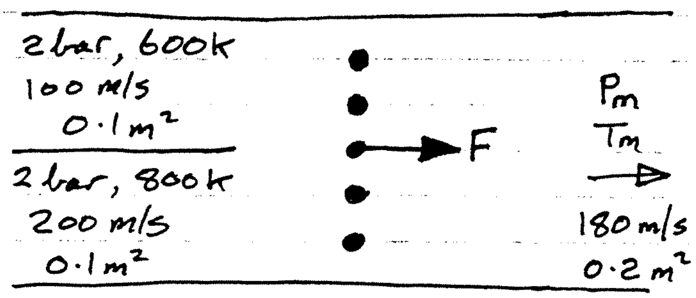
Crib for Engineering Tripos Part IA Paper 1 – Mechanical Engineering 2000

Section A: Thermofluid Mechanics

Q1



F = force on bars

a) Entry flows must be at the same pressure otherwise one will ~~reverse~~ slow down and possibly even reverse. SAME PRESSURE ALSO MEANS NO TRANSVERSE PRESSURE GRADIENT ⇒ PARALLEL STREAMLINES

b) MASS CONSERVATION $\dot{m} = \rho A v = \frac{PAv}{RT}$

$$\dot{m}_1 = \frac{2\times10^5\times0.1\times100}{287\times600} = 11.614 \text{ kg/s}$$

$$\dot{m}_2 = \frac{2\times10^5\times0.1\times200}{287\times800} = 17.422 \text{ kg/s}$$

$$\dot{m}_m = 11.614 + 17.422 = 29.036 \text{ kg/s}$$

$$\rho_m A_m v_m = 29.036 \Rightarrow \rho_m = \frac{29.036}{0.2\times180} = \underline{\underline{0.8066 \text{ kg/m}^3}}$$

c) ENERGY CONSERVATION $\dot{m}_1(c_pT_1+\tfrac{1}{2}V_1^2) + \dot{m}_2(c_pT_2+\tfrac{1}{2}V_2^2) = \dot{m}_m(c_pT_m+\tfrac{1}{2}V_m^2)$

$$c_pT_m + \tfrac{1}{2}V_m^2 = \frac{\dot{m}_1}{\dot{m}_m}(c_pT_1+\tfrac{1}{2}V_1^2) + \frac{\dot{m}_2}{\dot{m}_m}(c_pT_2+\tfrac{1}{2}V_2^2)$$

$$T_m = \frac{11.614}{29.036}\underbrace{\left(600 + \frac{100^2}{2\times1010}\right)}_{604.950} + \frac{17.422}{29.036}\underbrace{\left(800 + \frac{200^2}{2\times1010}\right)}_{819.802} - \frac{180^2}{2\times1010}$$

$$\underline{\underline{T_m = 717.82 \text{ K}}}$$

d) $P_m = \rho_m R T_m = 0.8066\times287\times717.82 = 1.6617 \text{ bar}$

FORCE – MOMENTUM CONSERVATION (F = Force on bar)

$$(P_1A_1 + \dot{m}_1V_1) + (P_2A_2 + \dot{m}_2V_2) - F = (P_mA_m + \dot{m}_mV_m)$$

$$F = (2\times10^5\times0.1 + 11.614\times100) + (2\times10^5\times0.1 + 17.422\times200) - (1.6617\times10^5\times0.2 + 29.036\times180)$$

$$F = 21161.4 + 23484.4 - 38460.5$$

$$\underline{\underline{F = 6185.3 \text{ N}}}$$

FORCE IN DIRECTION SHOWN IN DIAGRAM.

Q2 $Q - W = \Delta E \Rightarrow dQ - dW = c_v\,dT$ (perfect gases, unit mass)

a) ADIABATIC $\Rightarrow dQ = 0$

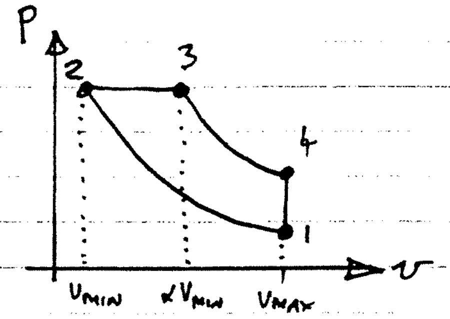
REVERSIBLE $\Rightarrow dW = p\,dv \quad \Rightarrow 0 - p\,dv = c_v\,dT$

$$\Rightarrow c_v\,dT + \frac{RT}{v}\,dv = 0$$

$$\frac{R}{c_v} = \frac{c_p - c_v}{c_v} = \gamma - 1 \qquad \Rightarrow \frac{dT}{T} + \frac{R}{c_v}\frac{dv}{v} = 0$$

$$\Rightarrow \underline{\underline{T v^{\gamma-1} = \text{const.}}}$$

b)



1-2 ISENTROPIC COMPRESSION
2-3 CONSTANT PRESSURE HEATING
3-4 ISENTROPIC EXPANSION
4-1 CONSTANT VOLUME COOLING.

