I A ANSWERS
2002

Engineering Tripos Part IA, June 2002, Paper 1: Mechanical Engineering

Section A: Thermofluid

Answers

1 (a) –

(b) $z_1 = 0.567$ m, $z_2 = 0.717$ m, $z_3 = 0.217$ m

(c) $F = 258$ N, acting to the left

2 (a) Uniform pressure distribution

(b) $p_1 = p_{atm} + \rho_0 g H - \frac{1}{2}\rho_0 V_1^2$

(c) $\dot{m}\, c_p\,(T - T_0) = \dot{Q}$

(d) $p_1 = p_{atm} + \rho g H$, $\dot{m} = 1.15$ kg s^{-1}

3 (a) $W_c = -117$ J

(b) $T_2 = 549$ K

(c) $T_3 = 2306$ K

(d) $W_{exp} = 408$ J, $W_{net} = 291$ J, $\eta = 0.484$

4 (a) –

(b) $F = 1$ in both cases

(c) –

(d) 512.5 K

(e) 845 K

5 (a) $\dot{m}_1 = 45.3$ kg s^{-1}, $\dot{m}_{1b} = 35$ kg s^{-1}, $\dot{m}_{1a} = 10.3$ kg s^{-1}

(b) $V_2 = 287$ m s^{-1}, $\rho_2 = 0.559$ kg m^{-3}, $T_2 = 623$ K

(c) Virtually zero compared to the large fluxes of energy involved

Engineering Tripos Part 1A
Paper 1 Mechanical Engineering
Solutions
Section B

6 (b) (iii) $0.93m$ (iv) Loss of $3.83mgR$
7 (b) 0.04 rad/s clockwise, 0.04 rad/s anticlockwise, 15 mm/s, 8.3 mm/s (c) $3.7mg$
8 (a) ρAu^2 (d) $2m_0\rho Au, m_0^2$
9 (b) (ii) $y = \frac{Q}{\lambda}\left(1 - e^{-t/T}\right)$, where $T = m/\lambda$ (iii) $y = \frac{Rt}{\lambda} + \frac{RT}{\lambda}\left(e^{-t/T} - 1\right)$
10 (b) $0.35 < \zeta < 1.25$ approx; $170 < \lambda < 610$ Ns/m (c) $\zeta > 2.3\times10^{-3}$, $\lambda > 1.1$ Ns/m

Part 1A Engineering Tripos 2002

Paper 2 Section A, Structures

Answers

1 (b) 500 kN, 15000 kNm
(c) 9656 mm^2
(d) $2.20.10^9$ mm^4

2 (a) 3125 kN
(b) varies from 0.377 to 1.00 N/mm^2
(c) 3.31 degrees
(d) 5.23 degrees
(e) 6.89 degrees

3 (a) $\frac{wL^2}{8\delta}$
(b) $\frac{\rho g L}{E}, \frac{d}{L}$ or $\frac{\rho g d}{E}, \frac{L}{d}$ (other combinations possible)
(c) $\frac{e}{L} = \frac{1}{8}\left(\frac{\rho g L}{E}\right)\left(\frac{L}{\delta}\right)$
(d) Steel

4 $0.802\frac{WL}{AE}$

5 (a) 222.7 N/mm^2
(b) $44.7.10^3$ mm^3
(c) 210 N/mm^2 (Al); 354 N/mm^2 (CFRP)
(d) 0.925 N/mm^2

ENGINEERING TRIPOS PART IA 2002
Paper 2 Section B: Materials

Answers

6. (b) Assumption valid.
   (c) (ii) $\nu = 1/3$. Assumption reasonable.
   (iii) Yes. Rubber.

7. (a) $\sigma_T = \sigma_N (A_0 / A)$, $\varepsilon_T = \ln(1 + \varepsilon_N)$. Not valid after necking.
   (c) (ii) n is the resistance of the material to necking.

8. (a) (i) $\sigma = \sigma_0$
   (ii) σ_0 is the tensile stress that allows 37% of the samples to survive.
   (c) (i) No.

9. (a) G_{IC} and K_{IC} are material properties. $G = G_{IC}$, $K = K_{IC}$.
   (b) $K_{IC} = Y\sigma_F \sqrt{\pi c}$.
   (d) $\sigma = 353\,\text{MPa}$.

10. (a) $I_1 = \sigma_f^{2/3} / \rho$.
    (b) PMMA.
    (c) $c = (1/\pi)(K_{IC} t^3 / 6Fl)^2$.
    (d) $I_2 = K_{IC}^{2/3} / \rho$.
    (e) Stainless steel.
    (f) Stainless steel.

