ENGINEERING TRIPOS
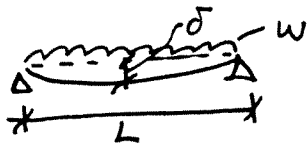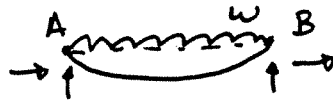## PART IA 2000
PAPER 2: STRUCTURES & MATERIALS SOLUTIONS
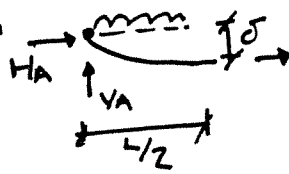
1 a)

find reactions at supports

vertical equilibrium → $V_A = V_B = \dfrac{wL}{2}$

free body diagram   take moments about mid-span $\circlearrowleft$ +ve

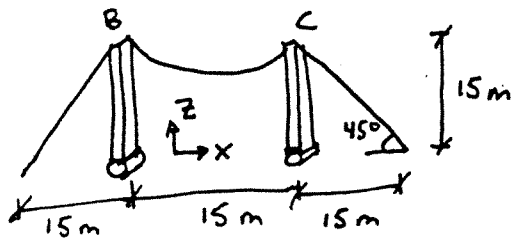$+H_A \cdot \delta + V_A \cdot \dfrac{L}{2} - \dfrac{wL}{2} \cdot \dfrac{L}{4} = 0$

$H_A \cdot \delta = -V_A \cdot \dfrac{L}{2} + \dfrac{wL^2}{8} = -\dfrac{wL^2}{4} + \dfrac{wL^2}{8} = -\dfrac{wL^2}{8}$ → $H_A = -\dfrac{wL^2}{8\delta}$

overall horizontal equilibrium → $H_A + H_B = 0$ → $H_B = -H_A$

∴ reactions at supports

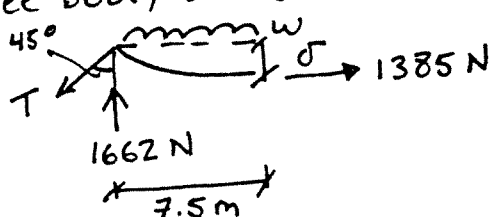

b) i)

column size UB 127×76×13
known parameters
- cable tension at midspan
- compressive force in columns

free body diagram

3 unknowns w, δ, T
combined weight of cable + ice   tension in guy rope

horizontal equilibrium $-T\sin 45° + 1385 = 0$ → $T = 1385\sqrt{2}\,N$
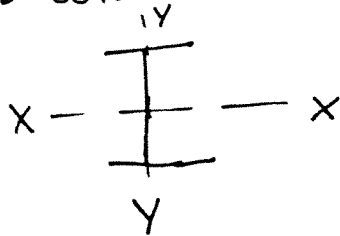
vertical equilibrium $-T\cos 45° + 1662 - w \cdot 7.5 = 0$

$w = \dfrac{-T\cos 45° + 1662}{7.5} = 36.9\,N/m$

1 b) i) continued

COMMENTS ON 1bi) - The most common mistake was not to include the influence of the guy rope in the calculations.

b) ii) consider possible buckling modes:

The columns are oriented as follows:

The columns can buckle about the X-X axis OR the Y-Y axis. The support conditions are different in the two directions so the effective lengths for each case need to be considered.

- **CASE 1 - OUT-OF PLANE BUCKLING (ABOUT X-X AXIS)**

The columns are effectively fixed at the base in this direction. It is possible to assume that either
a) the cables provide no restraint or
b) the cables provide some restraint at the top of the columns

Case 1a)   Case 1b)

$L_E = 2L$ $L_E = \frac{2}{3}L$
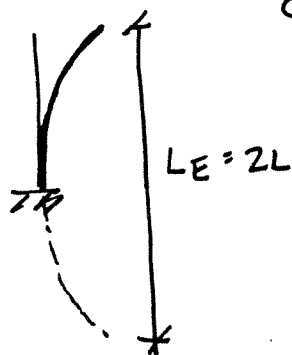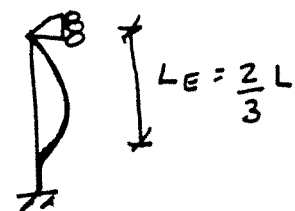
Of the two cases, Case 1a is the more critical.

$$P_{cr} = \frac{\pi^2 EI}{(L_E)^2}$$ where $L_E = 2L$
$I = I_{XX} = 473 \text{ cm}^4$ (from Data Book)

$$= \frac{\pi^2 (210 \times 10^3)_{,N/mm^2} \times (473 \times 10^4)_{,mm^4}}{(2 \times 15000)^2_{,mm}} = 10893 \text{ N}$$

- **CASE 2 - IN-PLANE BUCKLING (ABOUT Y-Y AXIS)**

The columns are pinned at the base in this direction. Assume the cables provide restraint in horizontal direction.
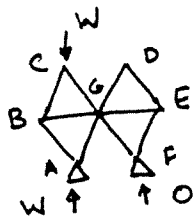
1 b) ii) continued
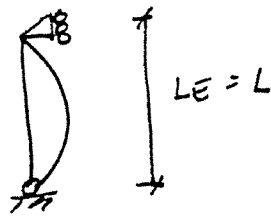
$$P_{cr} = \frac{\pi^2 EI}{(L_E)^2} \qquad \text{where} \qquad L_E = L$$

$$I = I_{yy} = 55.7 \text{ cm}^4 \ \left(\text{from Data Book}\right)$$

$$= \frac{\pi^2 (210 \times 10^3) \times (55.7 \times 10^4)}{(15000)^2} = 5131 \text{ N}$$

$\therefore$ In-plane buckling controls - safety margin

$$= 5131 / 1662 = 3.1 \leftarrow \left(\begin{array}{l}\text{not actually} \\ \text{asked for} \\ \text{in the} \\ \text{question}\end{array}\right)$$

$\uparrow$ applied

2 a) i)

all bars of equal length
$\therefore$ equilateral triangles

free body diagrams

- Joint C

$\Sigma H = 0 \quad T_{BC} \sin 30° = T_{CG} \sin 30° \rightarrow \quad T_{BC} = T_{CG}$

