ENGINEERING TRIPOS PART IA

Monday 11 June 2001 1.30 to 4.30

Paper 2

STRUCTURES AND MATERIALS

*Answer not more than **eight** questions, of which not more than **four** may be taken from Section A, and not more than **four** from Section B.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to Sections A and B should be tied together and handed in separately.

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SECTION A

*Answer not more than **four** questions from this section.*

1 Figure 1 shows a three-pinned arch ABCDEF, which has pins at A, B and F. It is subjected to vertical loads of W applied at B, C, D and E.

(a) Calculate the horizontal and vertical reactions at A and F. [4]

(b) Show that the bending moment is zero at C, D, and E. Comment on the bending moments elsewhere in the arch. [8]

(c) The loading is replaced by a single load W applied at B. Find the position and magnitude of the largest bending moment in the arch due to this loading. [8]

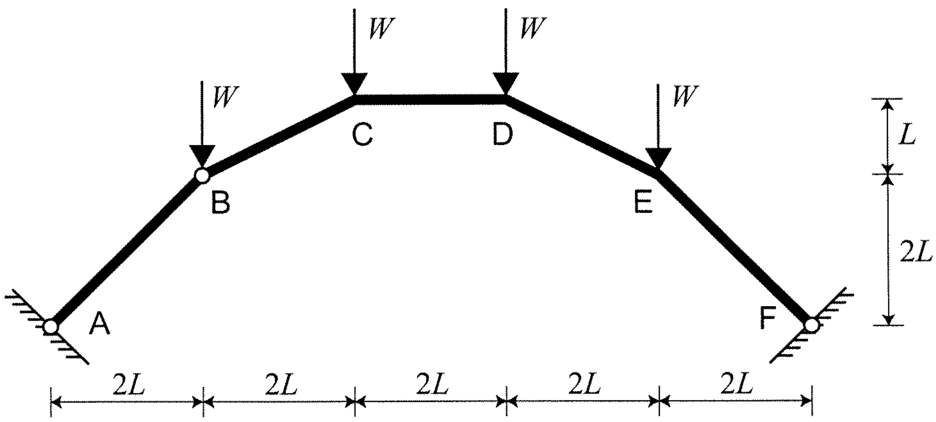

Fig. 1

2 Figure 2 shows a two-dimensional pin-jointed structure in which all the bars are made of material with a Young's modulus E. The cross-sectional area of the bars is A. A vertical downwards load of magnitude W is applied at S. The roller supports shown are able to take either a tensile or a compressive load.

(a) By initially considering the truss as a free-body, or otherwise:

(i) find the reactions at all of the supports; [4]

(ii) determine the forces in all of the bars. [4]

(b) Find the vertical deflection of the load point, at S, due to the applied load. [6]

(c) In addition to the applied load, bar QR is increased in length by 0.1%. Find the total vertical deflection of the support at R. [6]

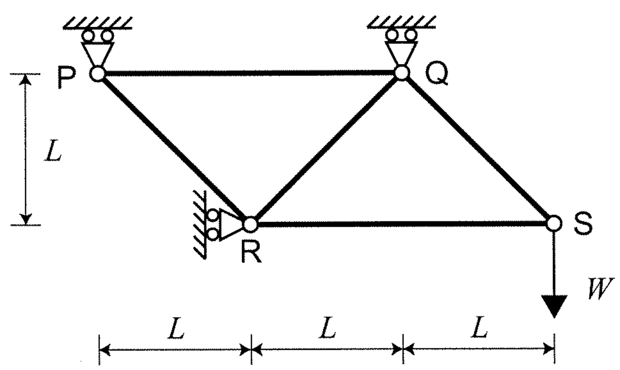

Fig. 2

3 The uniform cantilever shown in Fig. 3 is part of a support structure for a dam. Due to the water behind the dam, the cantilever is subjected to a load/unit length of:

$$q(z) = \frac{q_0}{L}(L - z)$$

(a) Show that the bending moment carried by the cantilever is:

$$M(z) = \frac{q_0}{6L}(L - z)^3$$

[6]

(b) The cantilever has a bending stiffness EI, and is straight and vertical when unloaded. Write down an expression for the curvature of the cantilever as a function of z, and hence find the deflection and rotation of the tip of the cantilever due to the load applied. [8]

(c) The water level behind the dam is reduced to half the original level. Calculate the deflection of the tip of the cantilever due to this load. [6]

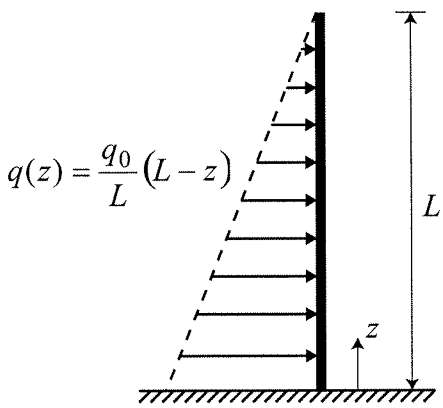

Fig. 3

4 A 406 mm × 178 mm × 60 kg/m Universal Beam (Structures Data Book, p. 13) carries, at a particular cross-section, a bending moment of 300 kNm about its major axis that causes tension at the bottom of the beam.