Engineering Tripos Part IA 2002

Paper 3, Electrical and Information Engineering

Section A - Numerical Answers

1. (b) $R_o = 2304\Omega$; $X_o = 758\ \Omega$; $R_t = 1.6\ \Omega$; $X_t = 8.9\ \Omega$

(c) (ii) $\theta_{TOTAL} = 530$ VAR ; $C = 29.3\mu F$

2. (b) (ii) $V_{oc} = 25$ V

(iii) $V_{meas} = 22.2$ V

(iv) $R_{in} \geq 24.9\ k\Omega$

(c) 11.2 % error

3. (c) (i) $Z_{in} = \dfrac{R_1 R_2}{R_1 + R_2}$

(ii) $Z_{out} = \dfrac{R_s r_d}{R_s + r_d + g_m R_s r_d}$

(iii) $\dfrac{v_{out}}{v_{in}} = \dfrac{g_m}{g_m + \left(\dfrac{R_s + r_d}{R_s r_d}\right)}$

(d) [e.g.] $R_1 = R_2 = 2\ M\Omega$; $R_S = 1978\ \Omega$

4. (c) gain = $\dfrac{-R_2}{R_1 + jwL}$

(d) $L = 15.9$ mH

(e) $A = 1.99$; $B = -1.47$ [or equivalent]

RJM/vg

15/8/02

Engineering IA 2002

Paper 3, Electrical and Information Engineering

Numerical answers

5) (c) 26 FA,

(d) (i) The program will add three numbers. The result is 2A,

(d)(ii) 32.5 μs.

6) (b) NAND gate

(c) (iv) $F = \overline{B} + \overline{C}\,\overline{D} + C\,D$, $I = C\,\overline{D} + \overline{B}\,\overline{D}$

7) (b) OR gate

8) (c) $J_A = BM$, $K_A = B + \overline{M}$, $J_B = \overline{A}\,P$, $K_B = A + M + \overline{P}$

Part 1A Electrical and Information Engineering Examination - Section C
June 2002

Numerical answers

9. a) $C = \dfrac{A\varepsilon_0\varepsilon_r}{x}$

b) $F = \dfrac{1}{2}V^2\dfrac{dC}{dx}$

c) 0.22mN (no edge or fringing effects)

d) 334 kV

10

b) 1.33 Tesla

c) 3.54 A

d) 140 N

11

c) $\dfrac{\sqrt{2}\mu_0 I}{\pi}\dfrac{n}{b-a}\ln\left(\dfrac{b}{a}\right)$ --- assuming n is large

Part IA Engineering Tripos Part 2002

Paper 4 Mathematical Methods Answers

1 (a) (ii) $\dfrac{\underline{a}.(\underline{b}\times\underline{c})}{|\underline{a}\times\underline{b}+\underline{b}\times\underline{c}+\underline{c}\times\underline{a}|}$ (iii) $\underline{a}.(\underline{b}\times\underline{c})\dfrac{\underline{a}\times\underline{b}+\underline{b}\times\underline{c}+\underline{c}\times\underline{a}}{|\underline{a}\times\underline{b}+\underline{b}\times\underline{c}+\underline{c}\times\underline{a}|^2}$

(b) any multiple of $\underline{b}=3\,\underline{i}+\underline{j}+5\,\underline{k}$;

$$\frac{x-0}{3}=\frac{y-\frac{4}{3}}{1}=\frac{z-\frac{2}{3}}{5}$$ (N.B. not unique)

2 (a) -1 (c) (i) $r=1.71$ and $\theta = 0.2145$ or 2.3089 or 4.4033

(c) (ii) $r=1$ and $\theta = 0.675$

3 (a) (i) $x_{CF}(t)=Ae^{-t/2}\sin\left(\dfrac{\sqrt{3}}{2}t+\phi\right)$ (ii) $x_{PI}(t)=C\sin\left(\dfrac{\sqrt{3}}{2}t\right)+D\cos\left(\dfrac{\sqrt{3}}{2}t\right)$

(b) $\dfrac{1}{\sqrt{5}}\left(\dfrac{1+\sqrt{5}}{2}\right)^n-\dfrac{1}{\sqrt{5}}\left(\dfrac{1-\sqrt{5}}{2}\right)^n$

4 (a) $\begin{bmatrix}\frac{13}{8} & \frac{3\sqrt{3}}{8}\\ \frac{3\sqrt{3}}{8} & \frac{7}{8}\end{bmatrix}$ (c) $\begin{bmatrix}-\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{bmatrix}$ (or any multiple) ; 2 and $\frac{1}{2}$ (d) $\dfrac{\sqrt{3}}{2}2^{100}\begin{bmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{bmatrix}$

5 (a) $g(t)=2e^{-2t}-e^{-t}\quad t>0;$ (b) $y(t)=\dfrac{e^{-t}}{1-\alpha}-\dfrac{2e^{-2t}}{2-\alpha}-\dfrac{\alpha e^{-\alpha t}}{(2-\alpha)(1-\alpha)}$

(c) $y(t)=e^{-t}-e^{-2t}$

6 (a) $\displaystyle\sum_{n=1}^{\infty}\frac{4}{n^3\pi}\left(1-(-1)^n\right)\sin nx$ (b) $-n^2a_n\quad -n^2b_n$

$$\sum_{n=1}^{\infty}\frac{4}{n^3\pi\left(k^2-n^2\right)}\left(1-(-1)^n\right)\sin nx$$

7 (b) (ii) .066 (iii) $.07\pm.01$

8 $x(t)=2e^t-\cos 2t-\dfrac{1}{2}\sin 2t$ $\quad y(t)=e^t-\cos 2t$

9 (a) 58.3 (b) $(0,0)$ & $(-2,0)$ are saddle points ; $\left(-1,-\frac{1}{2}\right)$ is a maximum.

