and D. [6]

(b) Find the bending moment in the beam at B, and hence sketch a bending moment diagram for AB. [4]

(c) Sketch a bending moment diagram for BC, marking extreme values. [10]

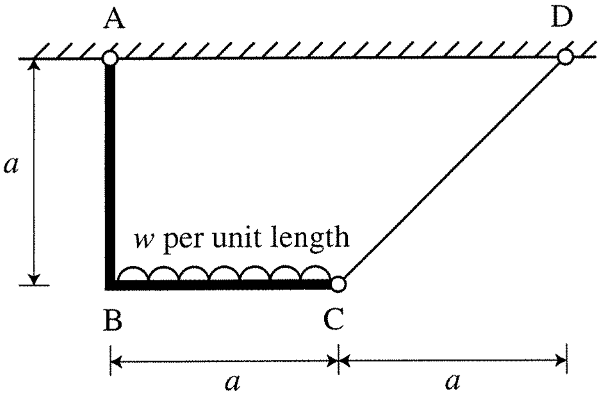


Fig. 3

4 Figure 4 shows a beam ABCD of uniform bending stiffness *EI*. It has supports at B and C and is loaded by a uniformly distributed load of *w* per unit length from B to D.

(a) Calculate the bending moment in the beam at C due to the applied load. [2]

(b) By superposition of Data Book cases, or otherwise, calculate the magnitude and direction of the rotation of the beam at both B and C due to the applied load. [12]

(c) Calculate the magnitude and direction of the deflection of the beam at A and D due to the applied load. [6]

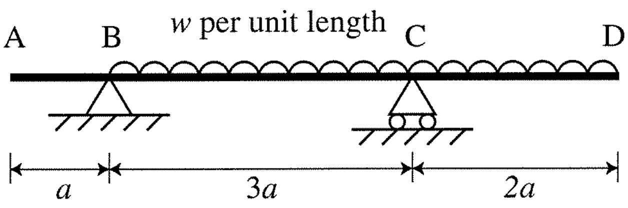


Fig. 4

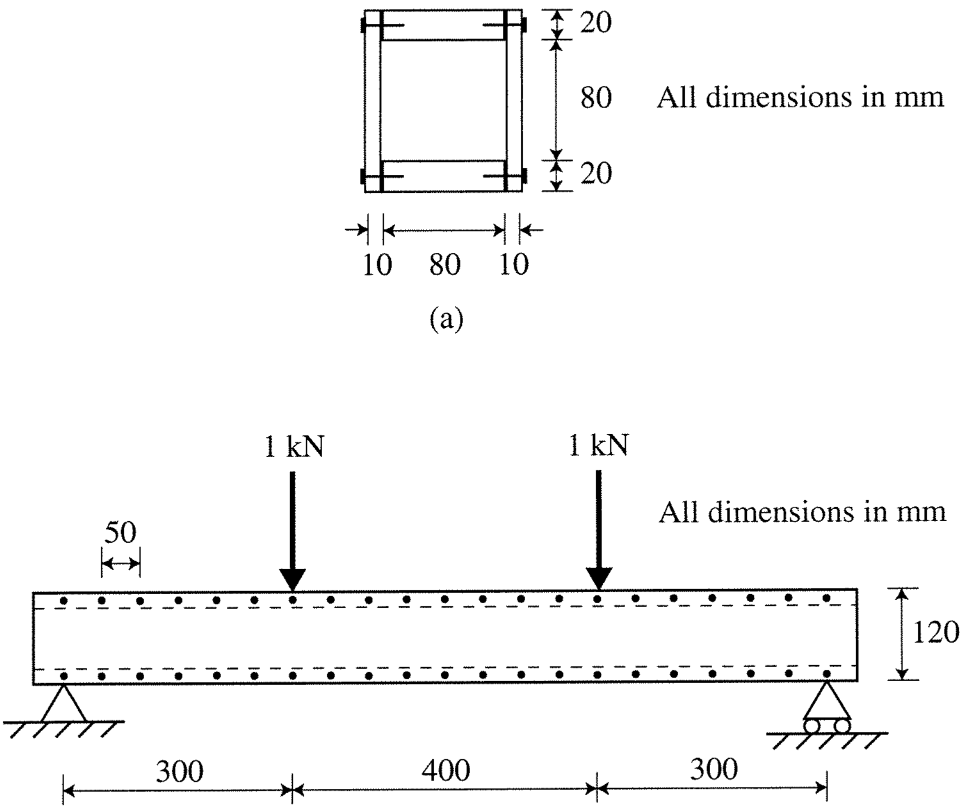
5 Figure 5(a) shows the cross-section of a beam which has been manufactured by nailing together four wooden planks, two of cross-section 120 mm × 10 mm, and two of cross-section 80 mm × 20 mm. Figure 5(a) shows a cross-section that cuts through the nails, which are placed every 50 mm along the beam, as shown in Fig. 5(b). A 1 m length of the beam is simply-supported, and is subjected to the loading shown in Fig. 5(b).

