ENGINEERING TRIPOS PART 1A

Monday 12 June 2000 1.30 to 4.30

Paper 2

STRUCTURES AND MATERIALS

Answer not more than **eight** *questions, of which not more than* **four** *may be taken from Section A, and not more than* **four** *from Section B.*

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

All questions carry the same number of marks.

Answers to Sections A and B should be tied together and handed in separately.

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SECTION A

Answer not more than **four** *questions from this section.*

1 (a) A flexible cable spans between two level supports that are a distance L apart. The cable is subjected to a uniform distributed load w, per metre of horizontal span. The central dip of the cable is δ. Derive an equation for the horizontal and vertical reactions at the supports in terms of L, w and δ. [4]

(b) Figure 1 shows two steel Universal beams, section size 127×76×13, that are connected at the base by frictionless hinges E and F that allow the columns to rotate only in the plane X-Z. The columns are used to support a flexible inextensional cable that spans from point B to point C. Additional guy ropes are used to attach the top of each column to the ground. During the winter, a uniform layer of ice forms on the cable BC. Assume that the combined weight of cable and ice applies a uniform load per metre of horizontal span and there is no ice on the guy ropes. Neglect the self weight of the columns and guy ropes.

(i) Determine the combined weight of the cable and ice that corresponds to the case where the tension in the cable at the midpoint between B and C is 1385 N *and* the compressive force in each of the columns BE and CF is 1662 N. [8]

(ii) Determine the buckling load of the columns. State any assumptions you make about the effective length of the columns. [8]

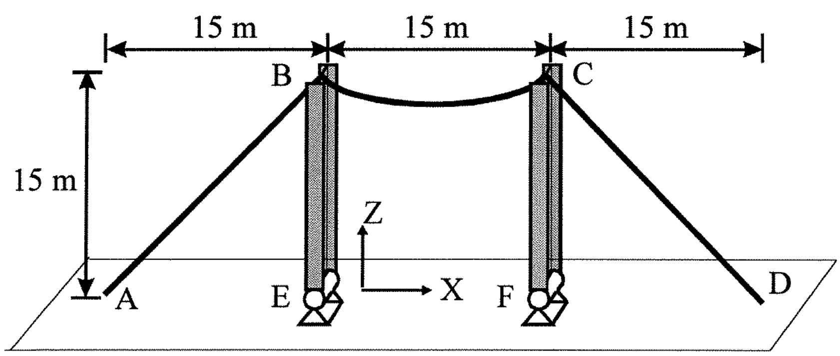

Fig. 1

2 Figure 2(a) shows a two-dimensional pin-jointed structure. The bars are of equal cross-sectional area, A. They are made of a linear elastic material with Young's modulus, E and are of length L.

(a) A vertical load W is applied at joint C.

(i) Calculate the bar forces in the structure. [4]

(ii) By virtual work or otherwise, determine the vertical displacements of joints C and D. [6]

(b) A beam of uniform bending stiffness EI is added to the structure shown in Fig. 2(a). The beam is supported on nodes C and D which act like roller supports (see Fig. 2(b)). The load W is placed at the mid-span of the beam. Calculate the total vertical displacement of the mid-point of the beam due to the load W. [6]

(c) The supports and joints of the structures are now welded. Without carrying out any additional calculations, discuss how you would expect:

(i) the vertical displacements calculated in part (a) to change; [2]

(ii) the midspan vertical displacement calculated in part (b) to change. [2]

