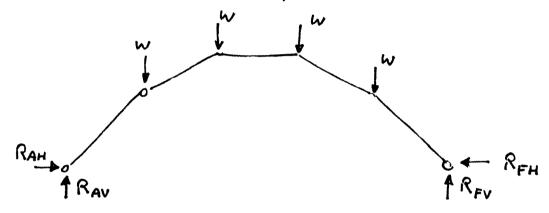
Engineering Tripos Part 1A 2001 Paper 2 Structures and Materials.

Solutions

Section A - Structures

1. (a) Consider overall equilibrium of the arch.



Moments about A:

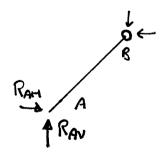
RFV x 10L = Wx(8L+6L+4L+2L)

Moments about F:

R_{Fv} = 2w R_{Av} = 2w

REV+RAN = Lyw

Consider AB



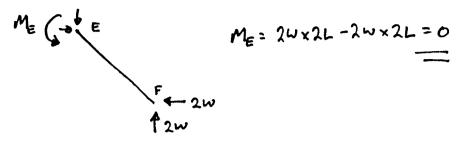
Moments about B

RAVX2L = RAHX2L

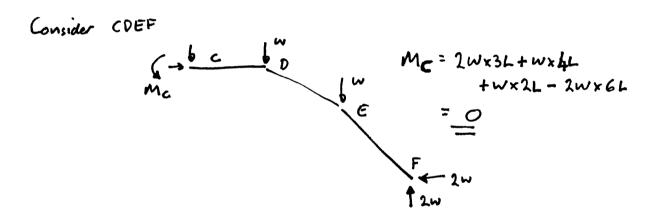
: RAH = 2W

(also, by overall equil, RFH = 2W)

(b) Consider EF

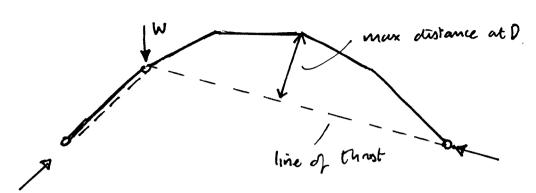


Consider DGF M_0 M_0



The moment is zoo at A,B,C,D,E,F. Between these points, the arch is straight and unloaded. The line of thoust therefore passes along the arch everywhere, and the moment is zoo everywhere.

(c)



From the even at D, and hence the maximum distance moment occurs here

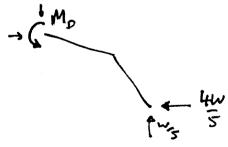
Find reactions at F:



Moments about A!

Moments about B:

Find moment at D - Mo

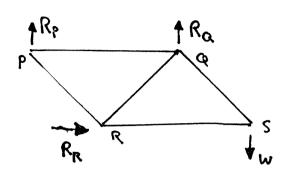


$$M_0 = -\frac{w}{5} \times 4L + 4w \times 3L$$

$$= 8wL$$

$$= \frac{5}{5}$$

2 (4) (1)



Horzontal equil => RR = 0

Moments about P,
$$R_{Q} \times 2L = w \times 3L$$
, $R_{Q} = \frac{3w}{2}$
Moments about Q, $-R_{p} \times 2L = w \times L$, $R_{p} = -\frac{w}{2}$

(ii) Ats
$$T_{RS} = \sqrt{2}w$$

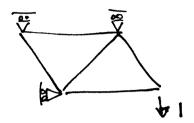
$$T_{RS} = -w$$

$$T_{RS} = -w$$

$$T_{RQ} = \sqrt{2}w$$

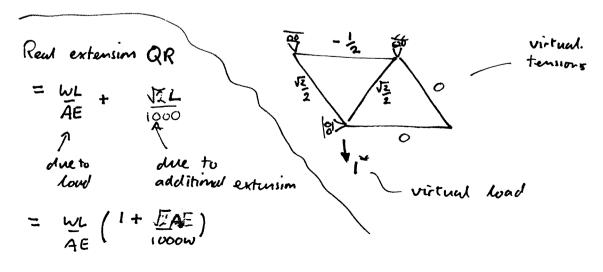
(b) Use virtual work, real extension, virtual tension

