CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT

EXAMINATION PAPERS

2000

ENGINEERING TRIPOS: 1A

ENGINEERING TRIPOS PART IA

Monday 12 June 2000 9 to 12

Paper 1

MECHANICAL ENGINEERING

Answer not more than **eight** *questions, of which not more than* **three** *may be taken from Section A, not more than* **three** *from Section B and not more than* **two** *from Section C.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

All questions carry the same number of marks.

Answers to questions in each section should be tied together and handed in separately.

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SECTION A

Answer not more than **three** *questions from this section.*

1 Two flows of air, which may be treated as a perfect gas, enter the mixing device shown in Fig. 1. The first flow has a pressure of 2 bar, temperature 600 K, and velocity 100 ms^{-1} and enters through a flow area of 0.1 m^2. The second flow has pressure 2 bar, temperature 800 K, velocity 200 ms^{-1} and has the same flow area of 0.1 m^2. Immediately downstream of the two flow inlets there is an array of bars to enhance mixing. Downstream of the bars the flow is well mixed and the velocity is 180 ms^{-1} through a flow area of 0.2 m^2.

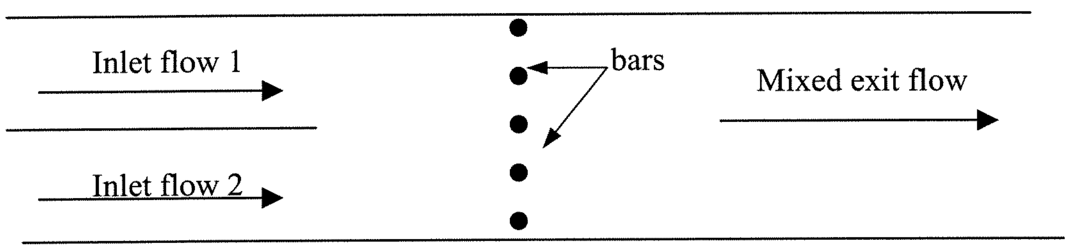

Fig. 1

(a) Given that the two entry flows have the same pressure, describe the shape of the streamlines just downstream of where the inlet flows first meet. [2]

(b) Calculate the density of the exit flow. [4]

(c) Assuming that the flow is adiabatic, calculate the temperature of the exit flow. [7]

(d) Ignoring friction on the walls of the mixing device, calculate the net force exerted by the flow on the bars and indicate clearly its direction. [7]

2 (a) Starting from the first law of thermodynamics applied to a system undergoing a small change of state show that, for an adiabatic, fully resisted (reversible) process involving a perfect gas:

$$Tv^{\gamma-1} = \text{constant}$$

where T is the temperature, v the specific volume and γ the adiabatic index. [4]

(b) Draw the pressure-volume diagram for the air-standard Diesel cycle and describe the four processes. [4]

(c) An air-standard Diesel cycle has a volumetric compression ratio of 20 and a cut-off ratio of 3. Given that the air temperature at the start of the cycle is 300 K, calculate the temperature at:

(i) the end of compression;
(ii) the end of heat input;
(iii) the end of expansion. [4]

(d) Evaluate the work done by the system per unit mass for each of the four processes. Calculate the net work done per unit mass. [4]

(e) Calculate the efficiency of the cycle. [2]

(f) Explain why the overall efficiency of a diesel engine is lower than that calculated above. [2]

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3 (a) Explain why, for a homogenous stationary fluid, pressure is only a function of vertical position. [4]

(b) Show that for a water-over-mercury manometer the pressure difference measured is given by:

$$P_1 - P_2 = (\rho_m - \rho_w)gh$$

where ρ_w and ρ_m are the densities of water and mercury respectively, g the acceleration due to gravity and h the vertical height of mercury supported. What constraint must be placed on the vertical position of locations 1 and 2? [6]

(c) In Fig. 2 water flows out of a large reservoir through a horizontal pipe of cross sectional area A which exits to the atmosphere. The height of water above the pipe is H. There is a venturi in the horizontal pipe with minimum area a. A water-over-mercury manometer is used to measure the pressure difference between the venturi and elsewhere in the horizontal pipe. Show that the height of mercury in the manometer is given by:

$$h = \left(\frac{\rho_w}{\rho_m - \rho_w}\right)\left(\frac{A^2}{a^2} - 1\right)H$$ [10]

