IA 2000 Answers

Part IA Paper 1 Section A

Question 1:

(b) 0.8066 kg/m^3
(c) 717.8 K
(d) 6185 N (in direction of flow)

Question 2:

(c) i) 994.3 K ii) 2983.0 K iii) 1396.7 K
(d) compression: -499.9 kJ/kg
constant pressure: 570.8 kJ/kg
expansion: 1142.2 kJ/kg
cooling: 0 kJ/kg
net work: 1213.0 kJ/kg
(e) 60.4 %

Question 3:

Question 4:

(c) 0.0115 K/W
(e) 0.043 kg/s

Section B: Mechanics

5. (a) 0.419s (b) 1.14ms^{-1} to the right
(c) 3.11 rad s^{-1}, 1.52 rad s^{-1}, 3.11 rad s^{-1}, all clockwise (d) 0.46Nm

6. (a) 10ms^{-1} (b) 0.00866 rad s^{-1} clockwise
(c) -0.696ms^{-2} (d) -0.000420 rad s^{-2}

7. (a) $10R$ (c) $2R\sqrt{gR}$ (d) $4R$ (e) extracted

8. (a) $\ddot{r} - 4r = 0$ (c) 0.658s, 1.32 rad (d) 41.6N (e) 36J

Section C: Linear Systems and Vibration

9. (a) $T\dot{v} + v = v_o$ where $T = \dfrac{m}{3\pi\mu D}$ and $v_o = \dfrac{(m - \frac{1}{6}\pi D^3\rho)g}{3\pi\mu D}$ (b) 9.86ms^{-1}

(c) $v = 9.86(1 - e^{-\frac{t}{T}})$, $T = 1.06$s, 9.30ms^{-2} (d) 11.9m

10. (a) $\dfrac{\omega^2 m}{k}$, $\dfrac{kY}{F}$, $\dfrac{\lambda^2}{km}$ (c) 25Nsm^{-1}, 16Hz, 12.5N (d) 8.5mm, 2.1mm

11. (a) $M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$ $K = \begin{bmatrix} 3k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 3k \end{bmatrix}$ $M\ddot{y} + Ky = 0$

(c) $\sqrt{\dfrac{3k}{m}}$, $\sqrt{\dfrac{k}{m}}$, $2\sqrt{\dfrac{k}{m}}$ (d) $\dfrac{\rho AL}{3}$

Engineering Tripos Part 1A 2000, Paper 2 Materials and Structures

SECTION A – STRUCTURES

1. a) $V_L = V_R = \frac{wL}{2} \uparrow \quad H_L = \frac{wL^2}{8\delta} \leftarrow \quad H_R = \frac{wL^2}{8\delta} \rightarrow$

   b) i) $w = 36.9$ N/m

   ii) out-of-plane buckling $P_{CR} = 10.9$ kN

   in-plane buckling $P_{CR} = 5.13$ kN

2. a) i) $T_{BC} = T_{CG} = T_{AB} = T_{AG} = -W/\sqrt{3}, \quad T_{BG} = W/\sqrt{3},$

   $T_{DG} = T_{DE} = T_{EF} = T_{FG} = T_{EG} = 0$

   ii) $\delta_{CV} = \frac{5WL}{3EA} \downarrow \quad \delta_{DV} = 0$

   b) $\delta_{MID} = \frac{5WL}{6EA} + \frac{WL^3}{48EI} \downarrow$

3 a) $V_A = 693$ N $\uparrow \quad H_A = 852$ N $\leftarrow$

4 a) i) $V_A = 480.2$ N $\uparrow$

   ii) $\delta_{VB} = 1.97$ mm $\downarrow \quad \delta_{HB} = 0$

   iii) $\delta_{VC} = 1.97$ mm $\downarrow \quad \delta_{HC} = 2.86$ mm $\leftarrow$

   b) $H_C = 120$ N $\rightarrow, \quad M_A = -300$ Nm, $\quad V_A = 480$ N $\uparrow, \quad H_A = 120$ N $\leftarrow$

5. a) width of 'transformed' concrete = 174.4 mm, depth = 20 mm

   b) i) $\bar{y} = 91.7$ mm

   ii) $EI = 1.084 \times 10^{11}$ Nmm^2

   c) i) $W = 12.3$ kN

   ii) $W = 98.7$ kN

PART 1A ENGINEERING TRIPOS
PAPER 2 STRUCTURES AND MATERIALS

SECTION B

6. (a) (i) CHOOSE MATERIAL B.

(ii) CHOOSE MATERIAL B.

(iii) CHOOSE MATERIAL A.

(b) (i) $E = \dfrac{100}{4.75x10^{-4}} = 210x10^{3} = 210GPa$

(ii) $G = \dfrac{E}{2(1+\nu)} = \dfrac{210}{2(1+0.295)} = 81.1GPa$

(iii) $\nu = \dfrac{\varepsilon_x}{\varepsilon_y} = -\dfrac{-1.4x10^{-4}}{4.75x10^{-4}} = 0.295.$

7. (c) (i) $\sigma_{ys} = 689.4$ MPa.