Use as virtual system



Member	Real T (xw)	Lengths L (×L)	Rent extensions (xwL)	Vistand T*	T'e (* WL)
PQ	1/2	2	1	1/2	1/2
PR	- 52/2	J2	-1	- 12/2	J2/2
QR	J2/2	J2	l	J2/2	J2/2
QS	J2	J2	2	V2	252
RS	-1	2	-2	-1	2

(C) Use as virtual system:



Member	Real extension (XWL/ALT)	Virtual TY	T*e (xwl AE)
PQ	1	-1/2	- 1/2
PR	-1	V2/2	- V ² /2
QR	1 + 5AE	52/2	J3 + AE
QS	2	0	2 1000 W
RS	-2	0	0

3 (a) Start with
$$M(z) = q_0 (L-3)^3$$
 ($m=0$ at $z=L$, matches $s.c.$;)

$$S = dM = q_0 (L-3)^2 \quad (s=0 \text{ at } z=L, matches } \frac{1}{2L} \quad \text{matches } s.c.$$
;)

$$q = dS = q_0 (L-3) - \text{Actual Loading} \quad dx = \frac{1}{L} \quad d$$

(b)
$$k(3) = M = 90 (L-3)^3$$

EI EIL 6

 $\Theta = \int k ds = \frac{q_0}{6EIL} \int (L-3)^3 d3$ $= -\frac{q_0}{6EIL} \cdot \frac{(L-3)^4}{4} + C$ $= \frac{q_0}{6EIL} \cdot \frac{(L-3)^4}{4} + C$

At 3=0, $\theta=0$, $\Rightarrow C = q_0L^3$ 24EI

$$\therefore \Theta = \frac{q_0}{24EIL} \left[L^4 - (L-3)^4 \right]$$

at z=L, $\Theta = q_0 L^3$ 24EI

$$S = \int \Theta ds = \frac{q_0}{24_{\text{EIL}}} \int L^4 - (L - 3)^4 d3$$

$$= \frac{q_0}{24_{\text{EIL}}} \left[\frac{L^4 3 + (L - 3)^5}{5} \right] + D$$

at 3=0, 8=0

$$.: O = \frac{q_0}{24EIL^2} \begin{bmatrix} L^5 \\ \bar{5} \end{bmatrix} + D = 7D = -\frac{q_0L^4}{120EI}$$

at z = L

$$S = \frac{q_0}{120EIL} \left[5L^5 + 0 - L^5 \right]$$

$$= \frac{q_0 L}{30EI}$$

(C) Replace L by
$$\frac{1}{12}$$
 to give $\frac{1}{9}$ $\frac{1}{384EI}$

Suz = $\frac{1}{90L^4}$

Suz = $\frac{1}{960EI}$

Suz = $\frac{1}{960EI}$

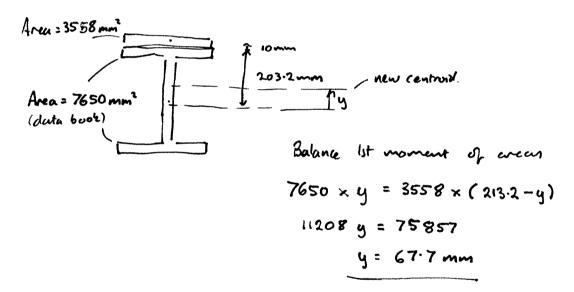
Suz = $\frac{1}{960EI}$

Suz = $\frac{1}{390EI}$

$$\therefore O_{max} = 282 \, N/mm^2 \, \text{(tensile)}$$

Suivery, Omin = -282 N/mm2 (compressive)