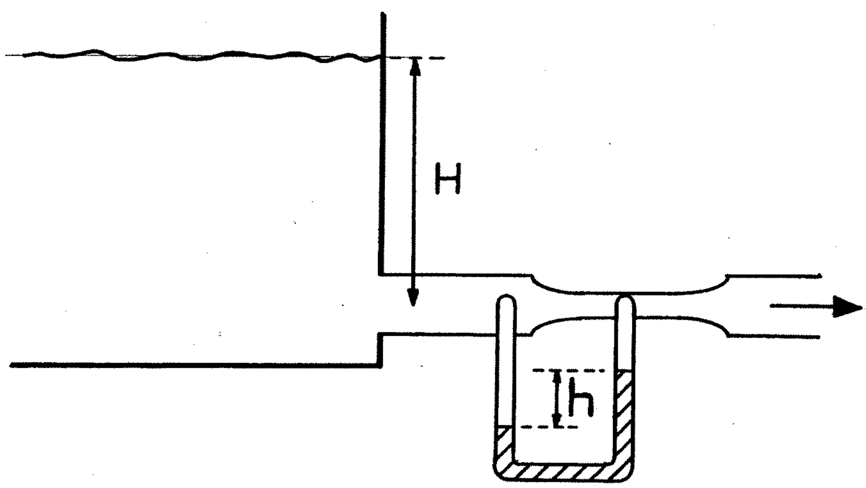

Fig. 2

4 (a) State Fourier's law of conduction for one-dimensional heat flow and show that for uniform thermal conductivity λ and fixed cross-sectional area A the thermal resistance can be written:

$$R = \frac{\Delta x}{\lambda A}$$

where Δx is the thickness of the material. [2]

(b) State the relationship between the heat flow and the surface heat transfer coefficient h for convection. Show that the thermal resistance can be written:

$$R = \frac{1}{hA}$$ [2]

(c) A room has a total wall area of 100 m^2 with an internal surface heat transfer coefficient of 10 $Wm^{-2}K^{-1}$ and external surface heat transfer coefficient of 20 $Wm^{-2}K^{-1}$. The walls consist of a 100 mm thick layer of insulation (thermal conductivity of 0.2 $Wm^{-1}K^{-1}$) sandwiched between two layers of brick (thermal conductivity of 0.72 $Wm^{-1}K^{-1}$) each 180 mm thick. Calculate the total thermal resistance R_{total} (you may ignore heat loss through the floor and ceiling). [6]

(d) A re-circulating warm-air furnace takes air from the room at temperature T_r and returns it at temperature T_f with negligible kinetic energy. If the temperature outside the room is T_c, show that the room temperature is:

$$T_r = \frac{T_c + R_{total}\, \dot{m}\, c_p T_f}{1 + R_{total}\, \dot{m}\, c_p}$$

where $\dot{m}$ is the mass flow rate of air through the furnace and c_p the specific heat capacity at constant pressure for the air. [6]

(e) The required room temperature is 20 °C, the furnace delivers warm-air at 40 °C and the temperature outside of the room is 10 °C. Calculate the required mass flow rate of warm-air. [4]

SECTION B

Answer not more than **three** *questions from this section.*

5 The quick-return mechanism shown in Fig. 3 is driven by the crank OB rotating clockwise at a constant angular velocity of 10 rad s^{-1} such that B slides along AD. The length of the crank OB is 100 mm, link CD is 200 mm, link AD is 400 mm and the total stroke of the slider C is 400 mm. At each end of the working stroke link CD is horizontal. O lies directly above A, as does C at the end of the working stroke. In Fig. 3(a) the mechanism is shown at the beginning of the working stroke when angle OAD is 30° and angle OBA is 90°.

(a) Find the time taken for the working stroke. [2]

(b) Consider the mechanism at the three-quarter position, shown in Fig. 3(b), when C has travelled 300 mm through its working stroke. Find the velocity of C at the three-quarter position. [6]

(c) Find the angular velocities of AD, DC and the slider at B when C is at the three-quarter position. [6]

(d) A frictional torque of 0.1 Nm acts at each of the pin joints at A, B, C and D and a frictional force of 2 N acts at each slider at B and C. Find the torque required at O to drive the mechanism when C is at the three-quarter position. [6]