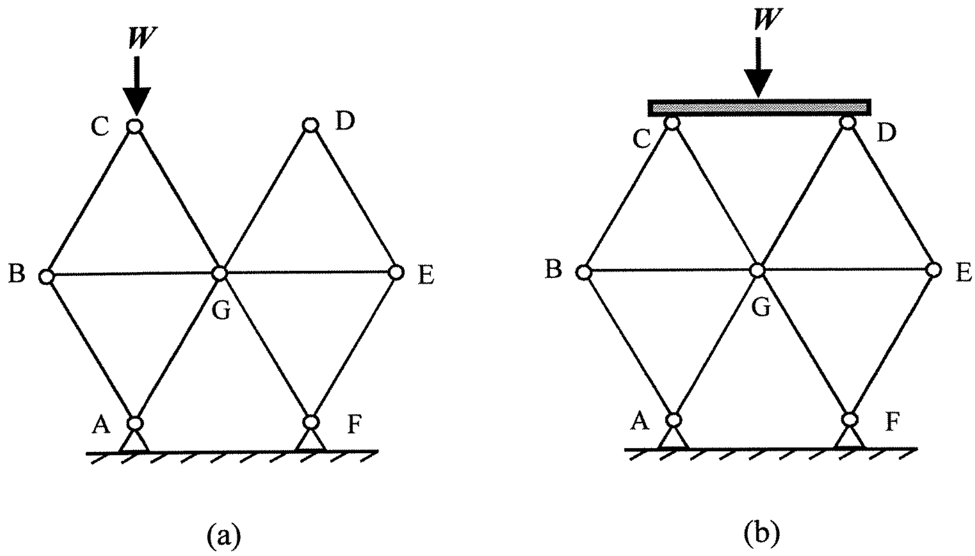

Fig. 2

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3 Figure 3 shows a uniform horizontal bar AB of length 5 m. The beam is pinned to a wall at A and connected to a flexible cable at point B. The cable passes over a small frictionless pulley and is attached to a weight W, where $W = 2$ kN. The weight rests on the beam AB. The self-weight of the beam and cable can be neglected.

A uniformly distributed weight of 150 Nm^{-1} is applied to the beam AB as shown in Fig. 3.

(a) Determine the reactions at A due to the loading shown. [12]

(b) Draw a free body diagram for beam AB and hence plot the shear force and bending moment diagrams for the beam, marking salient values. [8]

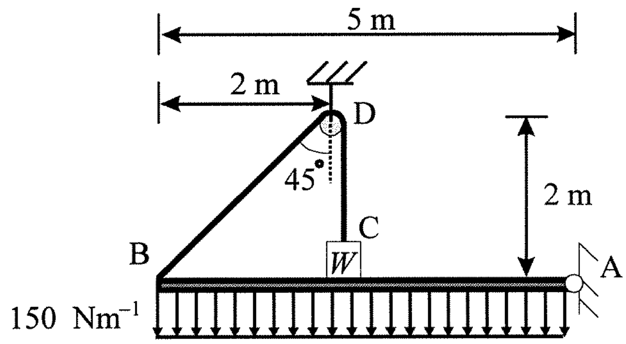

Fig. 3

4 Figure 4(a) shows a two-dimensional cantilever structure that lies in a vertical plane. The structure is made of steel and consists of two members each of length $L = 1.25$ m. The structure has rigid joints and the members are made of uniform square solid cross-section with sides of $a = 50$ mm.

(a) For the structure shown in Fig. 4(a):

(i) calculate the vertical reaction at A due to the self-weight of beams AB and BC; [2]

(ii) determine the deflections at B due to the self-weight of the structure; [5]

(iii) calculate the deflections at C due to the self-weight of the structure. [5]

(b) A roller support is now added to the structure at C as shown in Fig. 4(b). Calculate the horizontal force at C and the reactions at A due to the self-weight of the structure. [8]