α = CUT-OFF RATIO.

c) $T_2 = T_1\left(\frac{v_1}{v_2}\right)^{\gamma-1} = 300 \times 20^{0.4} = \underline{\underline{994.34\ K}}$

$T_3 = v_3 \frac{T_2}{v_2} = 994.34 \times 3 = \underline{\underline{2983.02\ K}}$

$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{\gamma-1} = 2983.02 \times \left(\frac{3}{20}\right)^{0.4} = \underline{\underline{1396.67\ K}}$

d) $W_{12} = c_v(T_1 - T_2) = 720(300 - 994.34) = -499.92$ kJ/kg

$W_{23} = p\Delta v = R(T_3 - T_2) = 287(2983.02 - 994.34) = 570.75$ kJ/kg

$W_{34} = c_v(T_3 - T_4) = 720(2983.02 - 1396.67) = 1142.17$ kJ/kg

$W_{41} = 0 \qquad\qquad = 0.00$ kJ/kg

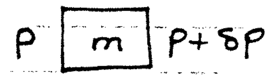
$W_{NET} = \underline{\underline{1213.00}}$ kJ/kg

e) $Q_{IN} = c_p(T_3 - T_2) = 1010(2983.02 - 994.34) = 2008.6$ kJ/kg

(INDICATED) $\eta = \frac{W_{NET}}{Q_{IN}} = \frac{1213.00}{2008.60} = \underline{\underline{60.4\ \%}}$

f) MECHANICAL LOSSES (FRICTION + AUXILIARY DRIVES)
HEAT LOSS — NOT TRUE ADIABATIC.
DIESELS HAVE VERY SMALL INTAKE LOSS (NO THROTTLE)

Q3 a) RESOLVE HORIZONTALLY. ANY PRESSURE DIFFERENCE WOULD CREATE AN ACCELERATION. FLUID IS STATIONARY SO NO ~~PRES~~ HORIZONTAL PRESSURE DIFFERENCES.



(CANNOT ASSUME FREE SURFACE, OR ANY STREAMLINES AS FLOW IS AT REST!)

b)

$$P_A = P_1 + \rho_w g d$$

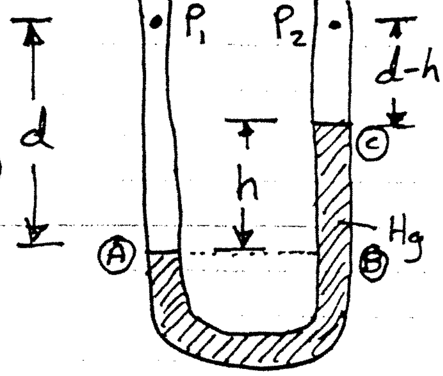
$$P_B = P_A = P_1 + \rho_w g d$$

$$P_C = P_B - \rho_m g h = P_1 + \rho_w g d - \rho_m g h$$

$$P_2 = P_C - \rho_w g(d-h) = P_1 + \rho_w g d - \rho_m g h - \rho_w g(d-h)$$

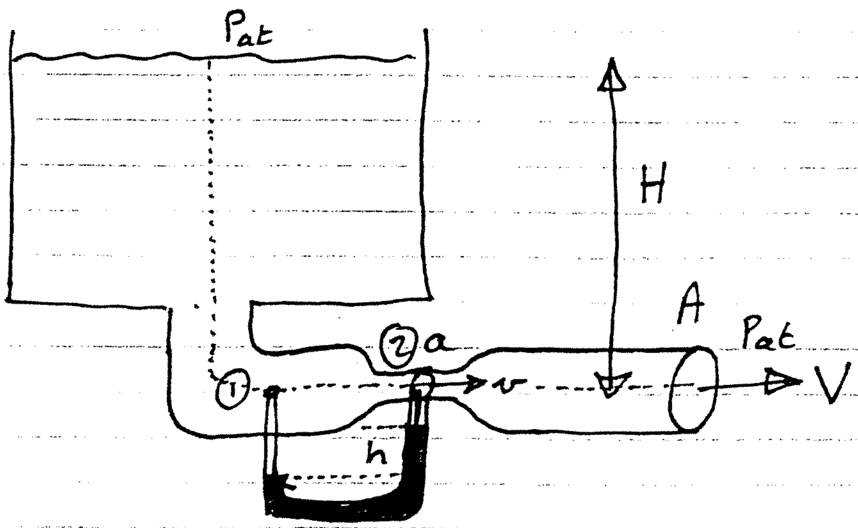
$$P_2 = P_1 - (\rho_m - \rho_w) g h$$

$$\underline{\underline{P_1 - P_2 = (\rho_m - \rho_w) g h}}$$



LOCATION 1 AND 2 MUST BE AT THE SAME HEIGHT.

c)