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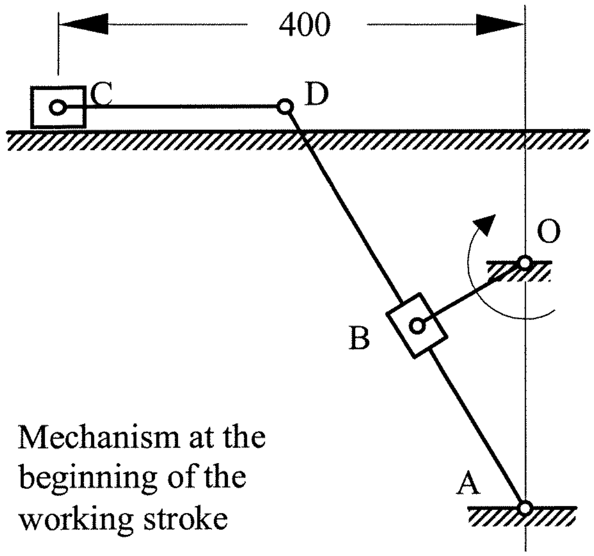
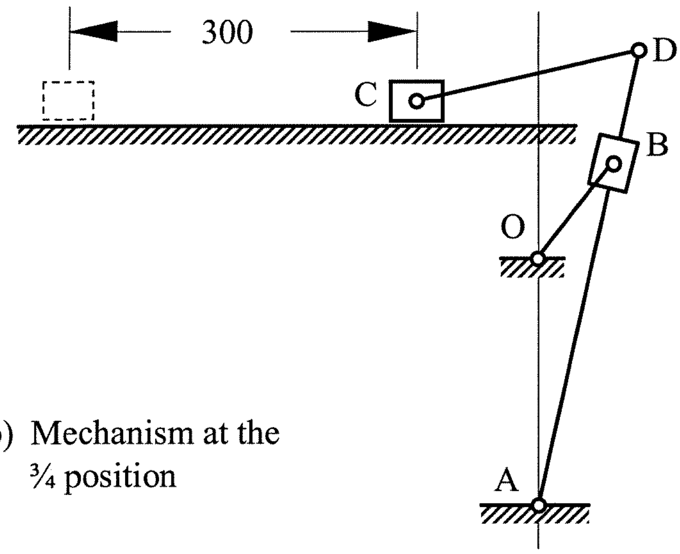


(a) Mechanism at the beginning of the working stroke



(b) Mechanism at the ¾ position

Fig. 3

(Not to Scale)

(A separate version of this figure, which is to scale, is supplied for your working).

6 A tracking radar detects a boat due north at a range $r = 2$ km. The radar points continually at the boat. The boat is moving at a speed of 72 kmh^{-1} on a curved path whose instantaneous centre of curvature is 1 km to the right of the boat. The course of the boat is 60° (measured clockwise from north) and its acceleration along its path is -1 ms^{-2}. Determine:

(a) the rate $\dot{r}$ at which the range is increasing; [4]

(b) the rate of turning $\dot{\theta}$ of the radar; [4]

(c) the acceleration rate $\ddot{r}$ at which the range is increasing; [6]

(d) the rate $\ddot{\theta}$ at which the turning rate is increasing. [6]

7 The perigee of an earth satellite is 2.5 R measured from the centre of the earth where R is the radius of the earth. The eccentricity of the orbit is 0.6 and the value of the acceleration due to gravity at the earth's surface is g .

(a) Determine the apogee of the orbit. [2]

(b) Show that the minimum speed of the satellite in orbit is:

$$0.2\sqrt{Rg}$$ [6]

(c) Find the moment of momentum per unit mass of the satellite. [2]

(d) Find the radius of a circular orbit whose moment of momentum per unit mass is equal to that of the satellite in its elliptical orbit and show both orbits on a single sketch. [6]

(e) Decide whether energy must be supplied or extracted in order to shift the satellite from the elliptical orbit into the circular orbit. Give reasons for your answer. [4]

8 A smooth rod OA of length 2 m is rotating in a horizontal plane about O at a constant angular velocity of 2 rad s^{-1}. A mass of 3 kg, ordinarily free to slide along the rod, is initially held in place on the rod at a distance of 1 m from O. At $t = 0$ the mass is released and it begins to slide freely along the rod.

(a) Find a differential equation which describes the free motion of the mass on the rod. [4]

(b) Show that the motion of the mass can be expressed as:

$$r = \cosh 2t$$

where r is the distance of the mass from O. [6]

(c) Find the time at which the mass leaves the end of the rod and the corresponding angle through which the rod has rotated since the mass was released. [2]

(d) Find the magnitude of the force acting on the mass just before it leaves the rod. [4]

(e) Evaluate the total amount of work done on the mass by the rod. [4]

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SECTION C

Answer not more than **two** *questions from this section.*

9 The force F required to propel a small sphere of diameter D at constant speed V through an oil of viscosity μ is given by Stokes' Law:

$$F = 3\pi \mu D V$$

(a) Obtain a differential equation which governs the general motion of a small sphere of mass m and diameter D sinking under the action of gravity through an oil of viscosity μ and density ρ. [4]