$\Sigma V = 0 \quad -T_{BC} \cos 30° - T_{CG} \cos 30° - W = 0$

$-2T_{BC} \cos 30° = W \quad \rightarrow \quad T_{BC} = \frac{-W}{2\cos 30°}$

- Joint B

$\Sigma V = 0 \quad T_{BC} \sin 60° - T_{BA} \sin 60° = 0 \rightarrow \quad T_{BC} = T_{BA} = \frac{-W}{2\cos 30°}$

$\Sigma H = 0 \quad T_{BC} \cos 60° + T_{BA} \cos 60° + T_{BG} = 0$

$2T_{BC} \cos 60° = -T_{BG} \quad \rightarrow \quad T_{BG} = \frac{W}{2\cos 30°}$

$T_{BC} = -T_{BG}$

$\therefore T_{BA} = T_{GA}$

The right hand side of the structure is unloaded

$\therefore T_{GD} = T_{DE} = T_{EF} = T_{GF} = T_{GE} = 0$

2 a) ii)

Member	Force	Length	real extension $\left(\frac{WL}{EA}\right)$	T^*	T^*e $\left(\frac{WL}{EA}\right)$
AB	$-W/\sqrt{3}$	L	$-1/\sqrt{3}$	$-1/\sqrt{3}$	$1/3$
BC	$-W/\sqrt{3}$	L	$-1/\sqrt{3}$	$-1/\sqrt{3}$	$1/3$
CG	$-W/\sqrt{3}$	L	$-1/\sqrt{3}$	$-1/\sqrt{3}$	$1/3$
BG	$W/\sqrt{3}$	L	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/3$
AG	$-W/\sqrt{3}$	L	$-1/\sqrt{3}$	$-1/\sqrt{3}$	$1/3$
GD					
DE					
EF	0				0
GF					
GE					

$$\Sigma T^*e = \frac{5}{3}\frac{WL}{EA}$$

virtual work

$$\underbrace{\overbrace{\Sigma P^* . \delta}^{virtual} = \underbrace{\Sigma T^* . e}_{real}}$$
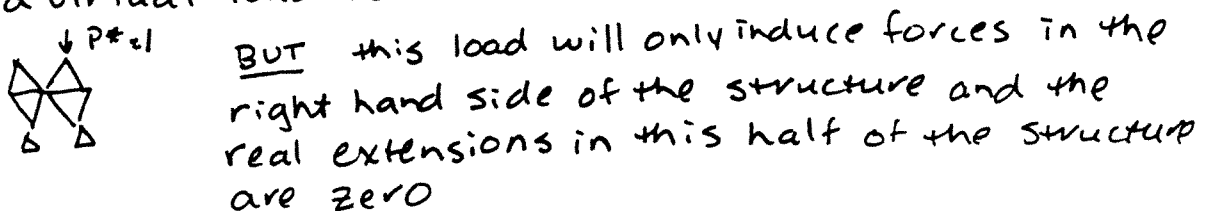
To find vertical displacement at C, apply a virtual unit load vertically at C then use method of virtual work to find real displacement at C.

$\downarrow P^* = 1$ (resulting bar forces T^* compiled in table above)
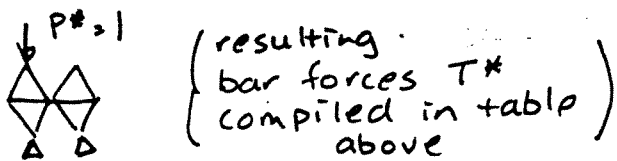
$$1 \cdot \delta_{CV} = \frac{5}{3}\frac{WL}{EA} \quad \Rightarrow \quad \delta_{CV} = \frac{5}{3}\frac{WL}{EA} \quad \text{vertically downwards} \downarrow$$

Consider the vertical displacement at D: apply a virtual load vertically at D.

$\downarrow P^* = 1$ BUT this load will only induce forces in the right hand side of the structure and the real extensions in this half of the structure are zero

$$\therefore \delta_{DV} = 0$$

OR by inspection $\delta_{DV} = 0$ from simple rigid body rotation argument

2a ii)