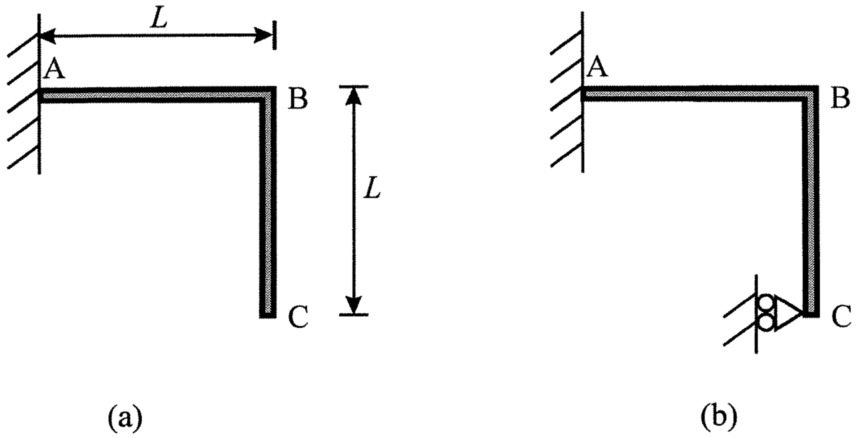

Fig. 4

5 The design of a 2.5 m long simply supported beam is to be considered. The beam cross-section is shown in Fig. 5 and consists of a glass fibre reinforced plastic (GFRP) section and concrete. The GFRP section is a square hollow 100 mm × 100 mm × 4 mm section, the dimensions and properties of which can be found in the Structures Data Book. The GFRP has a Young's modulus, *E*, of 17.2 GPa. The concrete is attached to the GFRP box using an epoxy glue of negligible thickness. Assume the concrete behaves elastically and has a Young's modulus of 30 GPa. The self-weight of the beam can be neglected.

(a) Sketch a cross-section of the beam 'transformed' to GFRP. [4]

(b) The properties about the horizontal axis of bending are of interest. For the transformed section:

(i) determine the distance of the neutral axis from the base of the beam; [3]

(ii) calculate the bending stiffness *EI*. [5]

(c) A vertical point load is applied at the midspan of the beam. Calculate the applied load at which the following events would occur:

(i) the concrete begins to crush at a compressive stress of 60 MPa; [3]

(ii) the epoxy glue begins to fail at a shear stress of 5 MPa. [5]

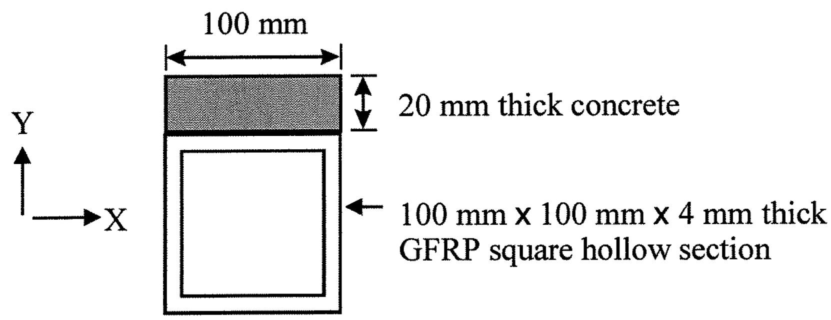

Fig. 5 (not to scale)

SECTION B

Answer not more than **four** *questions from this section.*

6 (a) Two bond-energy curves where r is distance between a pair of neighbour atoms for material A and material B are shown in Fig. 6.

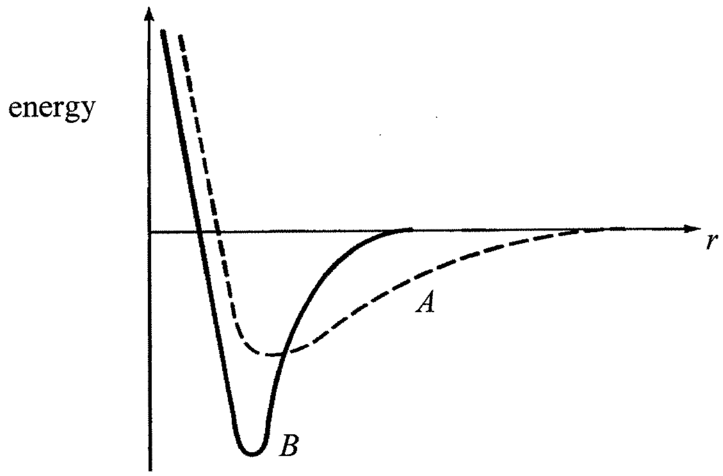

Fig. 6

By considering the shape only of the two bond-energy curves for material A and material B, select the better material in each of the following applications and account for your choice:

(i) a beam that deflects the least under moderate load; [3]

(ii) a crucible for use at high temperature; [4]

(iii) a device that detects fluctuations in temperature by changing its dimensions. [4]

(b) A tensile specimen of steel of diameter d and length l is subjected to an applied stress of 100 MPa. Before loading, d is 10.000 mm and l is 40.00 mm. When subjected to this stress d is 9.999 mm and l is 40.02 mm. By assuming that the applied stress is below the yield strength of the steel, calculate the following elastic properties:

(i) elastic modulus E; [3]

(ii) shear modulus G; [3]