(b) What is the value of the maximum steady velocity ultimately attained by a 0.01 kg sphere of diameter 10 mm sinking through oil of viscosity $\mu = 0.1$ kgm^{-1}s^{-1} and density 1000 kgm^{-3}? [4]

(c) The same sphere is released from rest in its oil. Find an expression for the subsequent variation of its speed with time. On a suitable sketch indicate the time constant T and the initial acceleration of the sphere. Give numerical values for both. [6]

(d) Find the distance travelled by the sphere after time $t = 2T$. [6]

10 An aluminium aircraft wing is flexible and behaves much like a cantilevered beam. For the purposes of studying its vibration characteristics, however, the wing is modelled more simply as a single-degree-of-freedom vibrating system comprising an equivalent mass m supported by a spring k in parallel with a viscous dashpot λ. The wing is acted upon by a steady sinusoidal force of amplitude F and angular frequency ω. The steady-state response amplitude is Y. These are the only variables of any significance.

(a) (i) Explain why there are three non-dimensional parameters required for this study of wing vibration. [2]

(ii) Identify three non-dimensional parameters convenient to the study of forced harmonic vibration. [4]

The parameters for a certain wing are m = 100 kg, $k = 4\times10^4$ Nm^{-1} and λ = 400 Nsm^{-1}. It is excited by a force of amplitude 200 N at a frequency of 4 Hz. The behaviour of the wing is to be investigated by studying an exact quarter-scale model, also in aluminium.

(b) Use physical reasoning to explain why the model should have an equivalent mass of 1.56 kg and a stiffness of 1×10^4 Nm^{-1}. You may wish to consult formulae for the deflection of beams given in the Structures Data Book. [4]

(c) For complete dynamic similitude determine for the model the required values of the dashpot rate λ, the excitation frequency ω and the force amplitude F. [6]

(d) Deduce values for the vibration amplitude Y of both the scale model and of the full-scale wing. [4]

11 A uniform bar of length L is constrained at each end by rigid supports as shown in Fig. 4(a). The bar has cross-sectional area A and is made of a material with density ρ and Young's Modulus E. Axial vibration of the bar is to be investigated approximately by using the three-degree-of-freedom discretized model shown in Fig. 4(b). The three masses shown each have mass m and the four springs have stiffnesses $2k$, k, k and $2k$ as shown.

(a) Obtain, in terms of m and k, the mass and stiffness matrices for the vibration of the discretized system and write down the matrix equation which governs its force-free motion. [6]

(b) Sketch *without calculation* the form of the mode shapes for the vibrating system. [4]

(c) In one of the modes the central mass remains at rest. Find by inspection the corresponding natural frequency and by matrix methods or otherwise find the remaining natural frequencies. [6]

(d) For the discrete system to be a reasonable model of the bar each mass is chosen to be equal to one third the total mass of the bar and the total axial stiffness of the bar and the model are the same. Show that $k = \frac{3EA}{L}$ and deduce a suitable value for m in terms of ρ, A and L. [4]

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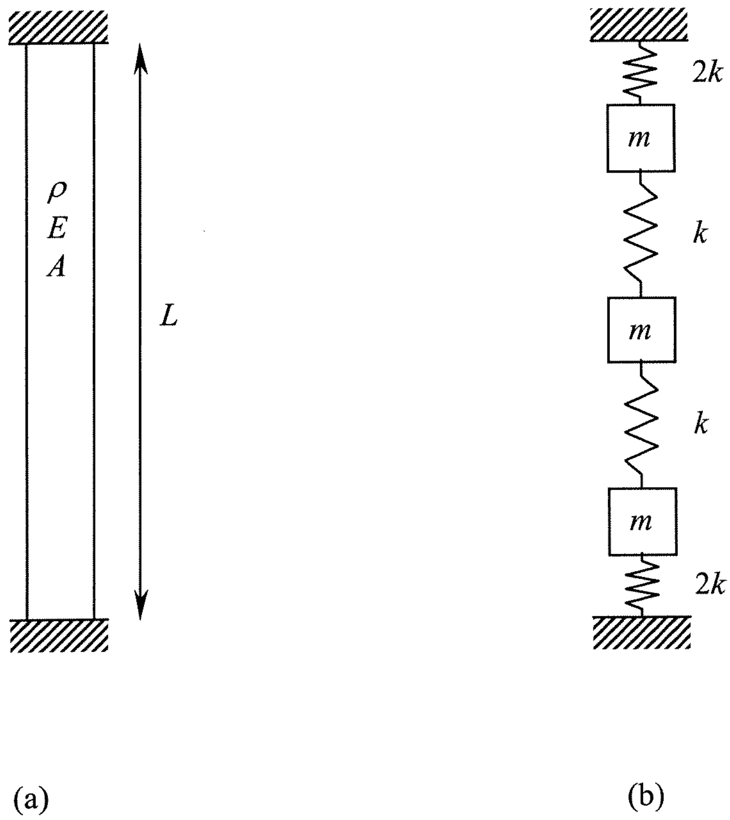