(b) Initially find new controld



Find I of total section

If (2nd moment of area of I-beam) = 21600 cm4 (from data book) = 216 × 106 mm4

As (area of I beam) = 7650 mm2

 I_{p} (2nd moment 4 avan of place) = bt^{3} = 177.9×20^{3} 12 = 0.12×10^{6} mm⁴

Ap (was 1) pute 1 = 3558 mm2

Ay = 3558 × 145.5 = 517.7 × 103 mm3

203.2-67.7+10 = 145.5 mm

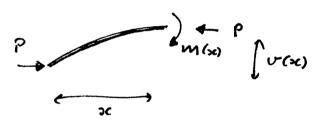
: force/unit length =
$$\frac{150 \times 10^3 \times 517.7 \times 10^3}{326 \times 10^6} = \frac{2382 \text{N/mm}}{326 \times 10^6}$$

$$P_{e} = \frac{\pi^{2} E \Gamma}{L^{2}}$$

$$I = \frac{25^4}{12} = 32.6 \times 10^3 \, \text{mm}^4$$

Pe = 269.9 kN

(b) Consider a general deflected shape



Geometry
$$K = -\frac{d^2v}{dx^2}$$
 $\Rightarrow JK = -\frac{d^2v}{dx^2} + \frac{d^2v_0}{dx^2}$

$$V_0 = S_0 \sin \Pi x$$
 (guiren), hence $d^2 V_0 = -\frac{\pi^2}{L^2} S_0 \sin \Pi x$

$$dx_2 = \frac{\pi^2}{L^2} S_0 \sin \Pi x$$

Sub back to give

$$P_{V} = -E_{I} d^{2}V - E_{I} \pi^{2} S_{o} \sin \pi x$$

$$dx^{2} L^{2} L$$

Rewrange to give

$$EI d^{2}v + Pv = -EI \left(\frac{\pi}{L}\right)^{2} \delta_{0} \sin \pi x$$

Rewrite, using $P_E = TI^2 EI$, as $\frac{L^2}{L^2}$ $EI d^2v + PV = -P_E S_0 sim IIx$ $\frac{1}{L^2}$

A perticular integral is given by $V = \frac{SoPE}{P_E - P} \cdot SinTx$

General solm, $V = A \sin kx + B \cos kx + \frac{Sole}{le-l} \cdot \sin \pi x$ where $d^2 = \frac{l}{le}$

Boundary conditions, V=0 at x=0 $\Rightarrow B=0$ V=0 at x=L $\Rightarrow A=0$ or $AL=\Pi, Z\Pi...$

For P<PE, A must be zero, hence

At midspan, sin Ttx, v= 8

(c) for P= 0.75 PE, S= 480 = 2mm

Max deflection occurs at midspan = 8

Max moment = PS = 0.75 PE S

= 405 ENmm

Average comprenive stress, $O_{AV} = 0.75 \, Pe = 324 \, N/mm^2$

Max compressive stress due to moment = My I

= 405 ×103 × 12.5 _ 155 N/mm2

Total peak compressive stres = 324 + 155 = 479 N/mm2

Very significant rise due to very small imperfection.

Examiner's Crib

ENGINEERING TRIPOS PART 1A Paper 2 Section B: MATERIALS

June 23rd 2001

Question 6

(a)

A unidirectional carbon fibre composite is highly anisotropic, i.e. its properties are directional which means that in this case it is very much stronger and stiffer in the orientation of the fibres than transverse to their length. The calculation of Young's modulus using the rule of mixtures (see Ashby and Jones Engineering Materials I page 63) shows the longitudinal value of E to be about 200GPa or so. The value of E across the fibres will be roughly of the order of the matrix modulus since in this direction the property is matrix-dominated, not fibre dominated. You could use the equation on page 64 to calculate this lower value of modulus. It turns out to be about twice the value of E of the pure epoxy matrix or 8GPa, orders of magnitude less the longitudinal modulus.

The strength of the composite would follow in a similar way, being very much stronger in the longitudinal direction.

The advantages of the high modulus (and strength) along the fibre length could be used in the example given in Question 6. Golf shafts and fishing rods would benefit also from this highly oriented composite. The problem, of course, arises if an off-axis stress exists which could case failure of the matrix and/or interface between fibre and matrix or between the plies of a multi-layer composite. In such instances, the composite would require additional layers of fibre at some angle to the aligned fibres, perhaps 45° or/and 90° .

Roughly speaking, $E_L = E_f V_f$ and $E_T = E_{matrix}$. Substituting the values given in the Table, the longitudinal Young's modulus is about 200 GPa whilst the transverse modulus is of the order of 3-4GPa, some 50 times or more smaller.

