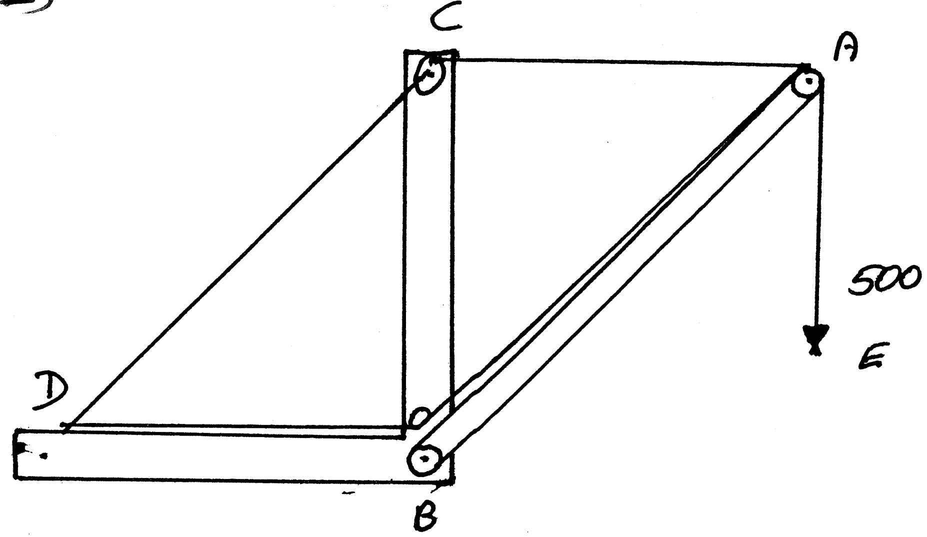
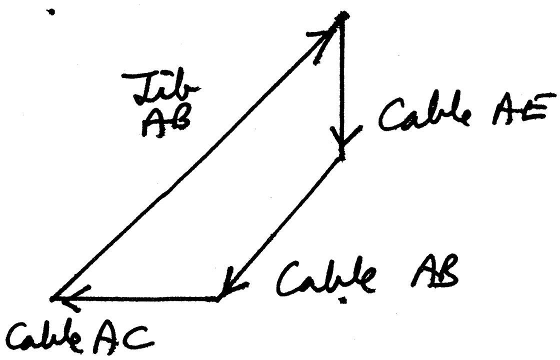
1.



Main lifting cable must carry 500 kN

Luffing cable must also carry 500 kN (by taking moments about B).

(a)

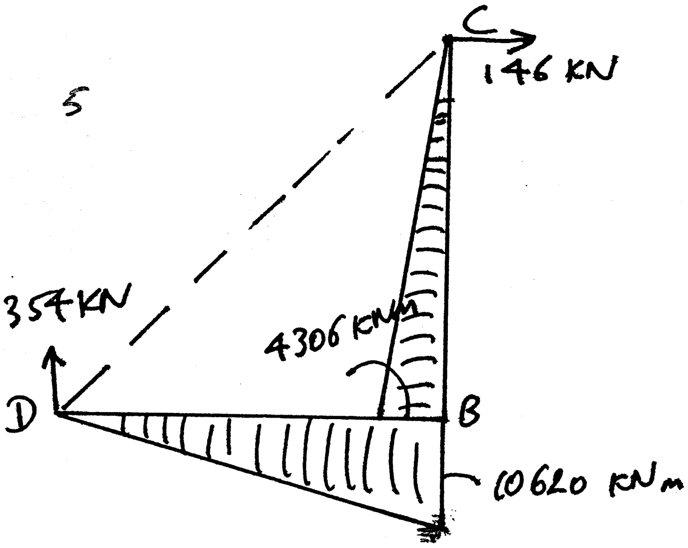


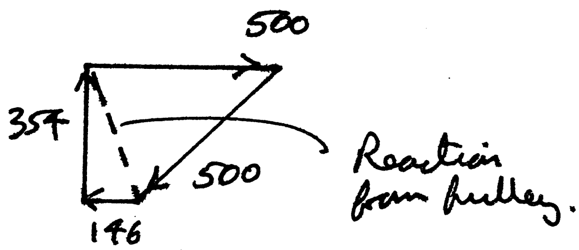
[4]

∴ Jib AB must carry $500 + \frac{2.500}{\sqrt{2}} = 1207$ kN.