ALONG NOTIONAL STREAMLINE

$$P_{at} + \rho_w g H = P_1 + \tfrac{1}{2}\rho_w V^2 = P_2 + \tfrac{1}{2}\rho_w v^2 = P_{at} + \tfrac{1}{2}\rho_w V^2$$

$$\Rightarrow V^2 = 2gH \qquad \underline{\underline{V = \sqrt{2gH}}}$$

MASS CONSERVATION $\dot{m} = \rho_w a v = \rho_w A V$ $\qquad \underline{\underline{v = AV/a}}$

NOW $P_1 - P_2 = \frac{1}{2}\rho_w (v^2 - V^2) = \frac{1}{2}\rho_w V^2 \left(\frac{A^2}{a^2} - 1\right) = (\rho_m - \rho_w) g h$

$$\Rightarrow (\rho_m - \rho_w) g h = \tfrac{1}{2}\rho_w 2gH \left(\frac{A^2}{a^2} - 1\right)$$

$$\underline{\underline{h = \left(\frac{\rho_w}{\rho_m - \rho_w}\right)\left(\frac{A^2}{a^2} - 1\right) H}}$$

Q4 a) FOURIER : $\dot{Q} = -\lambda A \dfrac{dT}{dx}$

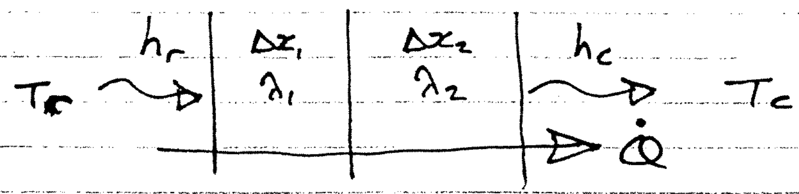
UNIFORM λ, CONSTANT A $\Rightarrow$ $\dot{Q} = -\lambda A (T_C - T_H)/\Delta x$

$\Rightarrow$ $T_H - T_C = \left(\dfrac{\Delta x}{\lambda A}\right)\dot{Q} = R\dot{Q}$ $\qquad \underline{\underline{R = \dfrac{\Delta x}{\lambda A}}}$

b) SURFACE HEAT TRANSFER $\dot{Q} = hA(T_H - T_C)$

$\Rightarrow T_H - T_C = \left(\dfrac{1}{hA}\right)\dot{Q} = R\dot{Q}$ $\qquad \underline{\underline{R = \dfrac{1}{hA}}}$

c)



$$R_{TOTAL} = \frac{1}{h_r A} + \frac{\Delta x_1}{\lambda_1 A} + \frac{\Delta x_2}{\lambda_2 A} + \frac{1}{h_c A}$$

$$= \frac{1}{A}\left(\frac{1}{h_r} + \frac{\Delta x_1}{\lambda_1} + \frac{\Delta x_2}{\lambda_2} + \frac{1}{h_c}\right)$$

$$R_{TOTAL} = \frac{1}{100}\left(\frac{1}{10} + \frac{0.1}{0.2} + \frac{0.36}{0.72} + \frac{1}{20}\right) = \underline{\underline{0.0115 \ \text{KW}^{-1}}}$$

d) SFEE FOR FURNACE $\dot{Q} - \dot{W}_x = \dot{m}(c_p T_f - c_p T_r)$

$\dot{W}_x = 0 \Rightarrow$ $\underline{\dot{Q} = \dot{m} c_p (T_f - T_r)}$

HEAT TRANSFER $\Rightarrow$ $\underline{\dot{Q} R_{TOTAL} = T_r - T_c}$

$\Rightarrow$ $\dot{m} c_p R_{TOTAL}(T_f - T_r) = T_r - T_c$

$T_c + \dot{m} c_p R_{TOTAL} T_f = T_r(1 + \dot{m} c_p R_{TOTAL})$

$$\Rightarrow \underline{\underline{T_r = \frac{T_c + \dot{m} c_p R_{TOTAL} T_f}{1 + \dot{m} c_p R_{TOTAL}}}}$$

e) $T_r = 20°C$, $T_f = 40°C$, $T_c = 10°C$

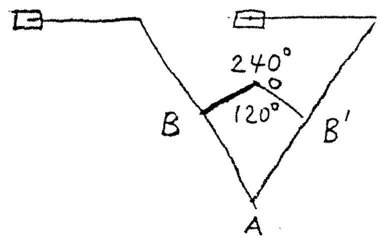
$\dot{m} c_p R_{TOTAL} = (T_r - T_c)/(T_f - T_r) = (20-10)/(40-20) = 0.5$

$\dot{m} c_p R_{TOTAL} = 0.5$

$\dot{m} = 0.5/(1010 \times 0.0115) = \underline{\underline{0.043 \ \text{kg/s}}}$

5. (a) In moving from start to finish of the working stroke OB moves through $240^\circ = \frac{4\pi}{3}$ rad