(a) Show that self-weight is not significant for this arrangement. [2]

(b) Draw a shear force and a bending moment diagram for the beam, marking salient values. [4]

(c) Calculate the largest longitudinal bending stress in the beam. [6]

(d) Calculate the maximum shear force carried by a single nail. [8]



(a)

(b)

Fig. 5

SECTION B

Answer not more than **four** *questions from this section.*

6 (a) Sketch the shape of a nominal stress-nominal strain curve and the shape of the corresponding true stress-true strain curve for a ductile metal specimen loaded in uniaxial tension. Indicate on both curves the yield stress σ_y, the tensile strength σ_{TS}, and the necking regime of the specimen. [7]

(i) Define true strain ε_t in terms of the initial A_o and reduced A cross-sectional area of the tensile specimen. [2]

(ii) Define tensile strength σ_{TS} in terms of true stress σ_t, A_o and A. [2]

(b) For a ductile metal specimen loaded in tension, a general relation for the true stress-true strain prior to necking is given by:

$$\sigma_t = \sigma_o(B + \varepsilon_t)^n$$

where the parameter B defines the metal's yield stress at $\varepsilon_t = 0$. A simple criterion for the onset of necking in terms of true stress and true strain is given by $d\sigma_t/d\varepsilon_t = \sigma_t$.

(i) Derive an expression for the true strain ε_t at the onset of necking in terms of n and B. [3]

(ii) Derive an expression for σ_{TS} in terms of σ_o, n and B. [3]

(iii) Determine the value of B if $n = 0.5$ and $\sigma_y = 0.5\sigma_{TS}$. [3]

7 When the new Addenbrooke's hospital in Cambridge was first opened all the ward doors had elastic hinges so that they could be pushed both ways and close automatically. Consider such a hinge as a thin ligament of material which flexes elastically (Fig. 6).

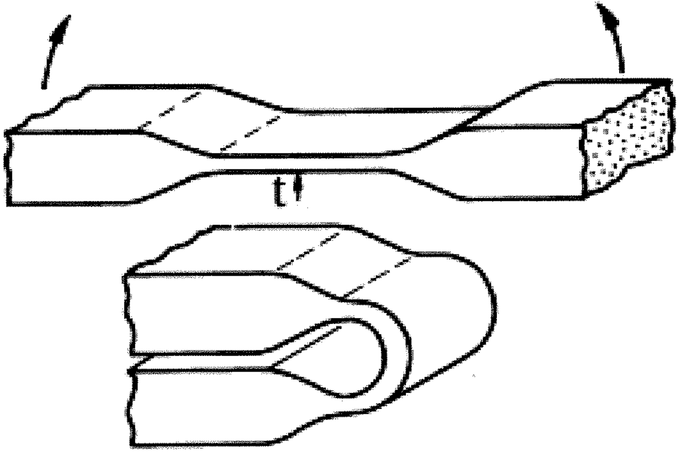


Fig. 6 Design of an elastic hinge

(a) Explain briefly why the best material for the elastic hinge (of a given design) would be one that bends to the smallest radius without yielding. [2]

(b) Show that the strain ε at the surface of a ligament of thickness t which bends elastically to a radius R is given by $\varepsilon = t/(2R)$. [3]

(c) Write down an expression for the maximum stress σ at the surface in terms of Young's modulus E, t and R. [3]