(iii) Poisson's ratio ν. [3]

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7 (a) For polycrystalline materials, give one example together with a brief description of each of the following classification of defect:

(i) point defect; [2]

(ii) line defect; [2]

(iii) planar defect. [2]

(b) Outline two methods for increasing the yield strength of metals and alloys. For each method, account for the effect of raising the temperature for a prolonged period of time on the mechanism controlling yield strength measured finally at room temperature. [8]

(c) The yield strength σ_y of plain carbon steel is moderately dependent on grain size d and the relation can be described by the equation:

$$\sigma_y = \sigma_o + k\sqrt{\frac{1}{d}}$$

where σ_o and k are material constants. The yield strength is 622 MPa for a grain size of 180 μm and 663 MPa for a grain size of 22 μm.

(i) Calculate the yield strength of the steel for a grain size of 11 μm. [4]

(ii) Explain briefly the physical significance of the constant σ_o. [2]

8 (a) Explain carefully what is meant by *solid-state diffusion.* Include in your answer a brief explanation of major factors that influence diffusion-controlled processes in engineering materials. [5]

(b) Diffusion-controlled processes that take place in engineering materials during their manufacture and application can affect the performance of the component either *detrimentally* or *beneficially*. Give an example of each of these two cases. [6]

(c) One face of a polished sample of copper in contact with a similar piece of zinc is heated to an elevated temperature for a sufficiently long time that a constant concentration gradient of zinc atoms through the thickness of the copper exists. The diffusion coefficient D of zinc in copper is 3.67 x 10^{-15} m^2s^{-1} at 1000 K and decreases to 8.32 x 10^{-22} m^2s^{-1} at 800 K.

(i) What is meant by the activation energy Q for diffusion? [4]

(ii) Determine Q ($kJmol^{-1}$) for the diffusion of zinc in copper at 600 K. [5]

9 (a) Write explanatory notes on the following corrosion process and method of corrosion prevention:

(i) galvanic corrosion; [4]

(ii) cathodic protection. [4]

(b) With respect to the high temperature oxidation of metals:

(i) Discuss briefly the requirements of an oxide film for it to be protective of the underlying metal or alloy substrate. [4]

(ii) A sample of nickel and a sample of magnesium are exposed to the same high temperature oxidising environment. Both metals are initially oxide-free and both have an oxide film of 0.17 μm after 1 minute of exposure time. By making certain assumptions of the oxide growth rates for the two metals, calculate the thickness of the oxide layer of both metals after an exposure time of 1 hour. You may assume that nickel oxide is protective whilst magnesium oxide is not. [8]

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10 (a) In relation to designing a high-strength aluminium alloy component against fatigue failure, explain briefly what is meant by the following terms:

(i) minimum detectable crack size; [3]

(ii) critical crack size; [3]

(iii) fracture toughness. [3]

(b) A pressure vessel for an aircraft is made of an aluminium alloy having a fracture toughness K_{IC} of 30 MPa$\sqrt{\text{m}}$. The vessel has an outside radius r of 10 cm, and the wall thickness t is 1 cm. The pressure vessel contains hydrogen gas at a maximum pressure p of 10 MPa, which gradually releases to 2 MPa over 4 hours. The vessel is then re-pressurised. The design life of the vessel is 5 years. The hoop stress σ in the wall of the pressure vessel is given by:

$$\sigma = \frac{pr}{t}.$$

The non-destructive inspection capability (NDT) of the aircraft's maintenance crew is such that there is a finite probability of an undetected longitudinal crack of 3.5 mm length present in the wall of the pressure vessel. The fatigue crack growth law of the aluminium alloy in hydrogen gas is given by:

$$da/dN = (8.5 \times 10^{-12})\,(\Delta K)^4 \qquad (\text{m cycle}^{-1}),$$

where a is crack length, N is the number of load cycles, and ΔK is the cyclic range of crack tip stress intensity factor. For the pressure vessel loaded cyclically:

$$\Delta K = 1.12\,\Delta\sigma\sqrt{\pi a} \qquad (\text{MPa}\sqrt{\text{m}}).$$

Determine a sensible inspection time interval during service to ensure the vessel completes its design life. Finally, propose a suitable NDT method to inspect the vessel. [11]

END OF PAPER