CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT

# EXAMINATION PAPERS

2001

ENGINEERING TRIPOS: 1A

ENGINEERING TRIPOS PART IA

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Monday 11 June 2001 9 to 12

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Paper 1

MECHANICAL ENGINEERING

*Answer not more than* **eight** *questions, of which not more than* **four** *may be taken from Section A and not more than* **four** *from Section B.*

*All questions carry the same number of marks.*

*The* **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

*Answers to questions in each section should be tied together and handed in separately.*

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SECTION A

*Answer not more than **four** questions from this section.*

1 A turbocharger is fitted to increase the output power of a petrol engine shown in Fig. 1. The air flow conditions at inlet to the turbocharger compressor are $T_1$ = 15 °C and $P_1$ = 1 bar. The flow at compressor exit, $T_2$ = 60 °C and $P_2$ = 1.5 bar, is fed directly into the inlet of the petrol engine. The exhaust from the petrol engine is connected to the turbocharger turbine inlet. The turbocharger turbine outlet pressure is $P_4 = 1.0$ bar , the gas velocity is negligible and all the turbine shaft power is used to drive the turbocharger compressor.

(a) The swept volume of the petrol engine is 500 $cm^3$ per cycle and there are 100 cycles per second. Calculate the mass flow rate of air into the petrol engine. [4]

(b) The petrol engine can be modelled as a steady flow device with heat input per unit mass flow rate $\dot{q}_{in} = 2.8$ $MJkg^{-1}s^{-1}$ which produces output shaft power $\dot{w}_x = 0.3\dot{q}_{in}$ . The amount of the heat input that is transferred to the cooling water is given by $\dot{q}_{cooling} = 0.35\dot{q}_{in}$. The mass flow rate of the fuel can be ignored and the exhaust gas may be treated as air. Calculate the output power of the engine and the exhaust temperature, $T_3$. [4]

(c) Calculate the shaft work transmitted between the turbocharger turbine and compressor and hence determine the air temperature, $T_4$ , at turbocharger turbine exit. [4]

(d) Assuming that the turbocharger turbine is adiabatic and reversible, calculate the turbine inlet pressure $P_3$. [3]

(e) What would be the output shaft power of the petrol engine without the turbocharger? [5]

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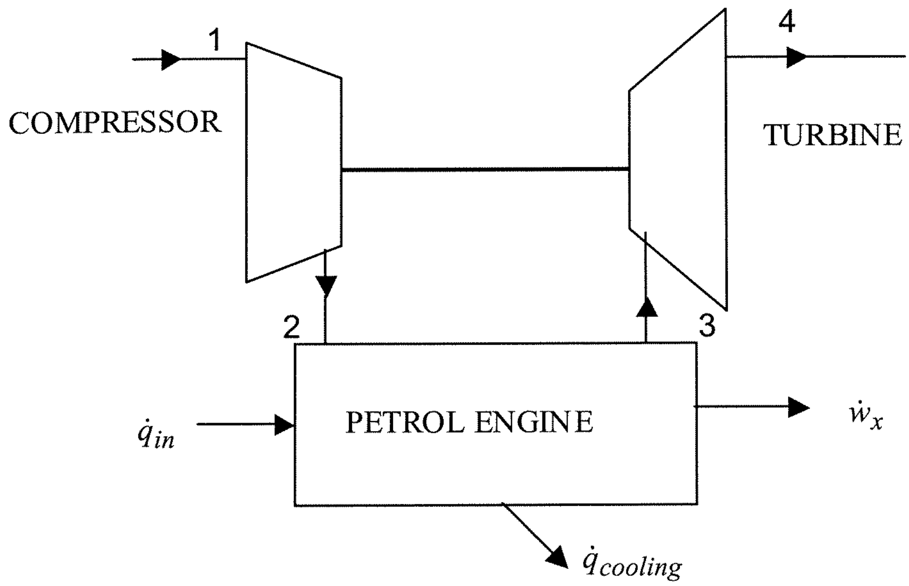


Fig. 1

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2 An inviscid liquid with density $\rho$ flows steadily through a nozzle of circular cross-section, shown in Fig. 2, producing a parallel jet with velocity $v_2$. The inlet gauge pressure is $P_1$ , the inlet area is $A$ and the exit area is $\alpha A$ $(0 < \alpha < 1)$. The force on the nozzle due to the fluid flow is in the direction shown in the figure.