(d) Deduce the minimum radius R of the ligament that can be formed without the material yielding in terms of t, E, and the yield stress σ_y. [3]

(e) Propose a Merit Index of the material in terms of σ_y and E. [3]

(f) Using the Materials Selection chart shown in Fig. 7, identify the best choices of material for the elastic hinge based on the Merit Index guideline. A separate chart is provided for this purpose and should be handed in with your answer. Make a short list of materials and from it select the best material giving reasons for discarding others. [3]

(g) Some 6 months following the opening of Addenbrooke's Hospital, many of the elastic hinges had failed. Propose one plausible explanation and suggest an additional factor that should have been taken into account. [3]

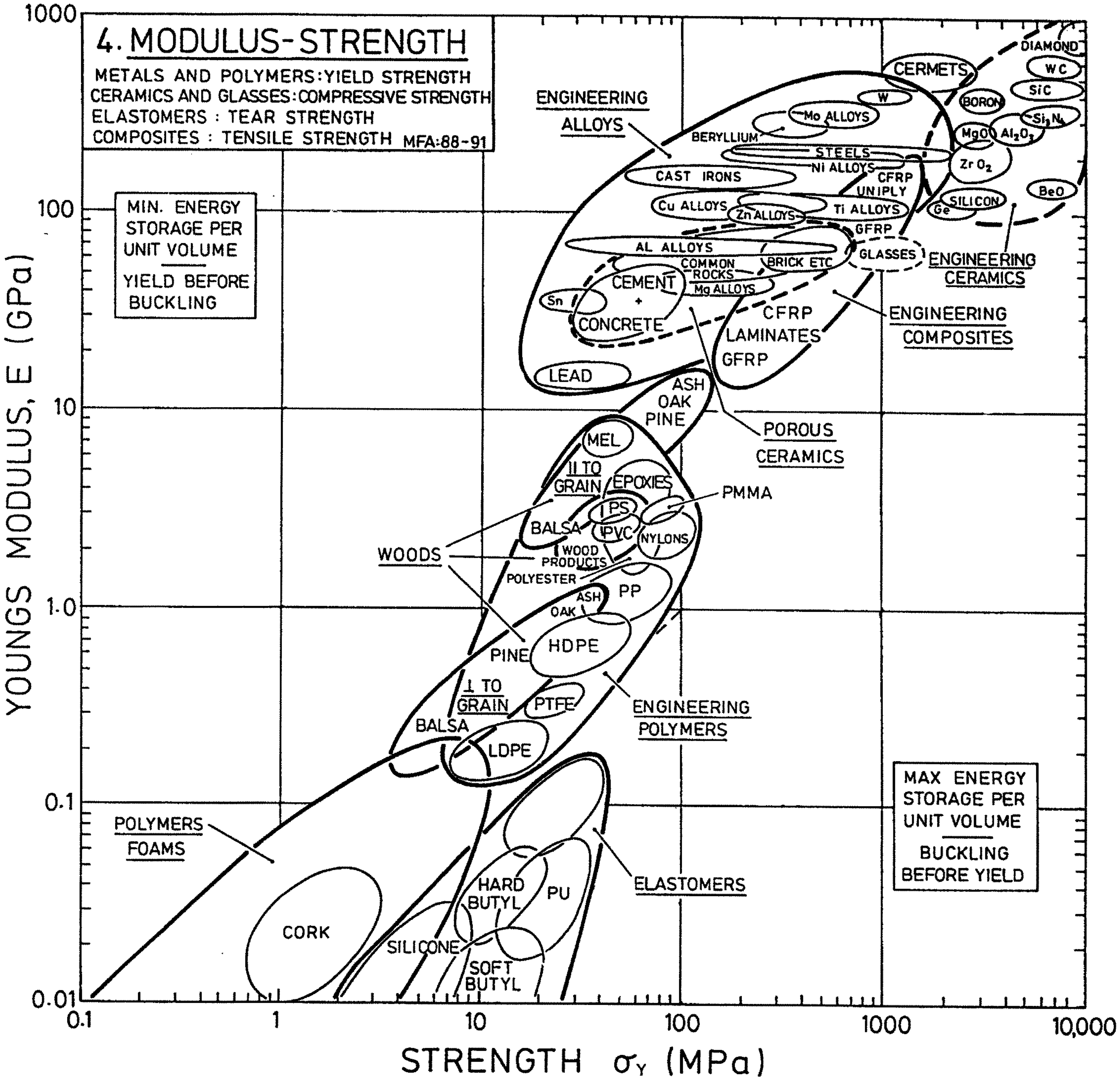


Fig. 7 A Materials Selection Chart

(TURN OVER

8 (a) Distinguish between the stress concentration factor K_t for a notch and the stress intensity factor K for a crack in a component under tensile load. [6]