⊥ED d-line

⊥FG g-line

⊥AB b-line

$\left(\begin{array}{c}c\text{-line}\\ \perp GC\end{array}\right)$

E

D

A,F

G

⊥EF

e-line

$\left(\begin{array}{c}d\text{-line}\\ \perp GD\end{array}\right)$ ⊥AG

g-line

B

C

⊥BC

c-line

$\underset{GB}{\perp}$ $\underset{GE}{\perp}$

b-line   e-line
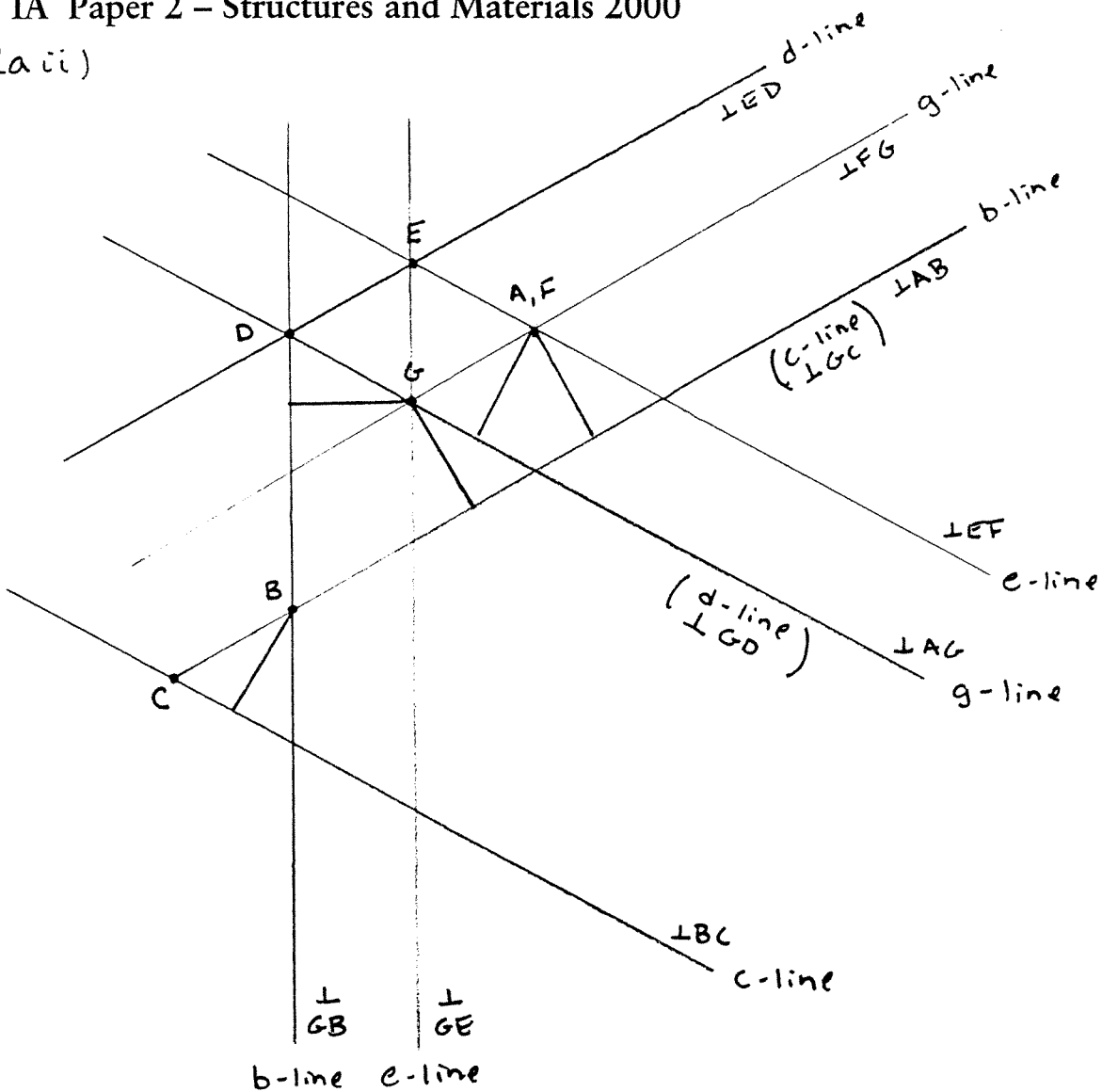
SCALE

$17\,mm : \dfrac{1}{\sqrt{3}}\dfrac{WL}{EA}$

• DISPLACEMENTS OF C

$48\,mm : \dfrac{2.82}{\sqrt{3}}\dfrac{WL}{EA} = 1.63\dfrac{WL}{EA}$ ↓

$51\,mm : \dfrac{3}{\sqrt{3}}\dfrac{WL}{EA} = \sqrt{3}\dfrac{WL}{EA}$ ← $\left(\begin{array}{l}\text{not actually asked}\\ \text{for in the question}\end{array}\right)$

• DISPLACEMENTS OF D

$0\,mm$ ↓

$34\,mm : \dfrac{2}{\sqrt{3}}\dfrac{WL}{EA} = 1.15\dfrac{WL}{EA}$ ← $\left(\begin{array}{l}\text{not actually asked}\\ \text{for in the question}\end{array}\right)$

POSSIBLE SOLUTION USING A DISPLACEMENT DIAGRAM $\left(\begin{array}{l}\text{for this particular problem the displacement}\\ \text{diagram is rather complex}\end{array}\right)$
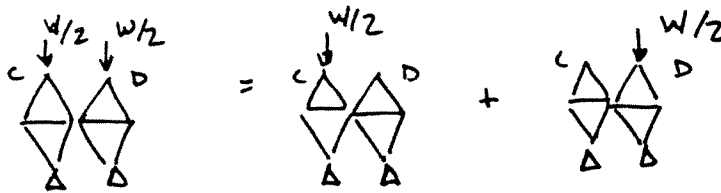
2b)
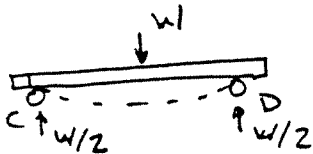
The displacement of the mid-point of the beam will consist of two components

1) the displacement of the supports C & D
2) the displacement due to beam bending.

From part a)ii) the displacements of C & D due to a load W at C were $\delta_{CV} = \frac{5}{3} \frac{WL}{EA}$ $\delta_{DV} = 0$

By superposition:

$\delta_{CV\ TOT} = \frac{1}{2}\left(\frac{5}{3}\frac{WL}{EA}\right) + 0 = \frac{5}{6}\frac{WL}{EA}$

$\delta_{DV\ TOT} = 0 + \frac{1}{2}\left(\frac{5}{3}\frac{WL}{EA}\right) = \frac{5}{6}\frac{WL}{EA}$

From the Structures Data Book, the mid-span deflection of a simply supported beam with a central point load is:

$$\delta = \frac{WL^3}{48EI}$$

$\therefore$ vertical displacement at mid-span $= \frac{5}{6}\frac{WL}{EA} + \frac{WL^3}{48EI}$ $\downarrow$

c)i) The structure is a triangulated structural framework loaded only at a node point. Hence the bars in the structure will primarily carry the loads in tension and compression only. Therefore, welding the joints will make little difference to the displacements. (as demonstrated in the first year lab on plane frameworks).

ii) The structure in part b) is somewhat different. The load W on the beam will cause moments at the fixed joints C & D. The flexural resistance of the joints will reduce the midspan vertical displacement of the beam.