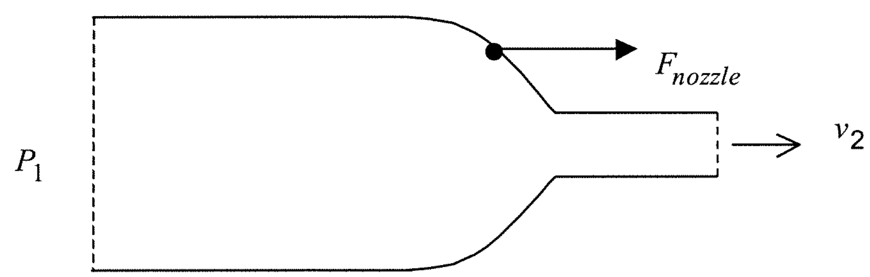


Fig. 2

(a) Explain why the pressure in the jet is ambient. [3]

(b) Show that the inlet gauge pressure is given by:

$$\frac{P_1}{\rho v_2^2} = \frac{1}{2}\left(1-\alpha^2\right)$$ [5]

(c) Calculate the force on the nozzle, $F_{nozzle}$, in terms of $\rho$, $v_2$, $\alpha$ and $A$. [7]

(d) Describe how dimensional analysis can be used to obtain a non-dimensional force coefficient. Explain why it is only a function of the geometry of the nozzle. [5]

3 Water is held in a reservoir by an inclined sluice gate of mass $M$ per unit width, as shown in Fig. 3. The sluice gate is at an angle $\theta$ to the horizontal, has length $L$ and is freely pivoted at $A$. The depth of water in reservoir is $D$, where $D > L\sin\theta$.

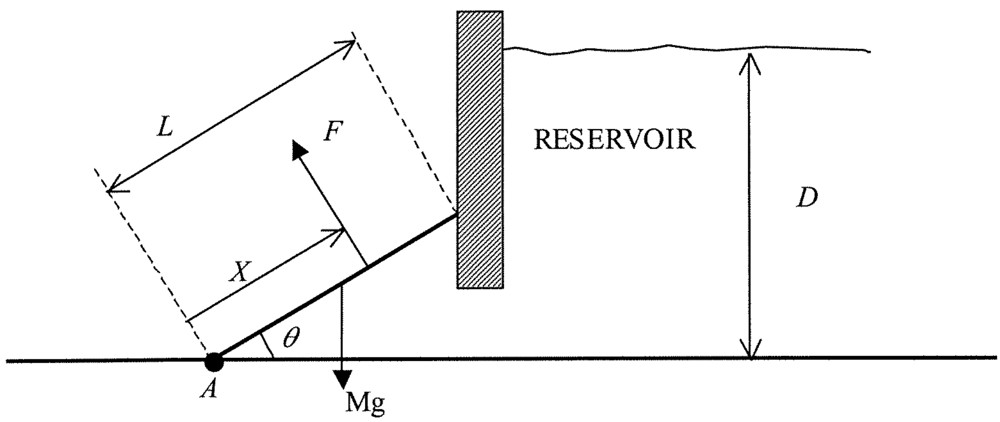


Fig. 3

(a) Show that the hydrostatic pressure gradient within the reservoir is given by:

$$\frac{dP}{dz} = \rho g$$

where $z$ is the depth below the surface, $\rho$ is the density of water and $g$ is the gravitational acceleration. [3]

(b) The resultant force on the sluice gate due to the hydrostatic pressure is $F$ per unit width and acts at a distance $X$ from the pivot $A$. Explain why $F$ acts perpendicular to the gate. [2]

(c) Show that the force $F$ on the sluice gate is:

$$F = \rho g L\left(D - \tfrac{1}{2} L\sin\theta\right)$$ [4]

(d) Calculate $X$, the distance of $F$ from the pivot $A$. [6]

(e) The sluice gate is made of uniform material so that its centre of mass is at the centre of the gate. Find an expression for the depth $D$ of the water at which the sluice gate will start to open. [5]

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4 A steady flow of hydrogen is burnt with excess air in an adiabatic combustion chamber. Both the hydrogen and air enter the combustion chamber at 25 °C with negligible kinetic energy.