(b) A company makes large industrial fans and buys in heavy steel rotor shafts that have been forged and then machined to final dimensions. Each forging is inspected by a magnetic particle method capable of detecting 1 mm deep surface cracks. An inspection policy of limiting the acceptable crack size to 1 mm would just ensure that all cracks are removed during the final machining operation. In practice, however, the supplier of these rotor shafts has been rejecting forgings that have contained detected cracks whatever their size and passing on the cost to the maker of the fan. The design life of a shaft is 30 years.

The shaft is supported at both ends and rotates at 1800 rpm. The principal stresses are due to bending resulting from the shaft's weight. In service, the shaft is cyclically loaded at an alternating stress of ± 25 MPa due to rotating bending. The yield stress of the steel is 300 MPa. Figure 8 shows appropriate fatigue crack growth data da/dN versus ΔK of the steel measured in separate experiments.

The stress intensity factor K for a surface flaw crack is given by:

$$K = 1.12\,\sigma\sqrt{\pi a}$$

where σ is applied stress and a is crack length.

(i) By making a rough estimate of the fatigue endurance limit of the steel shaft from its yield stress, would you expect fatigue cracks to nucleate in a shaft that is initially free of cracks? Briefly account for your answer. [3]

(ii) Make a sketch of Fig. 8 and indicate the fracture toughness K_{IC} and the threshold value of ΔK of the steel. What is the significance of $\Delta K_{threshold}$ and K_{IC}? [3]

(iii) Explain why, to *guarantee* that a shaft survives its design life, a pre-existing crack must effectively have zero crack growth. Apply the basic fracture mechanics formula to calculate the maximum allowable crack size if the shaft is to be guaranteed to complete its mission. [3]

(iv) Why is the supplier's actual inspection practice of rejecting all flawed forgings unreasonable? Propose and justify a straightforward inspection criterion based on the original policy that is fair to the fan manufacturer and one that is extremely unlikely to lead to premature failure of the rotor shaft in less than 30 years. [5]