(b)
$$S = EI, \text{ where } I = \pi d^4/64. \qquad \text{Mass } m = l \left(\pi d^2/4\right) \rho$$

Hence: $m^2 = (l^2 \rho^2 \pi S)/4E$ Thus: the mass m is proportional to $\rho/(E^{1/2})$.

(c)

Material $\rho/(E^{1/2})$ % wt change

Stainless 0.57 0

Epoxy 0.63 10.5 gain (heavier)

CFRP 0.11 80 saving (lighter)

Question 7

(a)

The precipitate or dispersion strengthened material may derive its addition strength from the dislocation bowing mechanism. This question shows schematically (fig. 6) the bowing of a dislocation between pinning points A, B. These points could be the precipitates which hinder the movement of the dislocation and subsequent dislocations. An applied shear stress has to push the dislocation between these 2 obstacles. Once a large enough stress is attained corresponding to a large enough bulge, the dislocation can easily expand further (see Ashby and Jones ibid page 106). The dislocation escapes and yielding occurs. Decreasing the spacing between 2 neighbouring precipitates results in a higher shear stress to bring about this phenomenon of dislocation bowing. If dislocation loops are left round each of the 2 precipitates, the effective spacing has decreased and an additional applied stress is necessary to get the second and subsequent dislocations to bulge through the gap.

Sketches shown on page 106 of Ashby and Jones would aid you in your answer.

(d)

If there is non-uniformity in precipitate spacing, those neighbour precipitates that are spaced furthest apart would allow segments of a dislocation to pass through more easily. The effect is that the alloy would have a reduced yield stress.

(b)

The force in the forward direction due to the applied stress is just balanced by the line tension until at the minimum (critical) radius of the dislocation loop corresponding to a maximum of the force due to line tension:

$$\tau bl = 2T$$

Hence: $\tau = 2Gb^2/bl$ Therefore: $\tau = 2Gb/l$

(c)

The area fraction of precipitate is equal to its volume fraction V_p or $(\pi d^2/4)/l^{*2}$ where l^* is the centre-to-centre spacing of the precipitate.

Since $V_p = 0.05$, then $l^* = (\pi^{1/2}) d / 2(0.05)^{1/2}$ which is equal to 0.395 x 10^{-6} m. The particle-to-particle spacing, l, is $(l^* - 2d)$.

Using the equation given and substituting values of G, b, d, l, where G the shear modulus of Al is 27GPa, then the applied critical shear stress τ is 80 MPa. The tensile strength is double this value, approx.; hence the tensile yield stress is 160 MPa plus the tensile yield strength of pure Al (100MPa), giving 260MPa.

Question 8

(a)

The elastic stress concentration factor K_t is simply the ratio of the local stress near to the tip of a notch or hole to the gross (applied) stress (that's applied load divided by gross section area). It depends on many factors such as the length of the notch and its sharpness (notch root radius), and the distance from the notch tip at which K_t is to be evaluated. Roughly, $K_t = 2(a/\rho)^{1/2}$ where a is notch length and ρ is notch root radius. K_t decreases with distance from the notch tip.

 K_I is a fracture mechanics term known as the stress intensity factor of a crack and is given by: $K_I = \sigma(\pi a)^{1/2}$ having the *units of stress times crack length*^{1/2}. Note that K_I has no units. Evidently, when the applied tensile stress σ attains the tensile strength of the component containing the crack, the crack size becomes critical and the component fractures. Alternatively, fast fracture occurs when K_I equals K_c . This latter term is known as the fracture toughness of the material, or critical stress intensity factor.

(b)

Maximum allowable stress is 360 MPa, at the edge of the hole. The remote stress σ is therefore $360/K_t$ which is 360/2 which equals 180 MPa. The minimum thickness t of the pressure vessel is given by:

$$t = p R/2\sigma = (20 \times 1000)/(2 \times 180) = 55.6 \text{ mm}.$$

which after substitution of data gives 55.6 mm. Now (56/1000) is less than 10 which is acceptable for a pressure vessel design.

(c)
$$\sigma = pr/2t = (40 \times 1000)/(2 \times 55.6) = 360 \text{ MPa.}$$

This is less than the yield stress of 500 MPa; yielding will not therefore occur. It be $2 \times 180 \text{ MPa} = 360 \text{ MPa}$, anyway!