(b)

5





Polygon of force at C

Support at B must apply 4380 + 10620
= 15000 kNm 3
(= 500 × 30 as expected)

1

(c) Axial force in jib $= 1207\,\text{kN}$

$\therefore$ Required area of jib $= \dfrac{1207.10^3}{250} \times 2 = \underline{\underline{9656\ \text{mm}^2}}$ 4

(d) Euler Buckling Load $= \dfrac{\pi^2 EI}{L^2}$

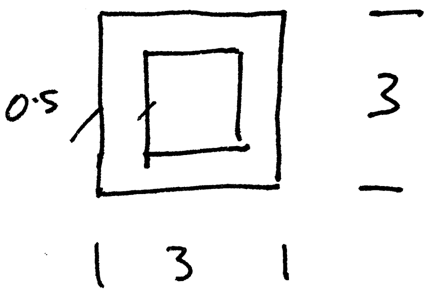
$$\therefore\ 1207.10^3 . 2 = \frac{\pi^2 . 200.10^3 . I}{(30.10^3 . \sqrt{2})^2}$$

4

$\Rightarrow \underline{I = 22{\cdot}0.10^8\ \text{mm}^4}$

2

(a)



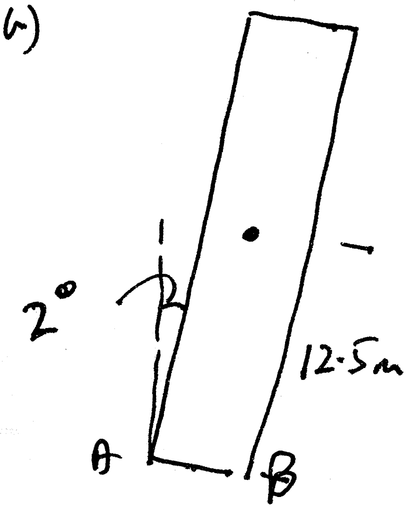
Cross sectional area
$= 9 - 4 = 5 \, m^2$

[3]

$\therefore$ Total weight of tower

$= 5 \times 25 \times 25 = \underline{3125 \text{ kN}}$

(b)

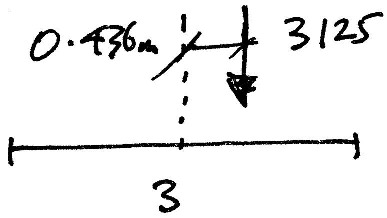


Offset due to 2° inclination

$= 12.5 \sin 2^\circ = 0.436 m$

Across base

[2]



$\therefore$ Applied moment on base $= 1362.5$ kNm

$$I \text{ of section} = \underbrace{\left(\frac{bd^3}{12}\right)}_{\text{outer}} - \underbrace{\left(\frac{bd^3}{12}\right)}_{\text{hole}} = \frac{3^4}{12} - \frac{2^4}{12}$$