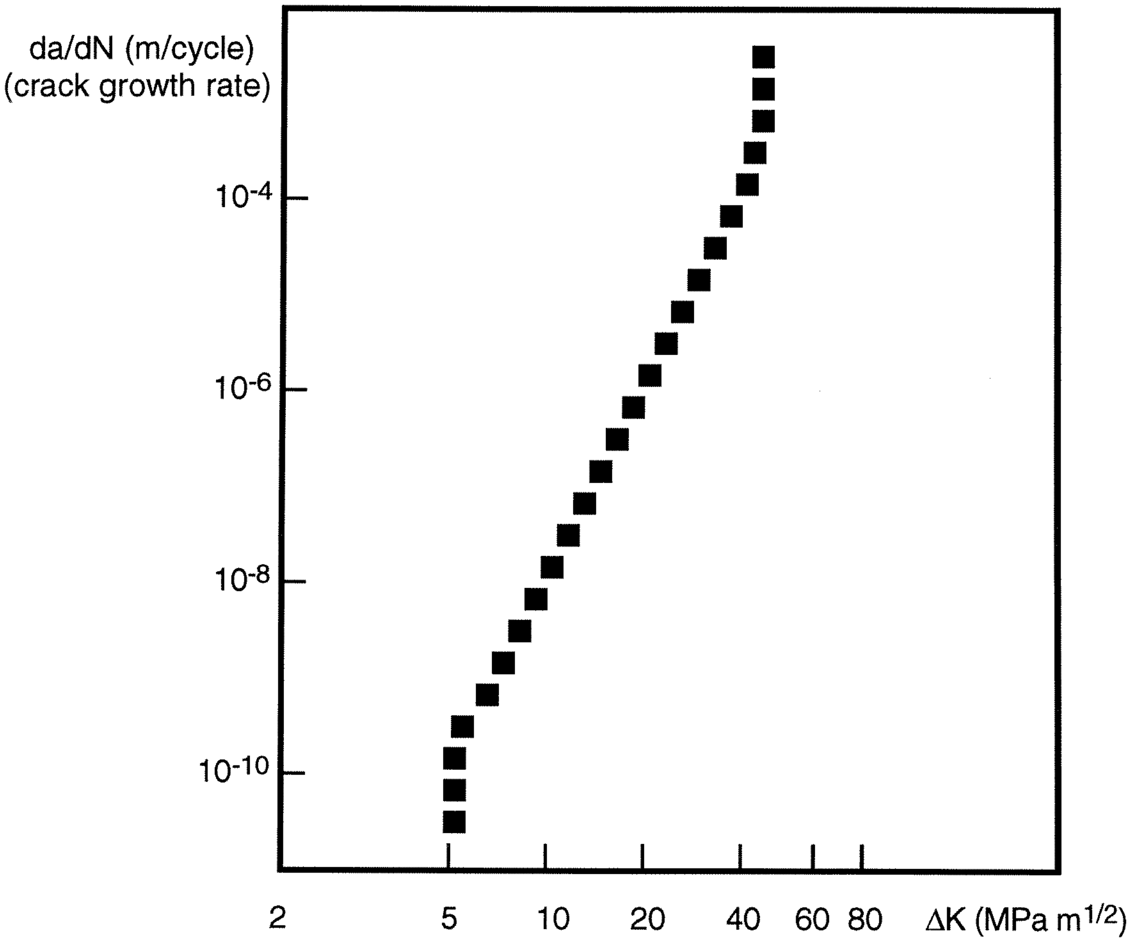


Fig. 8 Fatigue crack growth rate of a steel used in fan shafts

9 (a) Discuss briefly why a metal can be strengthened by obstructing the motion of dislocations with a dispersion of small, finely spaced precipitates. Comment on whether or not the enhanced yield stress is time or temperature dependent. [6]

(b) The tensile yield stress σ_y of iron is 175 MPa. Estimate the enhancement in yield stress if the iron now contains a uniform distribution of spherical iron carbide precipitates of diameter $5 \mathrm{x} 10^{-7}$ m and centre to centre spacing of 10^{-6} m. For iron, the shear modulus G is 105 GPa and the Burgers vector b is 3×10^{-10} m approximately. [6]

(c) (i) What is meant by the ductile-brittle transition in steel? [4]

(ii) The tensile yield stress σ_y of a low carbon steel at sub-zero temperature varies with absolute temperature T (K) over a particular temperature range according to:

$$\sigma_y = (1400 - 3.6T) \text{ MPa.}$$

The stress σ_f at which the steel fractures in a brittle manner increases linearly with temperature and is equal to 1190 MPa at 200 K. The ductile-brittle transition temperature T_D of this particular steel is 100 K. By precipitation strengthening, the steel's yield stress σ_y can be increased by 245 MPa. Assuming that σ_f and the temperature dependence of σ_y are unaffected by the presence of precipitates, estimate the change in transition temperature T_D of the steel due to the precipitation process. [4]

10 (a) Describe the principal mechanisms that can occur in metals when deformed at elevated temperature under creep conditions and which lead to the ultimate failure of a component. Explain ways in which the microstructure of a suitable high temperature alloy can be modified in order to supress creep in the component. [12]

(b) A high temperature alloy undergoes power law creep when the applied stress is 70 MPa and the temperature is 1000 $^{\circ}$C. The stress exponent n is 5 and the activation energy ΔQ is 300 kJmol^{-1}. Recent modification to the alloy's microstructure in manufacture is known to enhance its creep resistance. Such change enables the stress to be raised to 80 MPa at 1000 $^{\circ}$C without significantly affecting the material's creep rate.