$\therefore$ time taken $= \frac{4\pi}{3} \cdot \frac{1}{10} =$



(b) See velocity diagram $\therefore$ $v_c = 1.14 \text{ m s}^{-1} \rightarrow$

(c) See velocity diagram $\therefore$ $\omega_{AD} = 3.11 \text{ rad s}^{-1}$

$\omega_{DC} = 1.52 \text{ rad s}^{-1}$

and since the slider at B is constrained to move on AD

$\omega_{slider\,B} = \omega_{AD} = 3.11 \text{ rad s}^{-1}$

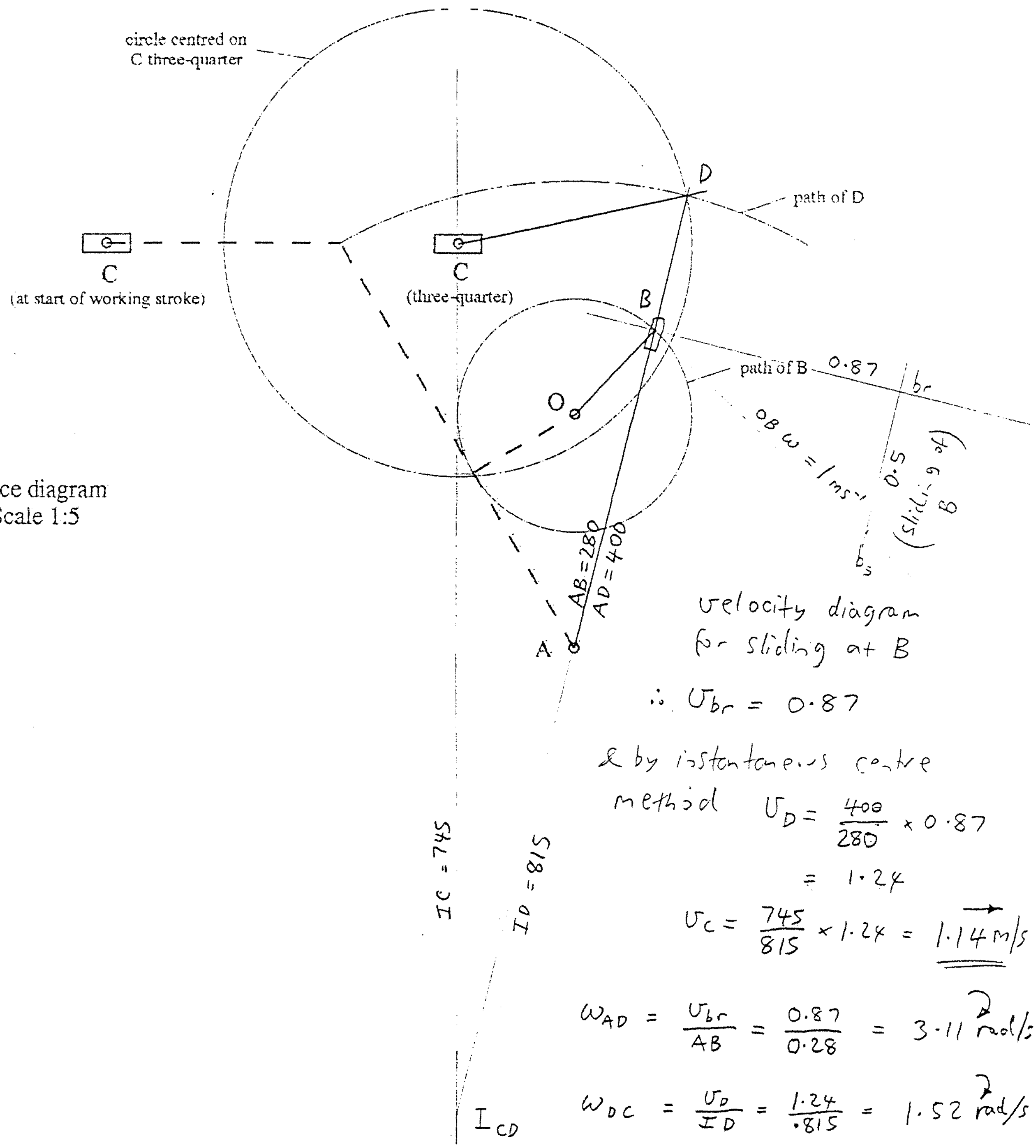
(d) Consider power : (take ω positive)

$$T\omega = 2 \times (|v_c| + |v_{\text{sliding at } B}|)$$
$$+\ 0.1 \times (|\omega_{AD}| + |\omega_{OA} - \omega_{AD}| + |\omega_{CD}| + |\omega_{CD} - \omega_{AD}|)$$

$$\therefore T = \frac{1}{10}\Big(2 \times (1.14 + 0.5)$$
$$+\ 0.1 \times \big(3.11 + (10 - 3.11) + 1.52 + |1.52 - 3.11|\big)\Big)$$