$$= 5.42 \, m^4$$

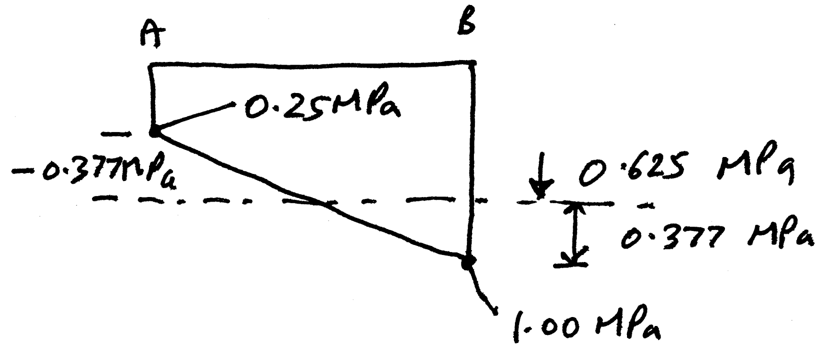
[2]

$$\therefore Z = \frac{5.42}{1.5} = 3.61 \, m^3$$

$$\therefore \text{Bending stress} = \frac{1362.5 \cdot 10^6}{3.61 \cdot 10^9} = 0.377 \text{ MPa}$$

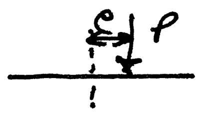
$$\text{Axial stress} = \frac{3125 \cdot 10^3}{5 \cdot 10^6} = 0.625 \text{ MPa}$$

∴ Stress distribution



[2]

(c) Stress at A will be zero when



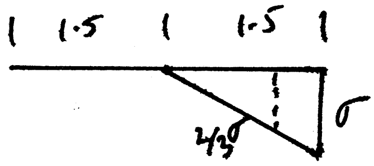
$$\frac{\cancel{P}.e}{Z} = \frac{\cancel{P}}{A}$$

$$\Rightarrow e = \frac{Z}{A} = \frac{3.61}{5} = 0.722 \text{ m}$$ [2]

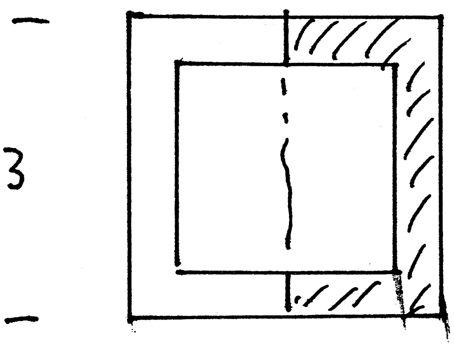
∴ Angle of inclination $= \sin^{-1} \frac{0.722}{12.5} = 3.3\phi$ degs

(d) If angle increases more, masonry cracks in tension. Stress distribution becomes triangular and the resultant needs to be found.

If zero stress at mid point



Centroid not at 2/3 point since section is not solid



Consider as section 3x1.5 m carrying maximum stress σ, <u>less</u> effect of section 2x1 carrying maximum stress 2/3σ

Force $= 3 \times 1.5 \times \frac{\sigma}{2} - 2 \times 1 \cdot \frac{\cancel{2}}{3}\sigma \cdot \frac{1}{\cancel{2}}$

$= 2.25\,\sigma - 0.67\sigma = 1.583\sigma$

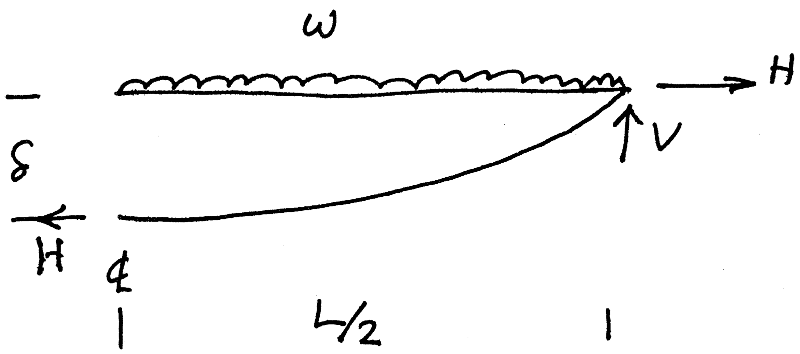
Centred at $\dfrac{2.25\sigma \cdot 1 - 0.67\sigma \cdot \frac{2}{3}}{1.583\sigma} = 1.14\text{ m}.$ [7]