3a)
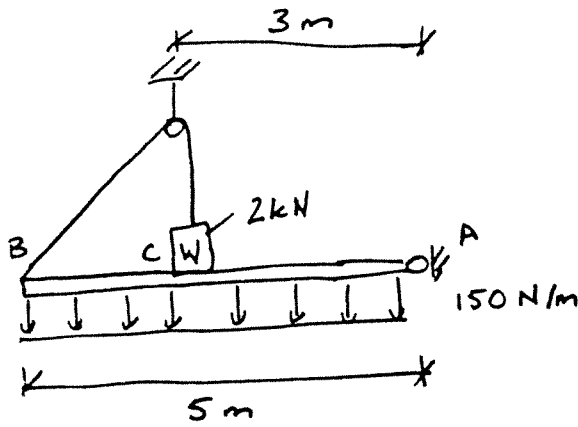
Free Body Diagram
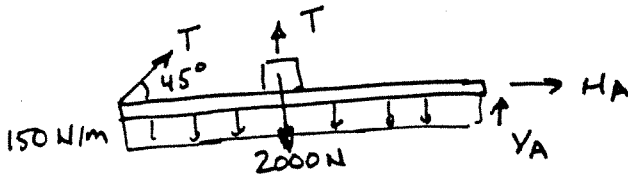
$\Sigma M_A = 0$ $T \sin 45° \times 5 + T \times 3 - 2000 \times 3 - 150 \times 5 \times 5/2 = 0$
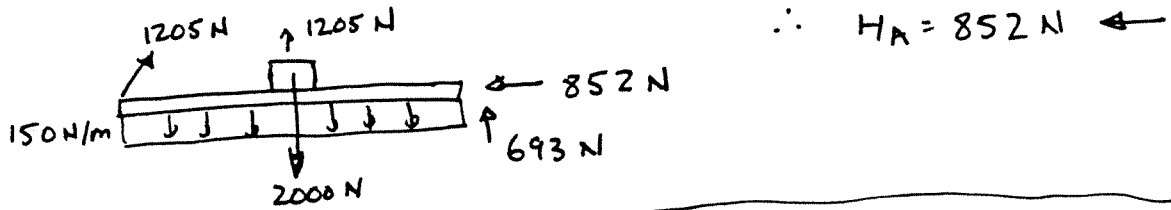
$\circlearrowright +ve$ $T(5/\sqrt{2} + 3) = 7875 N$ → $T = 1205 N$

$\Sigma V = 0$ $T \sin 45° - 150 \times 5 + T - 2000 + V_A = 0$ → $V_A = 693 N \uparrow$

$\Sigma H = 0$ $T \cos 45° + H_A = 0$ → $H_A = -852 N$

$\therefore H_A = 852 N \leftarrow$

NOTES - Common mistakes were to assume $T = W$ or to assume the force applied to the beam at C was equal to W.

b) Beam AB - Free body diagram
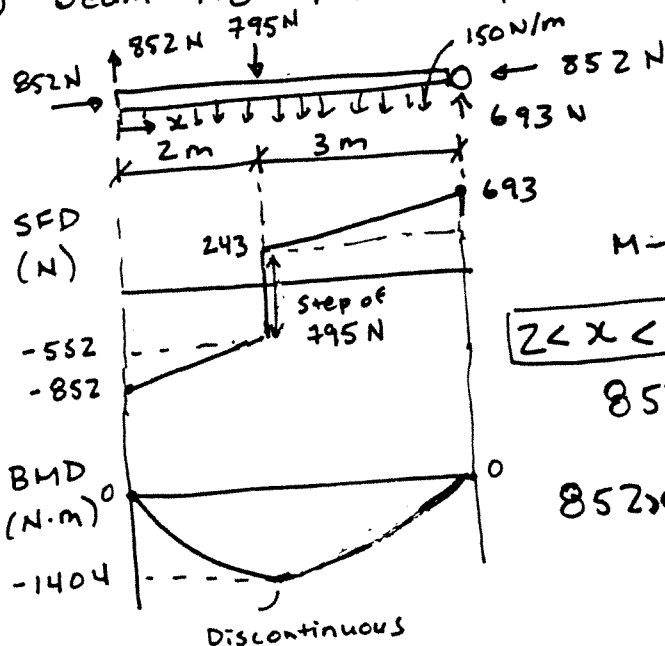
$\boxed{x < 2}$

$852 + V - 150x = 0$ → $V = 150x - 852$

$M - 150x^2/2 + 852x = 0$ → $M = \dfrac{150x^2}{2} - 852x$

$\boxed{2 < x < 5}$

$852 + V - 795 - 150x = 0$

→ $V = 150x - 57$

$852x - 795(x-2) - 150x^2/2 + M = 0$

→ $M = \dfrac{150}{2}x^2 - 57x - 1590$

3b) continued

NOTES - Part 3b) was answered very poorly. It is important to check that shear force diagrams and bending moment diagrams are consistent. Also the moment at a hinge location must be zero!

$\boxed{x < 2}$	$x = 0$	$x = 2$
$V = 150x - 852$	$V = -852$ N	$V = -552$ N
$M = 150\dfrac{x^2}{2} - 852x$	$M = 0$ N·m	$M = -1404$ N·m

$\boxed{2 < x < 5}$	$x = 2$	$x = 5$
$V = 150x - 57$	$V = 243$ N	$V = 693$ N
$M = 150\dfrac{x^2}{2} - 57x - 1590$	$M = -1404$ N·m	$M = 0$ N·m

4
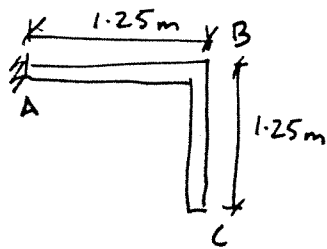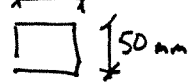
$I = \dfrac{bd^3}{12} = 520833 \text{ mm}^4$

$E_{steel} = 210000 \text{ N/mm}^2$

$\rho_{steel} = 7840 \text{ kg/m}^3$

a) i) vertical reaction at A

$$V_A = 1.25 \times 2 \times \underset{m^2}{0.05^2} \times \underset{kg/m^3}{7840} \times \underset{m/s^2}{9.8} = 480.2 \text{ N}$$

ii) self weight loads on span AB

192 N/m

240 N
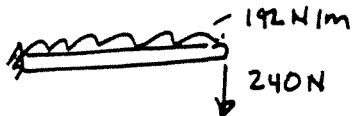
from Structures Data Book

$\downarrow W$

$+$

$W = wL$
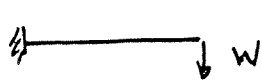
$\delta_1 = \dfrac{WL^3}{3EI} = \dfrac{240 \times (1250)^3}{3 \times 210000 \times 520833} = 1.43 \text{ mm} \downarrow$
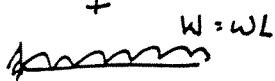
$\delta_2 = \dfrac{WL^3}{8EI} = 0.54 \text{ mm} \downarrow$