$$= \frac{1}{10}(3.28 + 1.311)$$

$$= 0.46 \text{ Nm}$$



Space diagram
Scale 1:5

$\therefore\ v_{br} = 0.87$

& by instantaneous centre method $v_D = \frac{400}{280} \times 0.87$

$= 1.24$

$v_C = \frac{745}{815} \times 1.24 = \underline{\underline{1.14\ \mathrm{m/s}}}$

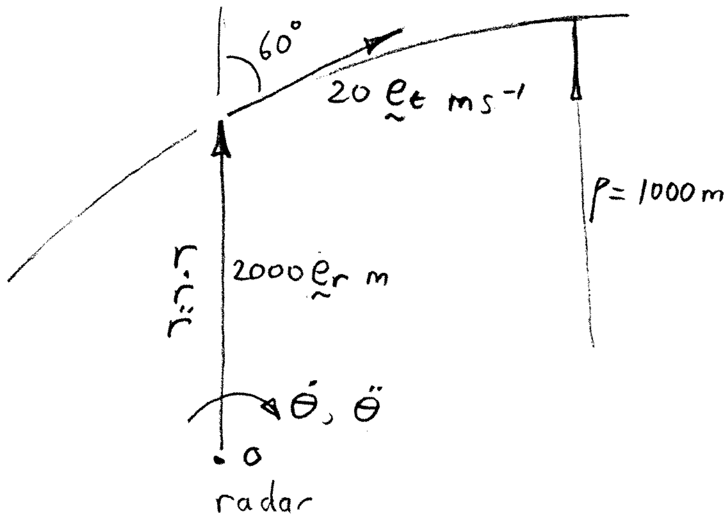
$\omega_{AD} = \frac{v_{br}}{AB} = \frac{0.87}{0.28} = 3.11\ \mathrm{rad/s}$

$\omega_{DC} = \frac{v_D}{ID} = \frac{1.24}{.815} = 1.52\ \mathrm{rad/s}$

Working sheet for Q 5
(to be handed in with your script)

6 (a) The speed of the boat is

$$\frac{72\,000}{3600}\ \mathrm{ms^{-1}} = 20\ \mathrm{ms^{-1}} = \dot{s}$$



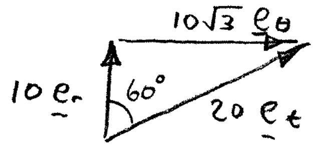
A position vector $\underline{r}$ from the radar to the boat is

$$\underset{\sim}{r} = r\,\underset{\sim}{e}_r \qquad r = 2000\ \mathrm{m}$$

The velocity vector is $\dot{\underline{r}}$

$$\dot{\underline{r}} = \dot{r}\,\underset{\sim}{e}_r + r\dot{\theta}\,\underset{\sim}{e}_\theta$$

$$= 20\ \underset{\sim}{e}_t$$



velocity

From the diagram, resolving, $\therefore\ \dot{r} = 10\ \mathrm{ms^{-1}}\ \uparrow$

(b) $$r\dot{\theta} = 10\sqrt{3}\ \mathrm{ms^{-1}} \qquad \therefore\ \dot{\theta} = \frac{10\sqrt{3}}{2000} = \frac{\sqrt{3}}{200}\ \mathrm{rad\,s^{-1}}\ \curvearrowright$$

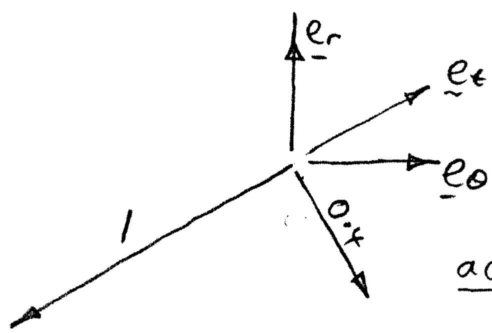
(c) The acceleration vector is $\ddot{\underline{r}}$

$$\ddot{\underline{r}} = (\ddot{r} - r\dot{\theta}^2)\,\underset{\sim}{e}_r + (2\dot{r}\dot{\theta} - r\ddot{\theta})\,\underset{\sim}{e}_\theta$$

$$= \ddot{s}\,\underset{\sim}{e}_t + \frac{\dot{s}^2}{\rho}\,\underset{\sim}{e}_n$$

$$= -1\,\underset{\sim}{e}_t + \frac{20^2}{1000}\,\underset{\sim}{e}_n$$

$$= -1\,\underset{\sim}{e}_t + 0.4\,\underset{\sim}{e}_n$$



acceleration

Resolving: $$\ddot{r} - r\dot{\theta}^2 = -1 \cdot \sin 30^\circ - 0.4\cos 30^\circ$$

$$\therefore\ \ddot{r} = -0.5 - 0.2\sqrt{3} + 2000 \cdot \frac{3}{200^2}$$

$$= -0.696\ \mathrm{ms^{-2}}$$

$$2\dot{r}\dot{\theta} + r\ddot{\theta} = -1\ \cos 30^\circ + 0.4\ \sin 30^\circ$$

$$\therefore\ \ddot{\theta} = \frac{1}{2000}\left(-2 \times 10 \times \frac{\sqrt{3}}{200} - \frac{\sqrt{3}}{2} + 0.2\right) = -0.420 \times 10^{-3}\ \mathrm{rad\,s^{-2}}$$