$\therefore$ Required angle $= \sin^{-1} \dfrac{1.14}{12.5} = \underline{5.23^\circ}$

(e) The tower will collapse when the resultant leaves the section

$\theta = \sin^{-1} \dfrac{1.5}{12.5} = \underline{\underline{6.89^\circ}}$ [2]

3 (a)



Vertical equilibrium

$$V = wL/2$$

[4]

moments about cable at ℄

$$H.\delta + wL.\frac{L}{4} = \frac{wL}{2}.\frac{L}{2} \Rightarrow H = \frac{wL^2}{8\delta}$$

(b) Relevant variables

diameter	d	$[L]$
density	ρ	$[ML^{-3}]$
sag ratio	δ/L	$[-]$
Youngs Mod.	E	$[ML^{-1}T^{-2}]$
gravity	g	$[LT^{-2}]$
span	L	$[L]$

[Buckingham :- 6 variables 3 dimensions ⇒ 3 non dimensional groups]

By inspection, to eliminate [M] and [T] from E, groups will be

$$\left(\frac{\delta}{L}\right) = f\left(\frac{\rho g L}{E}, \frac{d}{L}\right)$$

[4]

(Other combinations are feasible, such as $\left(\frac{\rho g d}{E}, \frac{L}{d}\right)$ etc.)

(c) Cable tension $= H = \dfrac{wL^2}{8\delta} = \dfrac{\rho g A . L^2}{8\delta}$

(where A = cross-sectional area)

Stress $= \dfrac{H}{A} = \dfrac{\rho g L^2}{8\delta}$

Strain $= \dfrac{\rho g L^2}{8\delta . E}$

extension $e = \dfrac{\rho g L^3}{8\delta E}$ [5]

$\left(\dfrac{e}{L}\right) = \dfrac{1}{8} \cdot \left(\dfrac{\rho g L}{E}\right) \cdot \left(\dfrac{L}{\delta}\right)$ [or similar if other N.D groups chosen]

$\left(\dfrac{\rho g d}{E}\right)\left(\dfrac{L}{\delta}\right)\left(\dfrac{L}{d}\right)$

(d) Effect of changing material is to alter ND group $\left(\dfrac{\rho g L}{E}\right)$.

Aramid $\rho = 1440$ kg/m³ $E = 126.10^9$ N/m²

Steel $\rho = 7860$ kg/m³ $E = 200.10^9$ N/m²

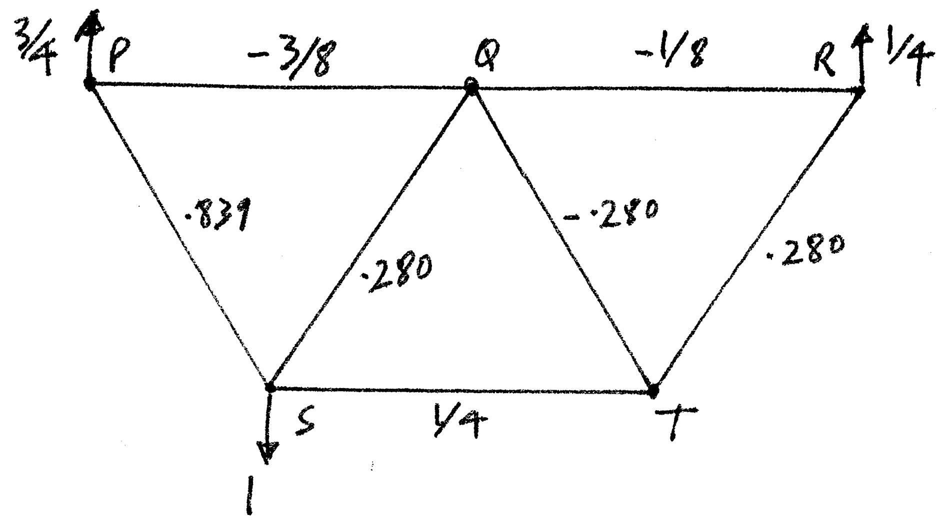
$\therefore \left(\dfrac{\rho g L}{E}\right)_{aramid} = \dfrac{1440}{126.10^9} gL = 11.4.10^{-9} . gL$

