IA ANSWERS

Engineering Tripos Part IA, June 2001, Paper 1 : Answers (Section A)

1 a) 0.0785 kgs^{-1}, ($\rho = 1.570$ kg/m^3)
b) 65.94 kW, 1303.3 K
c) 3.57 kW, 1258.3 K
d) 1.131 bar
e) 50.82 kW, ($\rho = 1.210$ kg/m^3)

2 a) Parallel Streamlines
b) -
c) $F_{nozzle} = \frac{1}{2} A\rho v_2^2 (1-\alpha)^2$
d) $5_{VAR} - 3_{DIM} = 2$ or more N-D groups

$\frac{F_{NOZZLE}}{A\rho v_2^2}$ depends on $\{\alpha\}$

3 d) $X = L\dfrac{\left(\frac{1}{2}D - \frac{1}{3}L\sin\theta\right)}{\left(D - \frac{1}{2}L\sin\theta\right)}$

e) $D \geq \dfrac{M\cos\theta}{\rho L} + \dfrac{2}{3}L\sin\theta$

4 a) 129% Excess Air
b) 10.8% Wet, 13.0% Dry
c) 159.2 kg/kmol H_2
d) 755 m/s

5 a) System: $Q - W = \Delta E$

c.v. $\dot{Q} - \dot{W}_x = \dot{m}\Delta\left(h + \frac{1}{2}V^2 + gZ\right)$

b) 9800 kJ, 2800 kJ
c) (i) 13.7 kg, $W = 4100$ kJ
(ii) $W_{DISP} = 1900$ kJ
$W_x = 2200$ kJ
d) None

Section B: Mechanics and Vibrations

6. (b) $\omega_{AB} = \dfrac{\omega R \mathrm{Cos}\theta}{L}$; $v_{\text{sliding}} = \omega R \sin\theta$

(c) $F_{(i)} = \dfrac{T}{R\sin\theta}$; $F_{(ii)} = \dfrac{T+Q}{R\sin\theta}$

7. (a) $J_X = \dfrac{3}{2}mR^2$

(b)(ii) $\mu \geq \dfrac{1}{3}$

8. (b) Height = $3.364R$

(c) $I = 0.16\sqrt{gR}$ tangential to the path, in the direction of motion.

9. (a) $\dfrac{\ddot{y}}{\omega_n^2} + \dfrac{2\zeta}{\omega_n}\dot{y} + y = \dfrac{2\zeta}{\omega_n}\dot{x} + x$, with $\zeta = \dfrac{\lambda}{2\sqrt{km}}$, $\omega_n^2 = \dfrac{k}{m}$

(b) $\left|\dfrac{F}{kH}\right| = \dfrac{\dfrac{\omega^2}{\omega_n^2}\sqrt{1+\left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\dfrac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}}$, with $\omega = \dfrac{2\pi v}{L}$

(c) (i) $|F_{\max}| \approx 880\text{N}$, at $v \approx 2\text{m/s}$; (ii) 512N

(d) At high frequencies mass becomes inertial, and $\left|\dfrac{F}{kH}\right| \to 2\zeta\dfrac{\omega}{\omega_n}$

10. (c) 0, $\sqrt{\dfrac{k}{J}}$, $\sqrt{3\dfrac{k}{J}}$

(d) $\sqrt{0.38\dfrac{k}{J}}$, $\sqrt{2.62\dfrac{k}{J}}$

Part 1A Engineering Tripos 2001

Paper 2 Section A, Structures

Answers

1 (a) $V_A = V_F = 2W$, vertically up; $H_A = H_F = 2W$, horizontally inwards.

(b) Bending moments are zero everywhere.

(c) Maximum moment at D $= 8WL/5$ (tension on the outside of the arch).

2 (a) (i) $R_P = W/2$ down; $R_Q = 3W/2$ up; $R_P = 0$.

(ii) $T_{PQ} = W/2$; $T_{PR} = -\sqrt{2}\,W/2$; $T_{QR} = \sqrt{2}\,W/2$; $T_{QS} = \sqrt{2}W$; $T_{RS} = -W$.

(b) $\delta_{SV} = \left(5/2 + 3\sqrt{2}\right)WL/AE$ (down).

(c) $\delta_{RV} = -WL/2AE + L/1000$ (down).

3 (b) $\theta = q_0 L^3/24EI$ (clockwise); $\delta = q_0 L^4/30EI$ (right).

(c) $\delta = 3q_0 L^4/1280EI$ (right).

4 (a) $\sigma_{\text{max tensile}} = 282\ \text{N/mm}^2$; $\sigma_{\text{max compressive}} = -282\ \text{N/mm}^2$.

(b) $\sigma_{\text{max tensile}} = 249\ \text{N/mm}^2$; $\sigma_{\text{max compressive}} = -143\ \text{N/mm}^2$.

(c) shearing force/bolt = 23820 N.

5 (a) $P_E = 269.9$ kN.