Fig. 4

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(Included also scale version of Fig 3)

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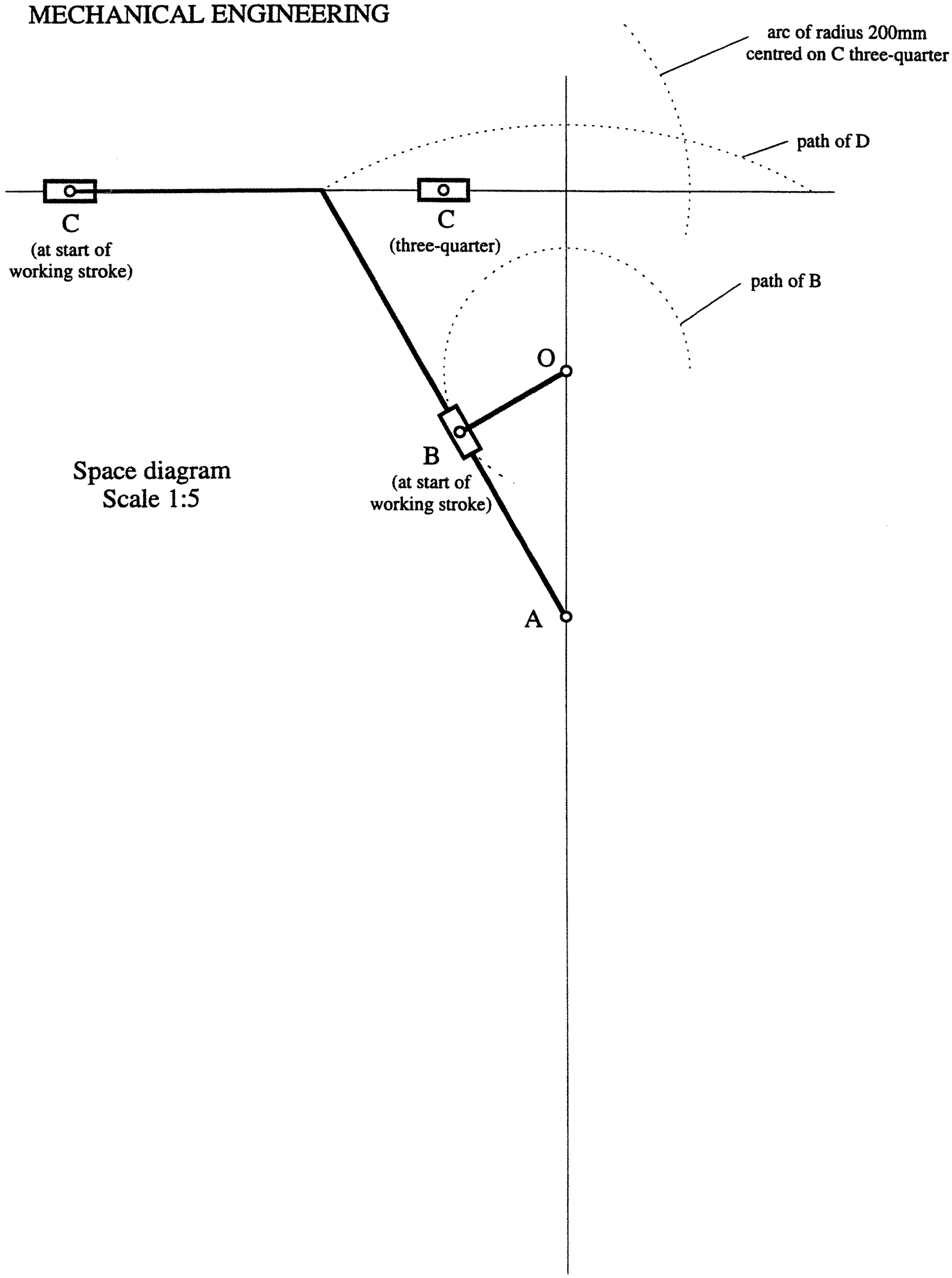

Working sheet for Q 5
(may be handed in with your script)