(a) Calculate the amount of excess air required for the combustion products to be at 1500 K, assuming that they have negligible kinetic energy. [10]

(b) On both a wet and dry basis calculate the molar fraction of $O_2$ in the combustion products. [2]

(c) Calculate the total mass of the combustion products for each kmol of hydrogen burnt. [2]

(d) The combustion products pass through an adiabatic nozzle. If the temperature at nozzle exit is 1300 K, calculate the exit velocity. [6]

(You may assume that the molar composition of air is 21.0% $O_2$ and 79.0% $N_2$)

5 (a) State the first law of thermodynamics both for a thermodynamic system and for steady flow through a control volume. For each, explain carefully the conditions under which they apply. [4]

(b) A vertical cylinder containing 20 kg of air has a frictionless piston that is weighted to maintain an absolute pressure of 5 bar within the cylinder. Initially the air is at 15 °C. Calculate the amount of heat that must be supplied and the displacement work done by the air when it is heated to 500 °C. [5]

(c) The piston in (b) is now locked in position, so the cylinder volume remains fixed. The air within the cylinder is allowed to escape through an adiabatic, reversible turbine that exhausts to an ambient pressure of 1 bar. The turbine and pipe-work have negligible volume.

(i) Calculate the mass of air that escapes through the turbine and the total work done. [5]

(ii) Calculate the shaft work produced by the turbine during the process. [4]

(d) Finally, the piston is unlocked and is allowed to purge the remaining air through the turbine to the same exit pressure of 1 bar in a slow reversible process. Calculate any additional work that is done during this process. [2]

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SECTION B

*Answer not more than **four** questions from this section.*

6 An 'air motor' is shown schematically in Fig. 4. It consists of a rigid crank OA, of length $R$, which is pinned to a rigid connecting rod AB. The connecting rod is rigidly connected to a piston at B. The piston slides freely in a cylinder which is pivoted to the ground at C. The distance from C to A in the position shown is $L$, and the angle between the crank and connecting rod is $\theta$. Air pressure inside the cylinder applies force $F$ to the piston. This force causes the crank to rotate clockwise with angular speed $\omega$ as shown.

(a) Draw a velocity diagram for the mechanism in the position shown. Use a scale of $\omega R$ = 50 mm. [8]

(b) Determine expressions for the angular speed of the connecting rod AB, and the sliding speed between the piston and cylinder at point B, in terms of $\omega$, $R$, $L$ and $\theta$. [6]

(c) A torque $T$ resists movement of the crank at O. Assume that all members have negligible mass. Determine the force $F$ required at B to move the mechanism:

(i) if there is no friction in the mechanism;

(ii) if there is a friction torque of $Q$ acting at hinges A and C. [6]