$\delta_{VB} = \delta_1 + \delta_2 = 1.43 + 0.54 = 1.97 \text{ mm} \downarrow$

$\delta_{HB} = 0$

$\left.\right\}$ Deflections of B

4 a iii)

end rotations due to loading

$\theta_1 = \dfrac{WL^2}{2EI} = \dfrac{240 \times (1250)^2}{2 \times 210000 \times 520833} = 0.00171 \text{ rad}$

240

+

W=wl

$\theta_2 = \dfrac{WL^2}{6EI} = 0.00057 \text{ rad}$

$\theta_B = \theta_1 + \theta_2 = 0.00229 \text{ rad}$
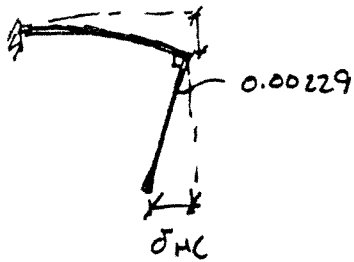
0.00229

$\delta_{HC} = \theta_B \cdot L = 0.00229 \times 1250 = 2.86 \text{mm} \leftarrow$

$\delta_{VC} = 1.97 \text{mm} \downarrow$

δ_{HC}

b)

A   B

B   C
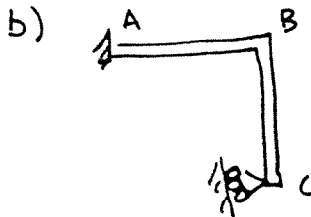
The structure is now statically indeterminate. Must consider compatibility.

A horizontal force at C would result in the following displacements:

P

$\big\}$ PL

① end rotation

P   ②
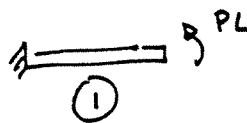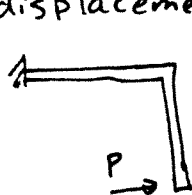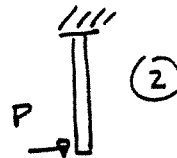
end deflection

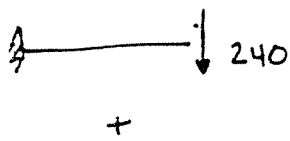$\theta_B = \dfrac{ML}{EI} = \dfrac{PL^2}{EI} \big\}$ $\delta_c = \dfrac{WL^3}{3EI} = \dfrac{PL^3}{3EI} \quad \rightarrow$

$\delta_{HC} = \theta_B \cdot L + \delta_c = \dfrac{PL^3}{EI} + \dfrac{PL^3}{3EI} = \dfrac{4PL^3}{3EI}$

For compatibility $\delta_{HC} = 0$

from before $\delta_{HC} = \left(\dfrac{WL^2}{2EI} + \dfrac{WL^2}{6EI} \right) L = \dfrac{2WL^3}{3EI}$
(part a iii)

$\therefore \dfrac{4PL^3}{3EI} = \dfrac{2WL^3}{3EI} \quad \Rightarrow \quad P = \dfrac{W}{2} = 120 \text{ N}$

Reactions at A + C

120 N   300 N·m

480 N

120 N

$= WL$ $= W/2$
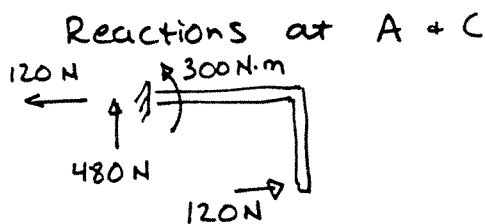
$M_A \big\}$ $M_A + \dfrac{WL^2}{2} + WL - PL = 0$

$M_A = -WL = -240 \times 1.25$
$= -300 \text{ N·m}$

5)

100mm ↕20   100 x 100 x 4
GFRP Square
Hollow Section
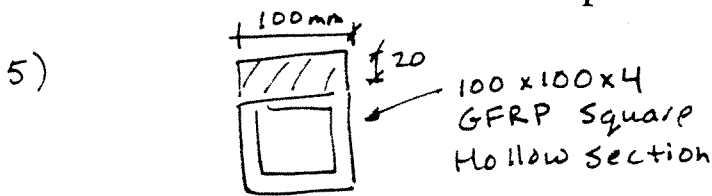
From Data Book
$I_{xx} = I_{yy} = 236 \text{ cm}^4$
$A_G = 15.3 \text{ cm}^2$

a) $E_{conc} = 30 \text{ MPa}$
$E_{GFRP} = 17.2 \text{ MPa}$

width of 'transformed' concrete $= 100\text{mm} \times \dfrac{30 \text{ MPa}}{17.2 \text{MPa}} = 174.4 \text{mm}$

174.4 mm
20 mm
4 mm
8 mm   92 mm
$\bar{y}$
4 mm
100 mm

$A_c = 20 \times 174.4 = 3488 \text{mm}^2$
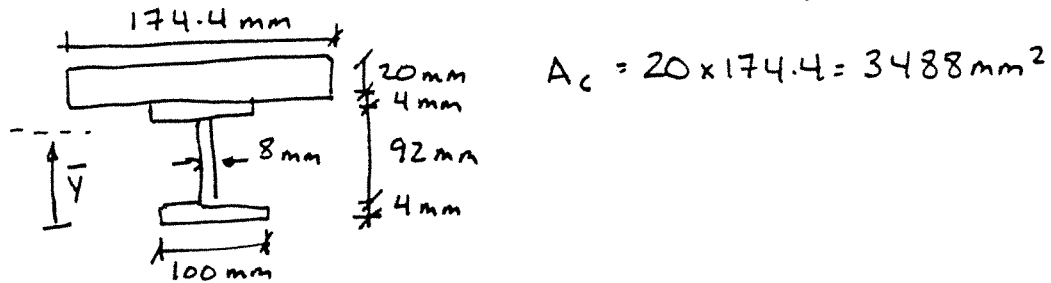
b) i) $\bar{y} = \dfrac{\Sigma y A}{A} = \dfrac{110 \times 3488 + 50 \times 1530}{3488 + 1530} = 91.7 \text{ mm}$

ii) $I_{xx} = I_{xxG} + A_G d_G^2 + I_{xxc} + A_c d_c^2$

$= 236 \times 10^4 + 1530(91.7 - 50)^2 + \dfrac{174.4 \times 20^3}{12} + 3488(110 - 91.7)^2$