Now:
$$K_1 = \sigma(\pi a)^{1/2} = K_c$$
 at fast fracture. $\sigma(\pi a)^{1/2} = 360 (\pi \times 0.005)^{1/2} = 45 \text{ MPa m}^{1/2}$

This is less than half the fracture toughness value of 100 MPa m $^{1/2}$. Fast fracture should not occur. (NB 2a = 10 mm which is much less than 56 mm so crack geometry is sensible.

(d)

Methods of detecting buried cracks include ultrasonics and X-radiography. The latter is an indirect method and the signal requires skill of interpretation. Only crack surfaces that reflect the sound waves will be detected. There is also skill required in the interpretation of an X-ray picture. The resolution of each technique depends on such things as the size of the transducer and the source of X-rays, angle of crack to the X-rays, etc.

Question 9 (a) and (b)

There are 2 mechanisms of creep: dislocation creep (power-law creep) and diffusional creep. Both are limited by diffusion, so both follow Arrhenius's Law. Diffusion starts noticeably at 0.3 T_m where T_m is melting point (⁰K). Sketches of dislocation creep are given on page 188 of Ashby and Jones ibid. They show the movement of dislocations controlled by diffusion which are affected by obstacles at the atomistic and microscopic levels of size, e.g., dissolved solute atoms and precipitates. Diffusion unlocks these dislocations from obstacles in their path and this leads to creep. The sketches asked for should show how this unlocking occurs.

If the applied stress is reduced, the rate of power-law creep falls quickly: recall that n in the power-law is between 3 and 8. Creep does not stop; instead an alternative mechanism takes over. An example of a diffusional creep mechanism is *grain elongation*. A sketch is shown on page 189 of Ashby and Jones ibid. The applied stress is the driving force and atoms diffuse from one grain face to another. Dislocations are not involved. At high temperature, this diffusion takes place through the grain itself, i.e. by bulk diffusion. Hence, the creep rate depends on the diffusion coefficient, the stress and is inversely proportional on grain size (larger grains means longer diffusion distances). Grain boundary diffusion dominates. Grain boundary sliding is a required accessory to this process.

(c)

To resist power-law creep, the criteria are:

Choose a high melting point material;

Maximise the obstructions to dislocation motion by alloying;

Choose a solid with a high intrinsic lattice resistance, e.g. covalent bonded solids such as silicates, carbides, nitrides, etc.

To resist diffusional flow:

Choose a high melting point material;

Arrange material to have a large grain size;

Arrange material to have precipitates at grain boundaries to impede grain boundary sliding.

creep-rate
$$\varepsilon = C e^{-Q/RT}$$
 (T in 0 K). Therefore: $\varepsilon_{560} = C e^{-Q/R833}$ and $\varepsilon_{500} = C e^{-Q/R733}$. Thus: $(\varepsilon_{500})/(\varepsilon_{560}) = (C e^{-Q/R733})/(C e^{-Q/R833})$. $= \exp(197 \times 10^{3} \text{ Jmol}^{-1})/(8.314 \text{ Jmol}^{-1}\text{k}^{-1}) \times (1/833 - 1/733) 1/\text{K}$ $= 0.110$ $\varepsilon_{500} = \varepsilon_{560} \times 0.110 = 1.1 \times 10^{-5} \text{ s}^{-1}$.

Question 10 (a), (b), and (c)

The galvanic series ranks the corrosion potentials of common metals in sea water. The metal with the more negative potential will become the anode when connected to another metal which becomes the cathode. Furthermore, the greater this difference between potentials of the 2 metals in contact with oneanother, the greater the initial thermodynamic driving force for the corrosion of the more anodic metal.

The severity of the problem is determined by:

Potential difference between the metals in contact;

Kinetic factors, such as passivation (where a protective oxide layer forms);

The ratio of the anodic surface area to the cathodic surface area.

So follow 2 general rules for corrosion protection:

Avoid unnecessary use of dissimilar metals in electrical contact; Keep the anode to cathode area ratio as large as possible.