8. (c) (ii) $Q = 190{,}800$ J/mol.

9. (b) (ii) It is reasonable to assume from the question that the oxide growth rate for MgO is linear whilst for NiO it is parabolic.

$y_{Mg}(t) = C_1 t$ (linear oxidation)

$[y_{Ni}(t)]^2 = C_2 t + C_3$ (parabolic oxidation).

$y_{(Mg)} = 10.2\mu m.$

$y_{Ni} = 1.32\mu m.$

10. N_i of about 2,000 cycles which corresponds to about 10 months.

The recommendation is to specify a NDT technique that is capable of detecting cracks as small as (approximately) 2mm and not to miss cracks deeper than 3.5mm. Also, the pressure vessel must be inspected using the same procedure no longer than 10 months.

Peter W R Beaumont
Easter Term, 2000

1A Paper 3 : Numerical Answers (Section A + Section C)

Section A

1(b) $R_3 = 5\ k\Omega$, $R_2 = 2\ M\Omega$
(c) $G = +0.958$, $Z_{out} = 192\Omega$, $Z_{in} = 1\ M\Omega$
(d) $C = 0.01\ \mu F$
(e) $P_{max} = 0.7\ mW$

2(a) $G_{approx} = -5$
(b) $G = -4.9665$
(c) f_{3dB} (lower) = 10 Hz
f_{3dB} (upper) = 10^4 Hz
(d) Midband phase = 180°, Lower 3dB phase = 135° (lagging), Upper 3dB phase = 225°

3(a) $v_A = 0.5 - j\,0.5$ V ; $v_B = j3$ V
(b) Thevenins: A voltage = $0.5 - j0.5$, A impedance = $0.5 + j1$
B voltage = $j3$, B impedance = $3 + j2$

Nortons: A current = $-0.2 - j0.6$, A impedance = $0.5 + j1$
B current = $0.46 + j0.69$, B impedance = $3 + j2$

(c) $V_{ABC} = 0.063 + j0.1856$
(e) V_{ABC} relative to $V_1 = 1.51 \angle 128°$

4(c) $V_{open\ circuit} = 257$ V

Section C

9(a) $C = 4\pi\varepsilon R$

(b) $E_r = \frac{Q}{2\pi\varepsilon} * \frac{1}{r}$

(c) $C_{AB} = \frac{\pi\varepsilon}{\ln\left(\frac{d-R}{R}\right)}$

(d) C/unit length = 5.25 pF/m $V_{max} = 53$ kV

10(b) $V_{rms} = 39$ V
(c) error = 10% ; if $\mu_r = 100$, error = 1%

11(a) $B_C = 1.29$ Tesla
(b) F = 66.2 Newtons
(c) $I_{release} = 1.85$ A

IA 2000 Paper 4 Mathematical Methods

Answers

1. N.B. The equations of lines and planes have many forms.

(a) $\mathbf{r} = (2,-3,-5)\lambda + (1,0,1)$ (b) (i) $2x+3y-z = 1 \pm \sqrt{14}$ (ii) 2

(c) $\mathbf{r} = (2,-3,-5)\lambda + \left(1 \pm \frac{3\sqrt{3}}{5} \pm \frac{\sqrt{14}}{5}, \mp \frac{2\sqrt{3}}{5} \pm \frac{\sqrt{14}}{5}, 1\right)$

2. (a) $\frac{2}{3}$ (b) $n\pi \pm i \sinh^{-1} 2$ (+ for n even, – for n odd))

(c) **a.c b.b c – a.c b.c b**

3. (a) $y = (Ax+B)e^x + \frac{1}{2}\cos x$ (b) $S_n = \frac{1}{2\varepsilon}(1+\varepsilon)^n - \frac{1}{2\varepsilon}(1-\varepsilon)^n$

(c) $An + B$

4. (b) **n** is perpendicular to page (*out* of page) (d) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$

5. (a) e-values 2; 5; –5 e-vectors (0,1,0); (2,0,1); (1,0,–2)

(c) Matrix **U** has $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ as its columns.

6. (a) (i) $f(t) = \frac{\pi}{2} - \sum_{\substack{1 \\ n\,odd}}^{\infty} \frac{4}{n^2\pi}\cos nt$

(b) (i) $d_n = (-1)^n c_n$ (ii) $e_n = \left(1+(-1)^n\right)c_n$ (iii) only even n present

7. (a) $\begin{matrix} (1-t)e^{-t} & t \geq 0 \\ 0 & \text{otherwise} \end{matrix}$ (b) $1 - e^{-t} - te^{-t}$ (c) integral of step response

8. (e) $\frac{9}{56}\left[\left(\frac{7}{8}\right)^n + 7\left(\frac{-1}{8}\right)^n\right]$

9. (a) $-\frac{e^{-3t}}{4} + \frac{5}{4}e^{-t} + \frac{t}{2}e^{-t}$ (c) $1 + \frac{t^2}{2}$

10. (a) (ii) $f = x^2y^3 - 2\ln x + 3y^3$ (+const) (b) (i) $\frac{(1-b-a)^2}{2}$