$= 6.305 \times 10^6 \text{ mm}^4$

$EI = 17.2 \times 10^3 \times 6.305 \times 10^6 \text{ mm}^4 = 1.084 \times 10^{11} \text{ N.mm}^2$
    (N/mm²)

c)

W
W/2   N/2
2500 mm

$S_{max} = W/2$
$M_{max} = W/2 \times L/2 = 625 W$
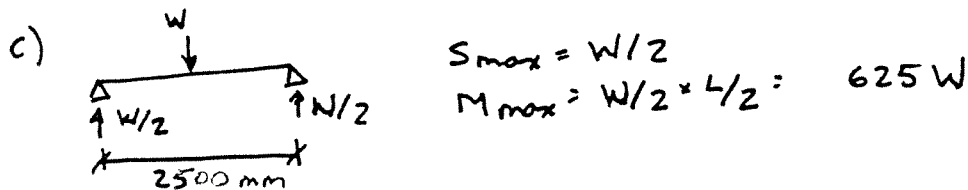
i) $\sigma_c = 60 \text{ MPa}$
convert using
$\dfrac{E_G}{E_c} = 0.573$

$\sigma = \dfrac{My}{I}$ where $y = (120 - 91.7) = 28.3 \text{ mm}$
$I = 6.305 \times 10^6 \text{ mm}^4$
$M = 625 W$

$M = \dfrac{\sigma I}{y} = \dfrac{60 \times 0.573 \times 6.305 \times 10^6}{28.3} = 7.660 \times 10^6 \text{ N.mm}$

∴ $W = M/625 = 12255 \text{ N} \rightarrow 12.3 \text{ kN}$

ii) $\tau = 5 \text{ MPa}$ $q = \tau b = \dfrac{S A_c \bar{y}}{I}$ where $\bar{y} = (110 - 91.7) = 18.3 \text{mm}$
$A_c = 3488 \text{ mm}^2$
$S = W/2$
$b = 100 \text{ mm}$
$I = 6.305 \times 10^6 \text{ mm}^4$

$S = \dfrac{I \tau b}{A_c \bar{y}} = \dfrac{6.305 \times 10^6 \times 5 \times 100}{3488 \times 18.3} = 49390 \text{ N}$

∴ $W = 2S = 2 \times 49390 = 98780 \text{ N} \rightarrow 98.7 \text{kN}$

6.   (a)   Any dislocation is an example of a linear or 1-D defect. A vacancy is a point defect.

An ordered arrangement of linear defects results in the formation of a 2-D defect. Thus, a low-angle boundary is a planar defect. The boundary between a precipitate and the matrix is a 2-D surface, so it is a planar defect.

A spherical cluster of vacancies takes on some 3-D qualities and becomes a volume defect. A local region of BCC structure within an FCC crystal structure represents a precipitate, which can be treated as a volume defect.

(b)   Cold working and precipitation strengthening, for example. In the former, the metal would harden due to interaction effects between an increasing density of dislocations; on heating, the metal would recrystallise and rearrangments of dislocations would result in softening (the dislocation density decreases). In the second case, precipitation of fine particles from a saturated solid solution increases the yield stress due to dislocation obstruction followed by dislocation bowing between precipitates. Increasing the temperature would result in growth of the precipitate (a corresponding increase in spacing between neighbouring particles). The yield stress falls.

(c)   Set up simultaneous equations:

$$663 = \sigma_0 + k\left(\frac{1}{\sqrt{22.5}}\right) \qquad (1)$$

$$622 = \sigma_0 + k\left(\frac{1}{\sqrt{180}}\right) \qquad (2)$$

Subtracting (2) from (1):

$$41 = k\left(\frac{1}{\sqrt{22.5}} - \frac{1}{\sqrt{180}}\right)$$

Solving for k gives 301 MPa$\sqrt{\mu m}$.

Substituting for k in (1) and solving for σ_0 gives

$$\sigma_0 = 663 - \left(301 \; x \; \frac{1}{\sqrt{22.5}}\right) = 599.5 \text{ MPa.}$$

Thus:

$$\sigma_{ys} = (599.5) + \left(301 \; x \; \frac{1}{\sqrt{d}}\right) \qquad \text{(MPa)}$$

Substituting for $d = 11.2\mu m$, then σ_{ys} is equal 689.4 MPa.

__7.__   (a)   Energy (or work) is the product of force and distance. For example, the attractive force does work as it pulls the atoms together from infinite separation:

$$U(x) = \int_x^\infty F dx \,.$$

(A similar integration exists for the repulsive force).

Alternatively, we could write:

$$F = \frac{dU}{dx} \,.$$

Since equilibrium is defined as the point at which the total force is zero, we can write:

$$F = \frac{dU}{dx} = 0 \text{ at equilibrium.}$$

dU/dx represents the slope of the bond-energy curve, and the slope is zero at the bottom of this curve. Hence, x_o corresponds to the minimum of the bond-energy curve.

(b)   (i) The magnitude of $U(x)$ at x_o or the depth of the energy well, is a measure of the strength of the bond, where x_o is the bond length. The values of bond strength and bond length are affected by external forces and energy (temperature change, for instance). The slope of the bond-energy curve at x_o is a measure of the force required to displace atoms which reflects the Young's modulus E of the material; the steeper the slope, the higher the modulus. The curvature of the curve at x_o is also proportional to E; the smaller the radius, the higher the bond stiffness.

(ii) Since bond strength is related to the depth of the bond-energy well, and the application requires high temperature, then we must select material B having the highest melting point.

(iii) As temperature T increases, the atoms gain energy and are able to "move up" the sides of the energy well. The average atomic separation distance increases as T increases. The curve for material B is nearly symmetric in the vicinity of x_o while that for material A is highly asymmetric. This means that coefficient of thermal expansion is greater for material A.