10 0 10010100 10000000000000000000000

Engineering Tripos Part IA 2002
Paper 4 (Mathematical Methods) Section C
Model Answers

Note that the text of the questions has been abbreviated. The official full text is available separately.

Question 10

C10 a i.

Convert 3145728.0 to IEEE float. [5 marks]

This is an integer. Converting it to binary yields 11,0000,0000,0000,0000,0000, which is $1.1 \cdot 2^{21}$. The IEEE exponent is 21+127=148 (bias to allow for negative exponents). Including one bit for the sign, and dropping the first 1 of the normalized mantissa, we write it all down as 0 10010100 10000000000000000000000.

C10 a ii.

Consider this C++ code segment. Why do strange things happen with `tan(x)` and `tan(y)`? [4 marks]

The number 1.5707963 can't be represented exactly, neither as float nor as double, although the double will be a more accurate representation since it has twice as many bits. Same goes for the tangent.

Having said that, that number is almost $\pi/2$, whose tangent is infinite. So this is an ill-conditioned numerical problem. This is why the two results are so wildly different.

C 10 b i.

Describe the algorithm used in the supplied `FindRoot()` function and identify the role of the main variables. [5 marks]

The bisection method is used to find the root.

The solution is always between `low` and `high`. At each iteration, the search interval is halved, and we check whether the root is in the lower or upper half. We exit when the size of the interval is smaller than the required `precision` or when (if ever) we land on the exact root.

The condition inside the `if` statement checks whether the signs of `f_low` and `f_mid` are same or opposite. Meaning: if opposite signs, then the root is between `low` and `mid`.

The main variables are as follows:

- `low` and `high` define the interval for the root.
- `mid` is the midpoint of that interval.
- `f_low` and `f_mid` are the images of `low` and `mid` via the function $f(x) = \texttt{square} - x^2$.
- `precision` is the upper bound for the error we accept on the solution.
- `square` is the number of which we wish to find the root.

C 10 b ii.

What is the accuracy of the solution and how many times will the `while`-loop execute? [3 marks]

Assuming the input is within range, the maximum distance between the solution and the actual root will be half the size of the final interval, i.e. 0.000005.

The initial interval is from 0.1 to 10. This is halved at every iteration, and the program stops when the size of the interval goes below 0.00001. This happens in $\lceil \log_2 \frac{10-0.1}{0.00001} \rceil = 20$ iterations. (It may stop earlier if it hits on the solution exactly, but this is statistically unlikely.)

Regardless of the input value, and even if the input is out of range, the program will still stop after at most 20 iterations—it won't loop forever.

C 10 b iii.

In what range must the root lie to get a solution? When will the algorithm fail to find the correct solution? [3 marks]

A solution will always be produced no matter what. However, for a *correct* one, the root r (not the input!) must be such that $0.1 < r < 10$.

Inputs (not roots!) outside the range 0.01 to 100, including of course negative ones, will fail to produce a correct solution.

Question 11

C 11 a i.

Identify bugs in the supplied `LeastSquares()` function and correct them. [8 marks]

- The variables `sum_...` are never initialized. They should be initialized to 0.
- The `for` loop has an off-by-one error: it should count from 0, not from 1, or it will miss the first element of the array.
- The semicolon at the end of the header (1st line) shouldn't be there.
- The semicolon at the end of the `for` line shouldn't be there, or the bit that follows will be taken to be outside the body of the loop.
- A close curly bracket is missing before the `data.a =...` section, to conclude the body of the loop.
- The `a` in the last expression is a typo for `data.a`.
- The `data` variable is passed by value, but should be passed by reference (`ModelData& data` in the header) otherwise no result will be returned to the calling program.

C 11 a ii.

Write a C++ definition for `ModelData`. [4 marks]

```
#define SIZE 100
struct ModelData {
    int n;
    float x[SIZE];
    float y[SIZE];
    float a;
    float b;
};
```

C 11 b.

What is meant by algorithmic complexity? [3 marks]

An estimate of the running time of the algorithm as a function of the size of the input when the size of the input tends to infinity. Only the dominant term is taken into account, ignoring smaller ones and multiplicative constants.

C 11 b i.

What is the complexity of the least-squares algorithm? [2 marks]

A constant amount of computation is performed for each of the n points, so $O(n)$.

C 11 b ii.

Describe an algorithm for sorting the pairs and state its complexity. [3 marks]

The exchange-sort algorithm sorts a list of n elements using n passes. In the first pass we look at the whole list, find its smallest element, and put it in the first position, exchanging it with whatever was there. At each successive pass we look at the rest of the list, find the minimum of that and put it in the first position of the sublist. The loop invariant is that, at the end of pass i, the first i elements are sorted.

The complexity is $O(n^2)$ because the operation of finding the smallest element of a sublist is $O(n)$ and it has to be repeated once for each of the n passes.