$\left(\dfrac{\rho g L}{E}\right)_{steel} = \dfrac{7860}{200.10^9} gL = 39.3.10^{-9} gL$

Steel cables are more susceptible to effects of axial extension than aramid cables. [4]

(c) The principal assumptions made are that the weight is uniformly distributed over the length, which is not the case because of the inclination of the cable, and that the bending stiffness is negligible. [3]

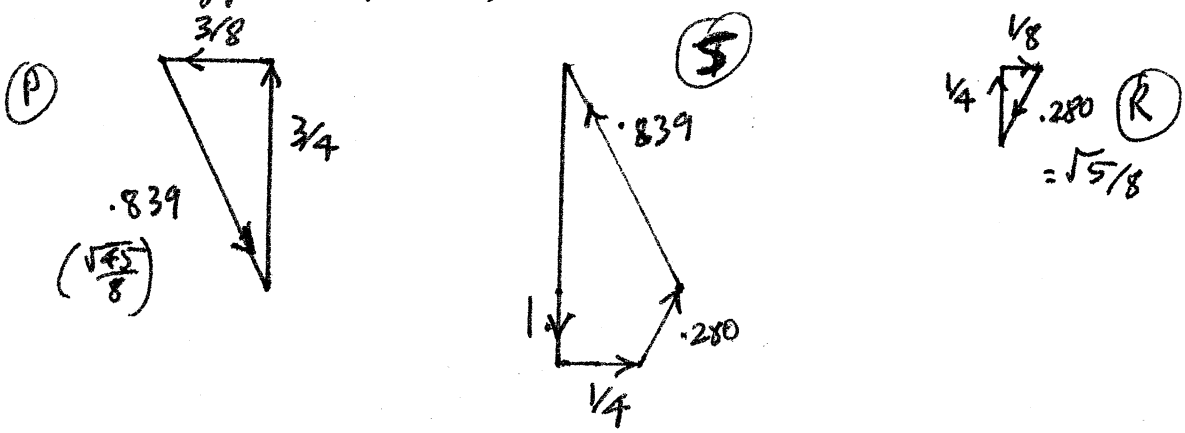
4



all × W

[10]

Force Polygons (all × W)

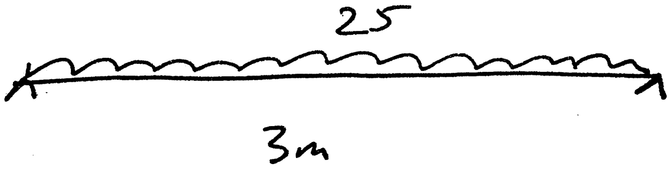


Vertical deflection at S

Member	Real F (×W)	Area (×A)	E (×E)	L (×L)	$e=\frac{FL}{AE}$ (×WL/AE)	T^* =F	T^*e
PQ	-·375	1	1	1	-·375		·141
QR	-·125	1	1	1	-·125		·016
PS	+·839	·5	4	1·12	+·470		·394
SQ	+·280	1	1	1·12	+·314		·088
QT	-·280	1	1	1·12	-·314		·088
RT	+·280	·5	4	1·12	+·157		·044
ST	+·250	·5	4	1	+·125		·031
		[7]					·802

$$\therefore \delta_S = \cdot 802 \frac{WL}{AE}$$ [3]