(c)   Assume the axis of the cylindrical specimen lie along the y axis and the x axis lie in a radial (transverse) direction. From information given:

$$\varepsilon_y = \frac{\Delta \ell}{\ell_o} = \frac{(40.019 - 40)}{40} = 4.75 \times 10^{-4}$$

$$\varepsilon_x = \frac{\Delta d}{d_o} = \frac{(9.9986 - 10)}{10} = -1.4 \times 10^{-4}$$

$$v = \frac{\varepsilon_x}{\varepsilon_y} = -\frac{-1.4x10^{-4}}{4.75x10^{-4}} = 0.295$$

$$E = \frac{100}{4.75x10^{-4}} = 210x10^3 = 210GPa$$

$$G = \frac{E}{2(1+v)} = \frac{210}{2(1+0.295)} = 81.1GPa$$

8.   (a)   If, for example, an iron poker is left in a burning hearth of coke, carbon atoms will diffuse (migrate) into the iron. This is solid state diffusion, a process of mass transport involving the movement of one atom species into another. It can occur by random jumps from position to position and becomes more rapid as the temperature T increases because of the higher thermal energy of the atoms. The rate of diffusion can be modelled using the Arrhenius equation. It becomes more rapid for more "open" structures.

(b)   Factors that affect rate of atomic diffusion include:

- Concentration difference between 2 planes (as this difference increases the diffusion flux increases
- As jump distance decreases, the diffusion flux increases
- Temperature.

Thus, diffusion is affected by the density of packing of the material's structure (it is more rapid for more open structures); and it is affected by temperature since vibrating atoms acquire more thermal energy to make "jumps" within the structure, from one atomic plane to another, for instance.

(c)   Diffusion can have adverse effects on engineering structures, e.g., diffusion of oxygen into the hot metallic components of a jet engine (thermal oxidation).

On the other hand, it can have beneficial effects, in the heat-treatment of precipitation hardening of alloys to raise yield strength. Here, the super-saturated solid solution "gives up" its solute atoms by precipitating very small particles of a new crystalline solid to hinder dislocation movement.

Another beneficial effect is diffusion of dopants (beneficial impurities in Si wafers the microelectronics industry. A change in spatial distribution of these impurities however by diffusion can have detrimental effects on performance of the device.

(d)   Combining these observations leads to Fick's first law of diffusion:

$$J = D(\frac{C_1 - C_2}{\Delta x})$$ where D is the diffusion coefficient (cm^2/s). D

contains information about the temperature dependence of the jump frequency as well as the interplanar distances, which depend on crystal structure.

This equation can be written as:

$$J = -D(\frac{dC}{dx}).$$

$$D = D_0 \exp(-\frac{Q}{RT}) \qquad (1)$$

where D_0 is a constant, Q is the activation energy in J/mol for the diffusion process.

The activation energy is the amount of free energy that an atom has to acquire (be supplied as thermal energy in the form of atomic vibrations, for instance) if the atom is to move from one position to another.

A mechanism for Zn diffusing in Cu is by substitutional diffusion (the exchange of positions of the two types of atoms on the copper lattice and by vacancy diffusion.

Calculation based on eqn. 1 and using the data given:

$$D(T_1) = 3.67 \times 10^{-15} = D_0 \exp(-\frac{Q}{RT_1})$$

and

$$D(T_2) = 8.32 \times 10^{-22} = D_0 \exp(-\frac{Q}{RT_2})$$

Dividing the expression evaluated at T_1 by that at T_2 and taking natural logs of both sides of the resulting eqn.:

$$\ln[\frac{D(T_1)}{D(T_2)}] = (-\frac{Q}{R})[(\frac{1}{T_1}) - (\frac{1}{T_2})]$$

Solving for Q:

$$Q = -(8.314)[\ln(\frac{3.67 \times 10^{-15}}{8.32 x_x^{-22}})] x (\frac{1}{\frac{1}{1000} - \frac{1}{600}}).$$

$$= 190,800 \text{ J/mol.}$$

9.   Below is a complete answer although it would not be expected in the allotted time for the candidate to include every point.

(a)   (i) There are usually specific areas of component or structure that are anodic relative to the bulk of the material. These regions corrode preferentially. Examples of "galvanic corrosion" include: compositional variations in the material, and when 2 dissimilar metals are in electrical contact (an electrolyte is required). The metal or alloy with the more negative corrosion potential will corrode preferentially. The galvanic series is a guide for predicting corrosion behavoiur. It ranks the corrosion potentials of common alloys in seawater. For example, Zn is anodic to steel and will protect the steel. In contrast, Cr is cathodic to steel, so once this coating breaks down (scratches), the steel corrodes

(oxidation stops). A coherent oxide structure to the metal substrate structure will generally make for a protective oxide film because of a strong bond.

(ii)   It is reasonable to assume from the question that the oxide growth rate for MgO is linear whilst for NiO it is parabolic. In mathematical terms:

$$y_{Mg}(t) = C_1 t \qquad \text{(linear oxidation)}$$

$$[y_{Ni}(t)]^2 = C_2 t + C_3 \qquad \text{(parabolic oxidation)}.$$

The value of C_1 can be found by noting that

$$0.17 \mu m = C_1 \times 60 \text{ s which gives } C_1 = 2.833 \times 10^{-3} \mu m.$$

Therefore, $y_{(Mg)}$ for 1 hour (3,600 s) exposure is given by:

$$y_{(Mg)} = (2.833 \times 10^{-3} \mu m/s)(3,600s) = 10.2 \mu m.$$

For nickel, $y_{Ni}(0) = 0$, implies that $C_3 = 0$. Thus, substituting $y_{Ni}(60) = 0.17 \mu m$, and solving for C_2:

$$C_2 = (0.17 \mu m)^2/60s = 4.82 \times 10^{-4} \ \mu m^2/s$$

Therefore, y_{Ni} for one hour exposure is:

$$[y_{Ni}(3,600)]^2 = (4.82 \times 10^{-4} \mu m/s)(3,600s) = 1.734 \mu m^2$$

$$y_{Ni}(3,600) = \sqrt{1.734 \ \mu m^2} = 1.32 \mu m.$$

(b)   Based on what was said on galvanic couples, the plate material chosen should be as least as anodic as nickel. The galvanic series shows that steel is the most anodic and is therefore the best material for the plate.

10.   (a)   The minimum detectable crack size, a_{min} is the smallest crack that can be detected by a specified technique. It is not a constant length and depends on the sensitivity (resolution) of the particular device used and the skill of the operator. There is always a possibility, however, that cracks of size greater than a_{min} can go undetected between routine inspections of components and structures.