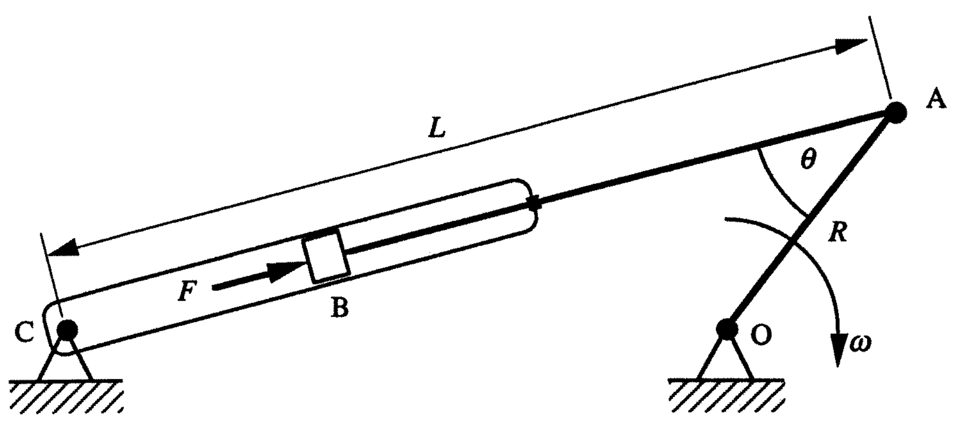


Fig. 4

7 (a) From first principles, show that the polar moment of inertia of a uniform solid disc of radius $R$ and mass $m$, about an axis through its centre is $J = mR^2/2$. Find also the polar moment of inertia about a point on the edge of the disc. [5]

(b) Such a disc rests on a rough horizontal table as shown in Fig. 5a. A light string is wound onto the disc and constant tension $T = mg$ is applied vertically downwards to the free end. There is no slip between the disc and the table.

(i) Show that the acceleration from rest of the centre of the disc is $2g/3$. [4]

(ii) Calculate the minimum coefficient of friction required between the disc and the table to ensure there is no slip. [4]

(c) The disc is now placed in a corner between a horizontal table and a vertical wall as shown in Fig. 5b. A force of $mg$ acts horizontally at the top of the disc as shown. The coefficient of friction at both contact points is $\mu$. Assuming that the disc slides at both contact points, write down the equations of motion for the disc and explain how you would solve them for its angular acceleration. [7]

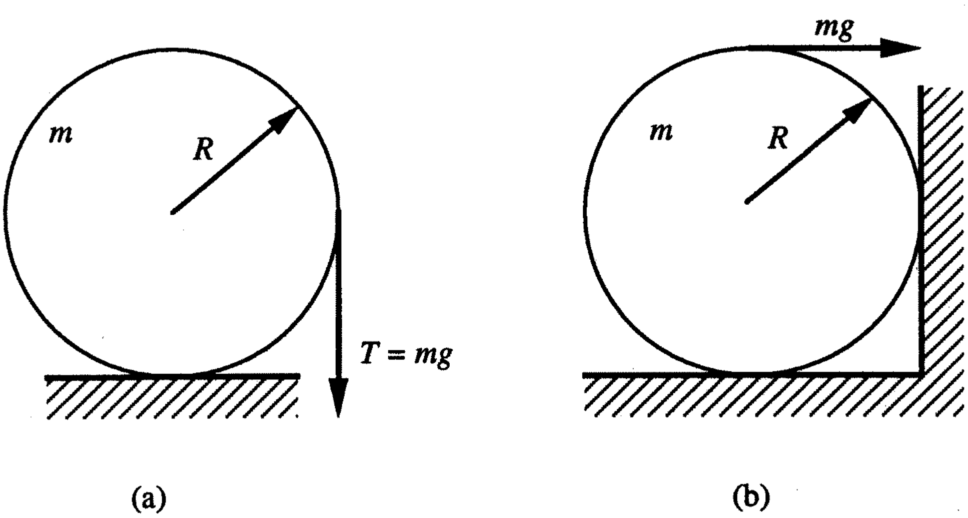


Fig. 5

8 A satellite is to be launched by taking it into orbit above the earth in a space shuttle and then firing the satellite's rockets to transfer it into a higher orbit. The radius of the Earth is $R$ and the acceleration due to gravity at the Earth's surface is $g$.

(a) Show that the speed $v_0$ at which the shuttle must fly in order to maintain a circular orbit at height of $0.25R$ above the Earth's surface is:

$$v_0 = \sqrt{gR/1.25}$$ [5]

(b) The satellite is released from the shuttle at this height, and the satellite's rockets are fired so that it has a velocity which is tangential to the initial orbit, but with a magnitude of $1.25\ v_0$. Determine the maximum height above the earth's surface in the satellite's subsequent motion, and its speed when it reaches this height, in terms of $g$ and $R$.