7 (a) From the data book : $r_p = (1-e)a$ at perigee

$r_a = (1+e)a$ at apogee

$$\therefore \quad r_a = \frac{1+e}{1-e} r_p = \frac{1.6}{0.4} \times 2.5\,R = 10R$$

(b) In orbit, moment of momentum is conserved

$$\therefore \quad r_a v_a = r_p v_p \quad \therefore \quad v_p = \frac{r_a}{r_p} v_a = 4 v_a$$

and energy is conserved

$$\therefore \quad \tfrac{1}{2} m v_p^2 - \frac{GMm}{r_p} = \tfrac{1}{2} m v_a^2 - \frac{GMm}{r_a}$$

(n.b. on Earth $\dfrac{GMm}{R^2} = mg \quad \therefore \quad GM = gR^2$)

$$\therefore \quad \tfrac{1}{2}(16-1) v_a^2 = gR^2 \left(\frac{1}{2.5R} - \frac{1}{10R} \right)$$

$$\therefore \quad v_a = \frac{\sqrt{gR}}{5}$$

(c) $$h = v_a r_a = \frac{\sqrt{gR}}{5} \times 10R = 2R\sqrt{gR}$$

(d) In a circular orbit, radius r_c, speed v_c

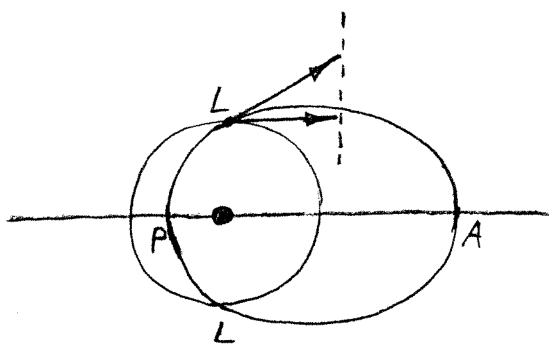
$$h = r_c v_c = 2R\sqrt{gR} \quad \therefore \quad v_c^2 = \frac{4R^3 g}{r_c^2}$$

$$\& \quad \frac{m v_c^2}{r_c} = \frac{GMm}{r_c^2} \quad \therefore \quad v_c^2 = \frac{gR^2}{r_c}$$

$\therefore \; r_c = 4R$

(e) Energy must be removed.

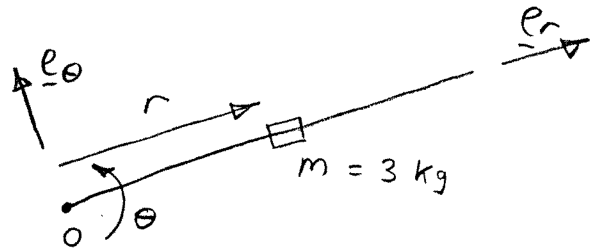
This is because at points L the PE is equal for the two orbits but the KE is greater for the ellipse. This in turn follows because the tangential component of velocity is the same in both orbits for equal moment of momentum.



8 (a) Polar acceleration: $\ddot{\underline{r}} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\underline{e}_\theta$

There is no radial force $\therefore$ no radial acceleration

$\therefore \ddot{r} - r\dot{\theta}^2 = 0$



and $\dot{\theta}$ is constant, 2 rad s^{-1}

$\therefore \ddot{r} - 4r = 0$

(b) The general solution is $r = A\cosh 2t + B\sinh 2t$

Boundary conditions: $r = 1$ at $t = 0$ $\therefore A = 1$

$\dot{r} = 0$ at $t = 0$ $\therefore 2B = 0$ $\therefore B = 0$

$\therefore r = \cosh 2t$

(c) Mass shoots off when $r = 2$ m

$\therefore \cosh 2t = 2$ $\therefore t = \frac{1}{2}\cosh^{-1} 2 = 0.658$ s

Angle rotated $\theta = 2t = 2 \times 0.658 = 1.32$ rad

(d) lateral force $\underline{F}_\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta})\underline{e}_\theta$

$r = \cosh 2t$, $\dot{r} = 2\sinh 2t$, $\dot{\theta} = 2$ rad s^{-1}

but $\cosh^2\theta - \sinh^2\theta = 1$ $\therefore \sinh 2t = \sqrt{3}$ when $\cosh 2t = 2$

$\therefore \dot{r} = 2\sqrt{3}$ at end of rod

& $\ddot{\theta} = 0$ $\therefore \underline{F}_\theta = 3 \times 2 \times 2\sqrt{3} \times 2\ \underline{e}_\theta$