An aero-engine manufacturer is more interested in increasing the operating temperature of turbine blades than increasing their working stress provided, of course, that the material's creep behaviour is unaffected. Estimate by how much the temperature of turbine blades can be increased by adopting the new manufacturing process. State any assumptions made. [8]

END OF PAPER

ENGINEERING TRIPOS PART 1A

Paper 2

STRUCTURES AND MATERIALS

Question 7 Fig. 7 To be included as part of your answer

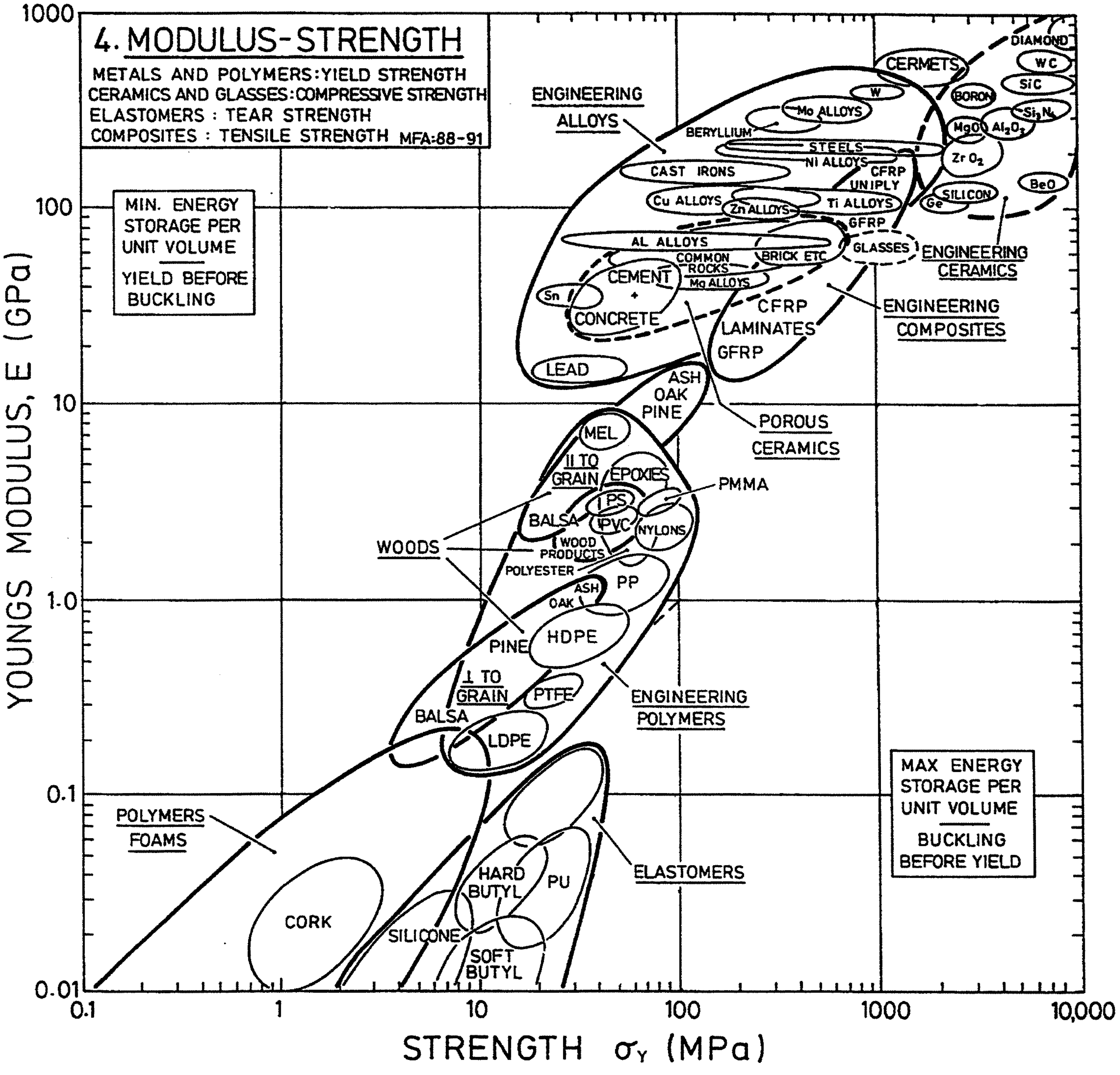


Fig. 7 A Materials Selection Chart

ENGINEERING TRIPOS PART 1A

Paper 2

STRUCTURES AND MATERIALS

Question 7 Fig. 7 To be included as part of your answer

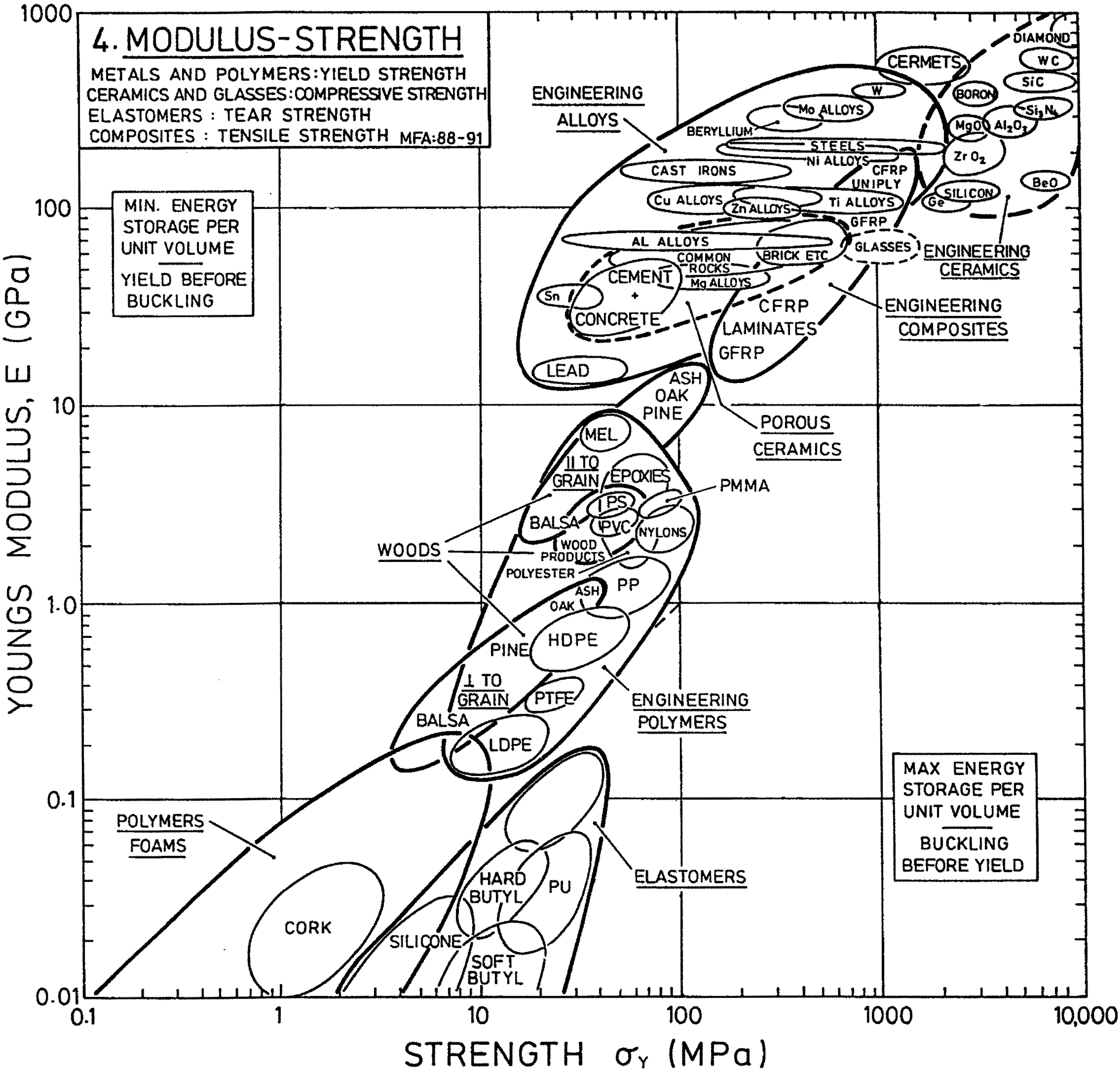


Fig. 7 A Materials Selection Chart