(c) $\delta_{\max} = 0.2$ mm; $M_{\max} = 405$ Nm; $\sigma_{\text{max compressive}} = -479\ \text{N/mm}^2$;
$\sigma_{\text{average}} = -324\ \text{N/mm}^2$.

ENGINEERING TRIPOS PART 1A
Paper 2 Section B: MATERIALS

Question 6

(a)

Roughly speaking, $E_L = E_f V_f$ and $E_T = E_{matrix}$.

E_L = 200 GPa

E_T = 3-4GPa, some 50 times or more smaller.

(b)

The mass m is proportional to $\rho/(E^{1/2})$.

(c)

Material	$\rho/(E^{1/2})$	***% wt change***
Stainless	0.57	0
Epoxy	0.63	10.5 gain (heavier)
CFRP	0.11	80 saving (lighter)

Question 7

(c)

The shear stress τ_y required to move a dislocation on a single slip plane is ~22 MPa. The total yield strength of the dispersion strengthened alloy is 30 MPa (yield strength of pure Al taken from Databook) *plus* 3×22 MPa ($3\tau_y$), giving a total of 96.5 MPa.

Question 8

(b)

$t = p\,R/2\sigma$ = (20 x 1000)/(2 x 180) = 55.6 mm.

(c)

$\sigma = pr/2t$ = (40 x 1000)/(2 x 55.6) = 360 MPa.

This is less than the yield stress of 500 MPa; yielding will not therefore occur.

$K_1 = \sigma(\pi a)^{1/2} = K_c$ at fast fracture.

$\sigma(\pi a)^{1/2}$ = 360 (π x 0.005)$^{1/2}$ = 45 MPa m$^{1/2}$

This is less than half the fracture toughness value of 100 MPa m$^{1/2}$. Fast fracture should not occur.

Question 9

(d)

$\varepsilon_{500} = \varepsilon_{560}$ x 0.110 = 1.1 x 10^{-5} s^{-1}.

Question 10

(d)

6.36 years.

Peter W R Beaumont
23rd June, 2001

1A Paper 3: Numerical Answers (Section A)

Section A

1(a) 1.262 A

(b) Power input to battery = 12 W
Power from battery as heat = 1 W
Power lost in ammeter = 0.1 W
Power lost in 50Ω resistor = 3.43 W
Fraction of power in battery = 72.6%

(c) 1.002 A

2 (a) $i/v_s = 1/R$

(b) $$i/v_s = \frac{1}{\left(R + \frac{R}{A}\right) + j\frac{\omega L}{A}}$$

(c) 3dB point is $f = 9.1 f_c$

3 (a) Power factor = 0.53

(b) C = 0.365 μF

(c) High voltage supply = 470 V, power factor = 0.545

4 (b)If $C_2 \rightarrow \infty$ then gain = $-g_m R_1$

(c) If $\omega \rightarrow 0$, gain = $-g_m R_1/(1+g_m R_2)$

(d) Gain = 2.09 with phase change of -120^0

11 (a) $H = I/2\pi r$

(b) $V = -d\phi/dt$

(c) $(\mu_o/2\pi)\ell n(b/a)$

IA 2001 Paper 4 Mathematical Methods

Answers:

1) distance to origin is $d/\sqrt{(a^2+b^2+c^2)}$

Shortest distance between two lines is $d(a,b,c)/(a^2+b^2+c^2)$

2) a) -$\pi/\sqrt{(3)}$, b) 1/3,

c) i) $y=(2n+1)\pi/2, x=(-1)^n \sinh^{-1}(1)$.

ii) $2^{1/6}(e^{i\pi/12}, e^{i9\pi/12}, e^{i17\pi/12})$,

and $2^{1/6}(e^{-i\pi/12}, e^{i7\pi/12}, e^{i15\pi/12})$.

3) a) $x(t)=(-3t-4)e^{-t/2}+2\sin(t/2)+4$.

b) $y_n=(n+1)(-1/2)^n$.

4) b) s=2, t=1.

c) l is and eigenvector with eigenvalue 1.

5) $\mathbf{A}^{-1}\mathbf{x}=\alpha\mathbf{u}_1/\lambda_1+\beta\mathbf{u}_2/\lambda_2+\gamma\mathbf{u}_3/\lambda_3$

$\mathbf{A}^N\mathbf{x}=\alpha\lambda_1^N\mathbf{u}_1+\beta\lambda_2^N\mathbf{u}_2+\gamma\lambda_3^N\mathbf{u}_3$

$(\mathbf{A}^{-1})^N\mathbf{x}=\alpha\mathbf{u}_1/\lambda_1^N+\beta\mathbf{u}_2/\lambda_2^N+\gamma\mathbf{u}_3/\lambda_3^N$

6) i) $t<a, y(t)=\frac{A}{b}(1-e^{tb})$

ii) $t>a, y(t)=\frac{A}{b}e^{-bt}(e^{ab}-1)$

7) $a_0=8/3, a_n=\frac{16(-1)^n}{n^2\pi^2}$.

9) (5,0) - minimum

(-5,0) - maximum

(3,4) - saddle point

(-3,-4) - saddle point