$= 24\sqrt{3}$ N

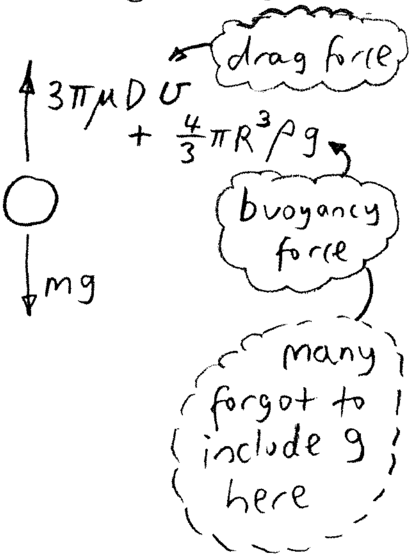
(e) Work done $= \Delta(KE) =$ Final KE $-$ initial KE

$= \frac{1}{2} \times m(\dot{r}^2 + (r\dot{\theta})^2)_{final} - \frac{1}{2}m(r\dot{\theta})^2_{initial}$

$= \frac{1}{2} \times 3((2\sqrt{3})^2 + (2\times 2)^2 - (1\times 2)^2)$

$= 36$ J

9 (a) $F = ma$



$$\therefore \quad mg - 3\pi\mu D v - \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \rho g = m\dot{v}$$

$$\therefore \quad T\dot{v} + v = v_0 \qquad (1)$$

where $T = \dfrac{m}{3\pi\mu D}$

& $v_0 = \dfrac{mg - \frac{1}{6}\pi D^3 \rho g}{3\pi\mu D}$

(b) Terminal velocity (when $\dot{v} = 0$) $= v_0$

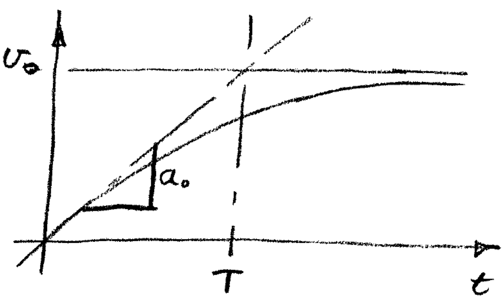
$$v_0 = \frac{(0.01 - \frac{1}{6}\pi(0.01)^3 \times 1000) \times 9.81}{3\pi \times 0.1 \times 0.01} = 9.86 \text{ ms}^{-1}$$

(c) The general solution to (1) is $v = v_0 + Ae^{-\frac{t}{T}}$

Boundary condition : $t = 0,\ v = 0 \quad \therefore A = -v_0$

$$\therefore \quad v = v_0\left(1 - e^{-\frac{t}{T}}\right)$$

$$T = \frac{m}{3\pi\mu D} = \frac{0.01}{3\pi \times 0.1 \times 0.01} = 1.06 \text{ s}$$



initial acceleration = gradient shown

$$= \frac{v_0}{T} = \frac{9.86}{1.06} = 9.30 \text{ ms}^{-2}$$

(d) $$x = \int_0^{2T} v\,dt = v_0\left(t + Te^{-\frac{t}{T}}\right)\Big|_0^{2T}$$

$$= v_0\left(2T + Te^{-2} - T\right)$$

$$= v_0 T\left(1 + e^{-2}\right)$$

$$= 9.86 \times 1.06 \times (1 + 0.135)$$

$$= 11.9 \text{ m}$$

10 (a) Parameters

m	kg	M
k	Nm^{-1}	MT^{-2}
λ	NSm^{-1}	MT^{-1}
F	N	MLT^{-2}
Y	m	L
ω	s^{-1}	T^{-2}

(i) Buckingham's pi theorem $\therefore$ 3 non-dimensional groups

(ii) This is a vibration problem so sensible groups are

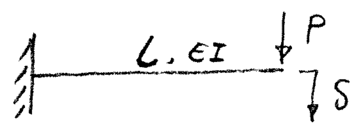
$$G_1 = \frac{\omega^2 m}{k}, \quad G_2 = \frac{kY}{F}, \quad G_3 = \frac{\lambda^2}{km}$$

but any other set of three will do

(b) Being an exact quarter-scale model, mass scales with L^3

$\therefore$ model mass $= \frac{100}{4^3} = 1.56$ kg

The deflection of a cantilever beam



$$\delta = \frac{PL^3}{3EI} \quad \text{so its stiffness } \frac{P}{\delta} = \frac{3EI}{L^3}$$

I has dimensions L^4 & L has dimensions L

so k scales with L

$\therefore$ model stiffness $= \frac{4\times10^4}{4} = 10^4\ Nm^{-1}$

(c) Require G_1, G_2 & G_3 of model & full scale to be identical

$$G_1 = \frac{(2\pi\times4)^2\times100}{4\times10^4} = \frac{(2\pi\times f_m)^2\times1.56}{10^4} \quad \therefore f_{model} = 16\ Hz$$

$$G_2 = \frac{4\times10^4\cdot Y}{200} = \frac{10^4\times\frac{Y}{4}}{F_m} \quad \therefore F_{model} = 12.5\ N$$

$$G_3 = \frac{400^2}{4\times10^4\times100} = \frac{\lambda_m^2}{10^4\times1.56} \quad \therefore \lambda_{model} = 25\ Nsm^{-1}$$