(a) Assuming that the beam is elastic and initially stress-free, find the greatest longitudinal tensile and compressive stresses in the section. [4]

(b) To reduce the peak compressive stress, a long steel plate of cross-section 177.9 mm × 20 mm is bolted to the outer surface of the top flange of the beam, as shown in Fig. 4, by pairs of bolts at a pitch of 200 mm along the length of the beam. If the beam carries the same moment of 300 kNm, find the greatest longitudinal tensile stress, and the greatest longitudinal compressive stress, in the new section. [10]

(c) The beam is also subject to a vertical shearing force of 150 kN. Calculate the shearing force in each bolt. [6]

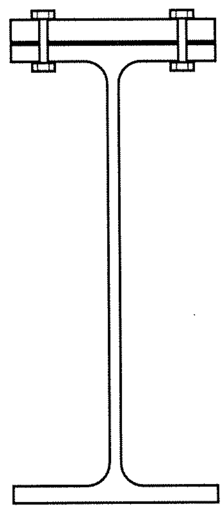

Fig. 4

5 The steel column shown in Fig. 5(a) has the square cross-section shown in Fig. 5(b). The column, of length $L = 500\ \text{mm}$, is subjected to a compressive load P. Consider only buckling in the x-y plane.

(a) Calculate the Euler buckling load, P_E, of the column [6]

(b) If the column has an initial sinusoidal imperfection in the x-y plane given by $v_0(x) = \delta_0 \sin(\pi x/L)$, show that the differential equation governing the shape of the column when loaded is given by:

$$EI\frac{d^2v}{dx^2} + Pv = -EI\left(\frac{\pi}{L}\right)^2 \delta_0 \sin\frac{\pi x}{L}$$

where E is the Young's Modulus, and I the relevant second moment of area. Hence show that the mid-span deflection for $P < P_E$ is given by:

$$\delta = \delta_0 \frac{P_E}{P_E - P}$$

[8]

(c) If the initial imperfection has magnitude $\delta_0 = 0.5\ \text{mm}$, find the maximum deflection of the column, and hence the peak bending moment in the column, when $P = 0.75P_E$. Find the peak compressive stress, and comment on the magnitude of this compared with the average compressive stress. [6]

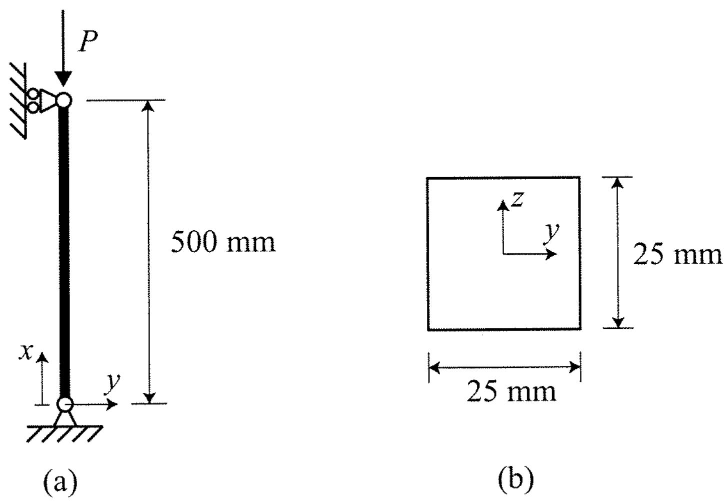

Fig. 5

SECTION B

*Answer not more than **four** questions from this section.*

6 (a) A unidirectional carbon fibre-epoxy composite has a fibre volume fraction of 60%. Using the data of Table 1, determine by calculation its longitudinal Young's modulus. Roughly by how much would you expect the longitudinal Young's modulus to differ from the transverse Young's modulus of the composite? [4]

Explain briefly how this difference between the two elastic moduli influences both beneficially and detrimentally the mechanical behaviour of a component made of a unidirectional fibre composite. [4]

(b) A robotic arm of solid circular cross-section is required for a space vehicle. Derive a performance index for selecting a material that will minimise the weight of the arm for a fixed bending stiffness.