[HINT: Simplify the equations using an expression for $g$ in terms of $G$, the universal gravitational constant; $M_e$, the mass of the Earth and $R$.] [10]

(c) When it reaches the maximum height, the satellite's orbit is converted to circular by firing its rockets. Determine the tangential impulse $I$ that the rockets need to apply to *per unit mass* of satellite to achieve this. In which direction must the impulse act? [5]

9 The body of a car and its suspension can be modelled by a mass $m$ supported by a spring of stiffness $k$ in parallel with a linear viscous damper of rate $\lambda$, as shown in Fig. 6. Gravity may be neglected.

(a) Derive an equation describing the displacement of the car body $y$ in terms of the displacement of the road surface $x$. [4]

(b) The car is driven at a speed $v$ over a 'corrugated' road surface, with a sinusoidal roughness profile of amplitude $H$ and wavelength $L$. Derive an expression for the amplitude of the 'dynamic tyre force' $F$ applied to the road surface by the car. [5]

(c) Sketch a graph of the amplitude of the dynamic tyre force $F$ as a function of the speed $v$, for speeds up to 4 m/s. Estimate and plot on this graph:

(i) the maximum force amplitude in this speed range and the speed at which it occurs;

(ii) the force amplitude at a speed of 4 m/s.

Assume: $m = 800$ kg, $k = 32$ kN/m, $\lambda = 2$ kNs/m, $L = 2$ m, and $H = 10$ mm. [8]

(d) Describe the motion at speeds considerably greater than 4 m/s. [3]

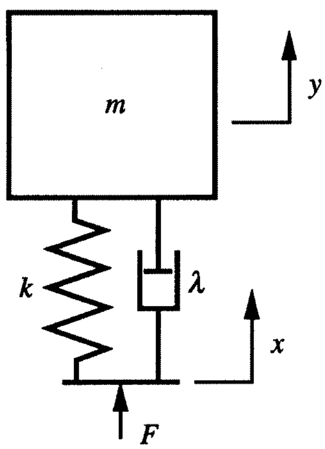


Fig. 6

10 A shaft of negligible mass has three rotors, each with polar moment of inertia $J$ as shown in Fig. 7. It is supported in frictionless bearings at A and B. The two central shaft segments have torsional stiffness $k$. The angular positions of the rotors are specified by coordinates $\theta_1$, $\theta_2$ and $\theta_3$.

(a) Show that the stiffness matrix of the system can be written:

$$\begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}$$ [3]

(b) Sketch the form of the three natural mode shapes of the system. [3]

(c) Determine the three natural frequencies of the system. [6]

(d) A sinusoidal torque $Q \sin \omega t$ is applied to rotor 1 as shown in the figure. Sketch a graph of the amplitude of the angular response of rotor 1, as a function of the frequency of the input $\omega$. Determine the frequencies at which the amplitude of the response of rotor 1 is zero. [8]

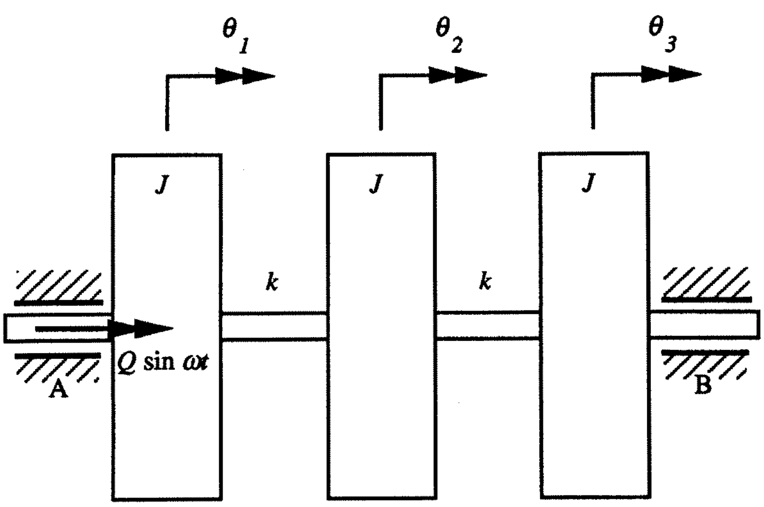


Fig. 7

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