10 (d) Use data book, Case (a)

$$c = \frac{\lambda}{2\sqrt{km}} = \frac{\sqrt{G_3}}{2} = 0.1$$

$$\frac{\omega}{\omega_n} = \sqrt{G_1} = 1.26$$

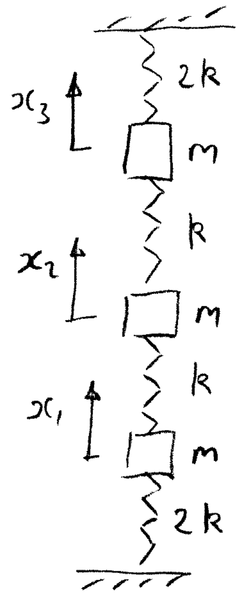
$$\therefore \frac{kY}{F} = 1.7 = G_2$$

Full size : $k = 4 \times 10^4$ N/m, $F = 200$ N $\therefore Y = 8.5$ mm

Model : $Y_m = \frac{Y}{4} = 2.1$ mm

11 (a)

$$[M] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$$



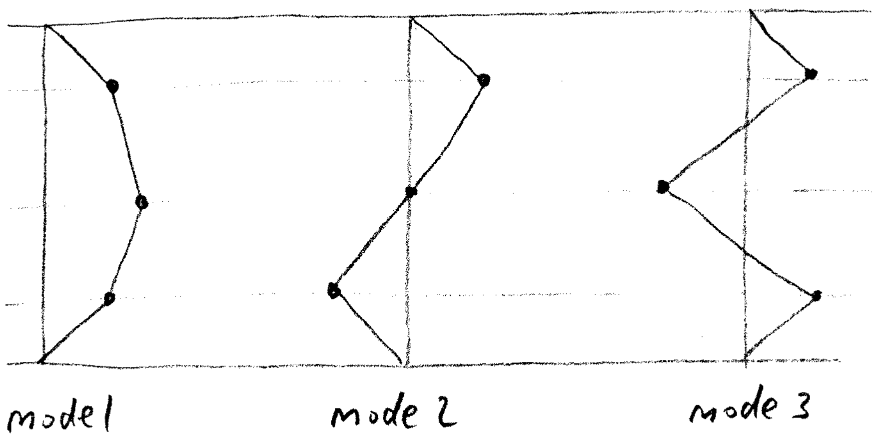
$$\& \quad [k] = \begin{bmatrix} 3k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 3k \end{bmatrix}$$

by inspection (!)

$$\underline{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

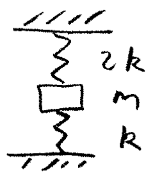
$$\& \quad [M]\,\ddot{\underline{x}} + [k]\,\underline{x} = \underline{0}$$

(b)



Many gave [1 1 1] as a mode 1 shape instead of [1 x 1]

(c) In mode 2 consider



$$\therefore \omega_2 = \sqrt{\frac{3k}{m}}$$

For the other frequencies use an eigenvalue approach

$$\left| [k] - [m]\,\omega^2 \right| = 0 \quad \therefore \quad \begin{vmatrix} 3-\lambda^2 & -1 & 0 \\ -1 & 2-\lambda^2 & -1 \\ 0 & -1 & 3-\lambda^2 \end{vmatrix} = 0, \quad \lambda = \frac{\omega}{\sqrt{k/m}}$$

$$\therefore (3-\lambda^2)\left((2-\lambda^2)(3-\lambda^2) - 1\right) - (3-\lambda^2) = 0$$

$$\therefore (3-\lambda^2)\left[6 - 5\lambda^2 + \lambda^4 - 1 - 1\right] = 0$$

$$\therefore (3-\lambda^2)\left[\lambda^4 - 5\lambda^2 + 4\right] = 0$$

$$\therefore (3-\lambda^2)(\lambda^2 - 4)(\lambda^2 - 1) = 0 \qquad \therefore \omega_1 = \sqrt{\frac{k}{m}}$$

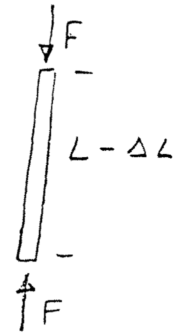
$$\omega_3 = 2\sqrt{\frac{k}{m}}$$

11 (d) Total mass of bar $= \rho A L$ $\therefore$ $m = \frac{\rho A L}{3}$

Total axial stiffness of the model :

$$\frac{1}{k_T} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{k} + \frac{1}{2k} = \frac{3}{k}$$

Axial stiffness of the bar $k_T = \frac{F}{\Delta L}$



Stress $\sigma = \frac{F}{A}$

Strain $\varepsilon = \frac{\Delta L}{L}$

& $\sigma = E\varepsilon$ $\therefore$ $k_T = \frac{EA}{L}$

$\therefore$ $k = \frac{3EA}{L}$