The bending stiffness S of the arm is given by $S = EI$, where E is the Young's modulus of the material, and the second moment of area I is given by $I = \pi d^4/64$, where d is the diameter of the arm. The density of the material is ρ. [6]

(c) A first choice material for the robotic arm in (b) might have been stainless steel. Using the data of Table 1, estimate the percentage weight change of the robotic arm if instead of stainless steel, it was made from:

(i) pure epoxy resin; [3]

(ii) carbon fibre-epoxy composite, as described in (a). [3]

Material	Density (Mg/m^3)	Elastic modulus (GPa)
Pure epoxy resin	1.2	3.65
Pure carbon fibre	1.9	350
Stainless steel	8.0	198
Carbon fibre-epoxy composite, as described in (a)	1.6	

Table 1

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7 (a) Explain briefly how impeding the motion of dislocations with dispersed precipitates in a metal can raise the yield stress. [5]

(b) A straight segment of dislocation line of length l is pinned at points A and B in a crystal. The application of a shear stress τ causes the dislocation to bow outwards, as shown in Fig. 6, decreasing its radius of curvature as it does so. The force f acting on the dislocation at its critical configuration of a semi-circle is τbl, where b is the Burgers vector. Given that the dislocation line tension $T = Gb^2/2$, show that the stress just large enough to fully bow out the dislocation segment is given by:

$$\tau = Gb/l$$ [5]

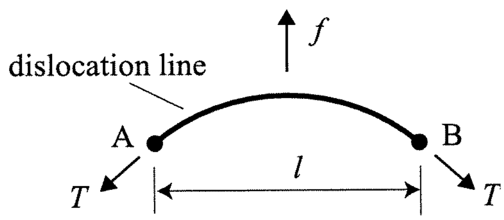

Fig. 6

(c) Estimate the tensile yield stress of aluminium containing an array of equally spaced second phase precipitates where the volume fraction of the second phase is 5% and the precipitate diameter is 0.1 μm. The Burgers vector for aluminium is 0.29 nm. [7]

(d) If incorrect heat-treatment of the aluminium alloy results in a non-uniform distribution of precipitates, explain the effect that this would have on the yield stress. [3]

8 (a) Explain the difference between the stress concentration factor K_t and the stress intensity factor K_1. [6]

(b) A thin-walled spherical pressure vessel of radius R = 1 m contains a hole of 25 mm radius to accommodate a safety valve. The vessel is made of a steel having a tensile yield strength of 500 MPa and a fracture toughness K_c of 100 MPa m$^{1/2}$. The vessel is designed for an internal working pressure p of 20 MPa. The maximum allowable stress in the wall at the working pressure is 360 MPa. Assuming that the stress concentration factor for the hole is equal to 2, calculate a suitable value for the wall thickness t of the pressure vessel. The stress in the wall of a uniform spherical pressure vessel is given by $\sigma = pR/2t$. [5]

(c) A similar pressure vessel with no hole is inspected by a non-destructive method, and an internal crack of length $2a$ = 10 mm is found in the wall, as shown in Fig. 7. Consequently, the pressure vessel is required to undergo a proof test at twice its normal working pressure. Determine by calculation whether the pressure vessel will survive the proof test, without yielding or fracturing catastrophically. [5]

(d) Describe two methods of non-destructively inspecting pressure vessels for buried cracks. In each case, indicate the limitations of the method. [4]

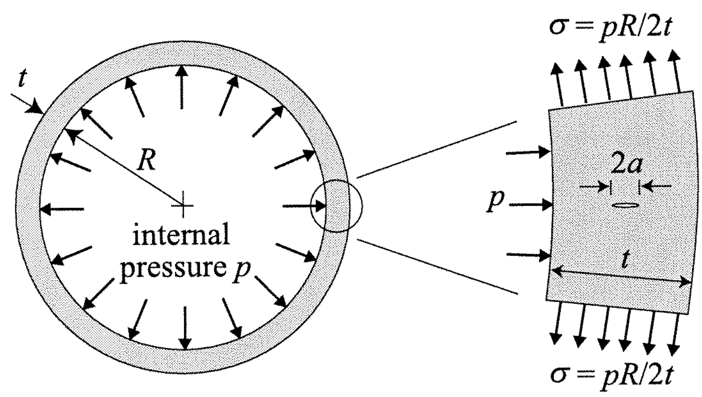

Fig. 7

9 (a) Describe the mechanisms of high temperature creep in polycrystalline materials using diagrams to illustrate your answer. [6]

(b) Account for the dependence of steady-state creep-rate on:

(i) temperature; [2]

(ii) stress; [2]

(iii) grain size. [2]

(c) List the important criteria for designing a creep-resistant material. [3]

(d) Polycrystalline copper has a steady-state creep-rate of 10^{-4} s^{-1} at 560 °C when subjected to a tensile stress of 50 MPa. The activation energy Q for self-diffusion in copper is 197 kJ/mol. Calculate the steady-state creep-rate of copper at 500 °C at the same applied stress. [5]

10 (a) Explain the usefulness of the galvanic series. [4]

(b) Indicate ways of eliminating or reducing the severity of galvanic corrosion. [4]

(c) Describe, giving specific examples, situations where galvanic corrosion can occur. [6]

(d) A ship's hull is protected from corrosion by connecting it to a zinc anode that initially weighs 136 kg. The average corrosion current is 2 A. Estimate how often the anode should be replaced.

The valency of zinc is 2. The atomic mass of zinc is 65.4 g/mol. Avogadro's number is 6.022×10^{23} atoms/mol and the charge on an electron is 1.602×10^{-19} C. [6]

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