Consider the protection of steel using zinc and chromium coatings. As long as the coating is continuous and defect-free, either metal protects. The difference is if a scratch in the coating occurs. The galvanic series shows that Zn is anodic to steel and should continue to protect the steel. In contrast, Cr is cathodic to steel and so the steel begins to corrode preferentially. The problem is amplified by the small exposed area of the steel which gives an unfavourable anode-to-cathode area ratio. Car bumpers are Cr plated; hence when rust first appears its volume increases rapidly. Zn-plated steel is a better choice if corrosion resistance is of prime concern. Steel trash cans are galvanised.

Galvanic couples may develop at the microscopic level, e.g. welded stainless steel. Whilst the passive chromium oxide layer is protective, unfortunately during welding some Cr can combine with carbon to produce Cr carbides at grain boundaries. The corresponding decrease in Cr content of the steel adjacent to the grain boundary renders the steel not stainless and anodic with respect to the bulk of the structure. The low anode-to-cathode area ratio is again unfavourable.

A possible remedy is to use a stainless steel with very low C content, or add Ti or Nb to the alloy to "mop" up the C. Also, a post-weld heat treatment can dissolve the Cr carbides and diffuse the Cr back into solution.

Here are several general rules that minimise this form of corrosion:

Select single-phase alloys;

Solid solution strengthen rather work harden an alloy. (Work hardening increases the energy of level of atoms at defects rendering the local region anodic);

Avoid small grain sizes if possible;

Ensure a homogeneous distribution of the alloying elements;

Eliminate any residual stresses;

Produce a fully recrystallised large grain structure;

Use plastic washers to separate dissimilar metals, between steel and copper.

Local anodic regions can form because of oxygen concentration gradients in the electrolyte. A region of higher oxygen content becomes cathodic with respect to the surrounding material. A single drop of water on a metal surface results in the formation of an oxygen concentration cell. On steel, rust can form just outside the waterline.

Pitting and crevice corrosion are localised forms of attack involving oxygen concentration gradients. Presence of chlorine ions increases the severity of the problem. Pits often form under debris, and places of stagnant pools of electrolyte on horizontal sections. Crevice corrosion can occur between metal sheets joined using bolts, screws, or rivets. These forms of corrosion which are difficult to detect, are best overcome by design considerations. Welded joints are preferred in aggressive environments. Avoid sharp corners, etc that favour the formation of stagnant pools. Design to permit free drainage and avoid sediment buildup.

Other methods that can be used to prevent galvanic attack include:

Cathodic protection where the current flow is reversed, i.e. reverse the anode and cathode. The metal to be protected is connected to the negative terminal of a DC power supply. This can be applied to buried steel pipes. A piece of scrap iron is connected to the other side of the voltage source and becomes the anode. Another version involves the use of a sacrificial anode (see Question 10 calculation). A galvanic cell is formed by attaching a more anodic metal in electrical contact with the metal to be protected. The anodic metal is sacrificed, e.g. large Zn or Mg plates attached to ocean vessels below the waterline; also, Zn slugs in car radiators.

Protective coatings based on cathodic (galvanic) protection or coatings to physically separate 2 metals in contact. They include grease, oil, waxes; also, porcelain enamels, tar, PTFE, paints, lacquers. Passive oxides, eg., those formed in the anodisation of Al also protect.

(d)
$$Zn \text{ forms } Zn^{2+} + 2e$$

If 1 mol of zinc dissolves, the number of electrons released is: $6.022 \times 10^{-23} \times 2$.

The charge released is
$$6.022 \times 10^{23} \times 2 \times 1.602 \times 10^{-19} \text{ C}$$
 which
= $6.022 \times 2 \times 1.602 \times 10^{4} \text{ C}$
= $1.93 \times 10^{-5} \text{ C}$.

The number of coulombs liberated by the anode is:

$$(136 \text{ kg})/(0.0654 \text{ kg}) \times 1.93 \times 10^5 \text{ C}$$

= $(1.36/65.4) \times 1.93 \times 10^8 \text{ C}$
= $4.01 \times 10^8 \text{ C}$.

If the current is 1A, 1C passes per second. Therefore, the time is: = $(4.01 \times 10^8)/(2 \times 60 \times 60 \times 24 \times 365)$ years = 6.36 years.

END

Peter W R Beaumont 23rd July, 2001