A crack reaches critical size a_c the following relation is satisfied:

$$a_c = \frac{K_c^2}{Y \pi \sigma_f} \qquad (1)$$

preferentially. The problem of the scratched layer of Cr is that it represents a highly unfavourable anode-to cathode area ratio. On the microscopic level, for example, Cr carbides can form at grain boundaries when stainless steel in regions of a weldment. Adjacent are depleted regions in Cr. A region less than 13% Cr is not stainless and becomes anodic with respect to the bulk material. Adding Ti and Nb to the alloy helps since these elements have a stronger affinity for C. Post-welding heat-treatment can redissolve the carbides and diffuse Cr back into solution. In this respect, whenever possible, select single phase alloys for use in aggressive environments. Solid solution strengthened a;loys is a more judicial choice than either cold working or the addition of a second phase. It is good practice to subject single-phase alloys to an additional solution treatment to promote diffusion and chemical homogenisation.

The severity of the problem depends on:

(1)   potential difference between the 2 half-cell reactions;

(2)   kinetic factors, e.g., polarisation and passivation effects;

(3)   ratio of the anodic surface area to the cathodic surface area.

The corrosion rate scales with the current density. This form of corrosion can be avoided by unnecessary coupling of dissimilar materials, or by keeping the anode-to-cathode area ratio as large as possible.

(a)   (ii) Cathodic protection is based on reversing the direction of the metal corrosion reaction, i.e., "force" the metal to be the cathode. One way is to set up an external voltage source to supply the metal with extra electrons. Do this by connecting the metal to the negative terminal of a DC power supply. It also requires that a piece of scrap iron, for example, is used as the anode in the impressed voltage cell. (Here, a sketch can be shown of, for example, a buried steel pipe connected to the negative terminal and the piece of scrap to the positive terminal of the power supply. The design suffers from an unfavourabl anode to cathode area ratio, and the scrap iron will heve to be regularly replaced. The problem is reduced by wrapping the pipe in protective plastic o minimise the amount of exposed pipe to minimise the area of the cathode.

Another version involves the use of a sacrificial anode. A galvanic cell is formed intentionally by joining a more anodic metal in electrical contact to the metal to be protected. Hence, the sacrificial anode corrodes since the protected metal is supplied with electrons. Zn and Mg can be chosen as sacrificial anodes. Ocean vessels can be protected in this way. So are car radiators using Zn slugs.

(b)   (i) For an oxide to be protective, it must be continuous, adherent, and completely cover the material. If the specific volume of the oxide is much larger than the specific volume of metal consumed to create this oxide, a compressive stress may lead to cracking or spalling of the brittle oxide layer. In addition, the oxide must have a similar coefficient of thermal expansion to the metal substrate to prevent spalling during possible thermal cycling. The rate limiting step in the formation of an oxide layer is dependent on the diffusion of either oxygen ions metal ions or both through the oxide thickness, and their relative diffusivities. The parabolic growth rate of an oxide is generally but not always a characteristic of a protective oxide. If electron motion is the rate limiting step (e.g., aluminium oxide), then the oxide is likely to be protective soon after it has initially formed

where K_c is the fracture toughness of the material, σ_f is the fracture stress of the component (for elastic fracture), and Y is a geometrical factor that depends essentially on the shape of the component containing the crack. (More often than not, π is included in Y).

The fracture toughness K_c is the critical value of the crack tip stress intensity factor K at fast (unstable) fracture. In fracture mechanics terminology, it is related to the toughness of the material G_c or *critical strain energy release rate:*

$$K_c = \sqrt{E G_c}.$$

The design life of a component is some fraction of the time it would take for that component to fail by fast crack propagation if it were left in service. For example, the design life could be determined by the time it takes for the crack to attain 50% of the critical crack size. The critical crack size could be calculated using, eqn (1) by inserting the working (design) stress for σ and known value of K_c. When combined with a crack growth law and a knowledge of the initial (inherent) crack size in the component determined by NDT, we can calculate the time to failure. The design life might then be fixed as the time it takes for the largest pre-existing crack in the structure to reach a particular length (less than the critical crack length) that can readily be detected in service. A safety factor is often included in the design calculation.

(b)   For the pressure vessel containing a longitudinal crack loaded cyclically,

$$\Delta K = 1.12 \, \Delta\sigma \sqrt{\pi a} \qquad (\text{MPa} \sqrt{m}).$$

Hence, using the data provided:

$$\Delta\sigma = \frac{(10-2)(0.1)}{0.01} = 80 MPa$$

The expected design life is ($6 \times 365 \times 5 = 10,950$ cycles.

The critical crack size at fracture, a_c, is obtained as follows:

$$a_c = \frac{(30^2)}{(1.12^2)(100)^2 \pi}$$

(where σ_{max} (= 100MPa) is the max. stress that occurs at max. pressure (= 10MPa).

This gives a critical crack size of about 5mm.

Next, estimate an initial crack size a that will permit a design life of 10, 950 cycles which requires integrating the crack growth law:

$$\int_0^{10950} dN = \int_{a_o}^{0.005} \frac{da}{8.5x10^{-12}(1.12\Delta\sigma\sqrt{\pi a})^4}$$

or

$$59.2 = \int_{a_o}^{0.005} \frac{da}{a^2} \qquad \text{which gives } a_o \text{ of about 2mm.}$$

Recall: the non-destructive inspection capability of the aircraft's maintenance crew is such that there is a finite probability of an undetected longitudinal crack of 3.5mm length present in the pressure vessel. Therefore, in-service inspection should be scheduled at intervals such that a crack 3.5mm deep does not become critical between successive inspections. To estimate the number of cycles between inspections, N_I, the following equation should be solved:

$$\int_{0}^{N_I} dN = \int_{3.5x10^{-3}}^{6x10^{-3}} \frac{da}{8.5x10^{-12}(1.12\Delta\sigma\sqrt{\pi a})}$$

or

$$5.406 \times 10^{-3} N_I = \int_{3.5x10^{-3}}^{6x10^{-3}} \frac{da}{a^2}$$

This gives N_i of about 2,000 cycles which corresponds to about 10 months.

In summary, the recommendation is to specify a NDT technique that is capable of detecting cracks as small as (approximately) 2mm and not to miss cracks deeper than 3.5mm. Also, the pressure vessel must be inspected using the same procedure no longer than 10 months.