5



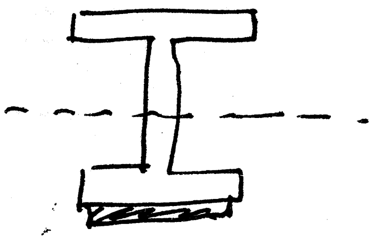
(a) Bending moment $= \frac{WL}{8} = \frac{25.3}{8} = 9.375$ kNm

Z Elastic section modulus $= 42.1 \cdot 10^3$ mm^3 (data book)

$\therefore$ Bending stress $= \frac{M}{Z} = \frac{9.375 \cdot 10^6}{42.1 \cdot 10^3}$ [4]

$= \underline{\underline{222.7 \text{ MPa}}}$

(b)



$E_{Al} = 70$ GPa

$E_{CFRP} = 140$ GPa.

$\therefore$ Effective transformed width of CFRP = 80mm.

Depth of centroid below top $= \frac{50 \times 1370 + 101 \times 80 \times 2}{1370 + 160}$

$= \frac{84660}{1530} = \underline{\underline{55.33 \text{ mm}}}$ [2]

New I $= 210 \cdot 10^4 + 1370 \cdot (5.33)^2 + \frac{80 \cdot 2^3}{12} + 160 \cdot (45.67)^2$

$= 2100000 + 38920 + 53.33 + 333720$

$= 247 \cdot 10^4$ mm^4 [2]

Z_{al} (N.B - top surface)

$$= \frac{247.10^4}{55.33} = 44.7.10^3 \text{ mm}^3$$ [4]

(c)

$\therefore$ Max stress is now $\frac{9.375.10^6}{44.7.10^3} = \underline{\underline{210 \text{ MPa}}}$ [2]

$$\left(Z_{CFRP}\right)_{Al\ units} = \frac{247.10^4}{(102-55.33)} = 52.9.10^3 \text{ mm}^3$$

$$\left(Z_{CFRP}\right)_{CFRP\ units} = 52.9.10^3 . \frac{70}{140}$$

$$= 26.5.10^3 \text{ mm}^3$$

[4]

Max stress = CFRP = $\frac{9.375.10^6}{26.5.10^3} = \underline{\underline{353.8 \text{ MPa}}}$.

(d) Max shear force = 12.5 kN.

$$\tau t = \frac{FA\bar{y}}{I}$$

$A\bar{y}$ = 160.(101 − 55.33) = 7307 mm^3 (Al units)

I = 247.10^4 mm^4 (Al units)

$$\therefore \tau t = \frac{12.5.10^3 . 7307}{247.10^4} = 37.0 \text{ N/mm}.$$

t = 40 mm.

$\therefore \tau = \underline{\underline{0.925 \text{ MPa}}}$ [5]

Part 1A Structures Paper 2002

Examiner's Comments to be attached to the crib

There were a number of lost marks because candidates failed to show units, and more worryingly because they have used the results of their calculator which use m, k, M and μ in place of units or of powers ten. Thus there were ludicrous things calculated such as moments of "23 μMNmm". A significant number could not distinguish their sines from their cosines, nor recognised when they used reciprocals of these quantities and resolved forces into components that were larger than the original force.

1. Forces and moments in a crane. A large number failed to make sense of the forces at the tip of the jib where there are two co-linear forces, one in the jib itself and the other in the cable running along the jib. A significant number did not keep the force in each cable constant along its length. A number confused action and reaction, with inevitable sign errors as a result. Very few drew free-body diagrams, or if they did had more than one cut or did not put in all the forces across the cut, and as a consequence the drawing of the bending moment diagrams was done very badly. It seems nonsensical that they can calculate the right forces on a body and then be unable to calculate the internal moments. The external reactions were almost invariably calculated wrongly by those who tried to resolve the internal forces, but correctly by people who considered overall equilibrium. The calculations of cross sectional area and inertia required for the jib were done well, apart from those who doubled the allowable stress to give a factor of safety, rather than halving it. Let us hope they become electricians.

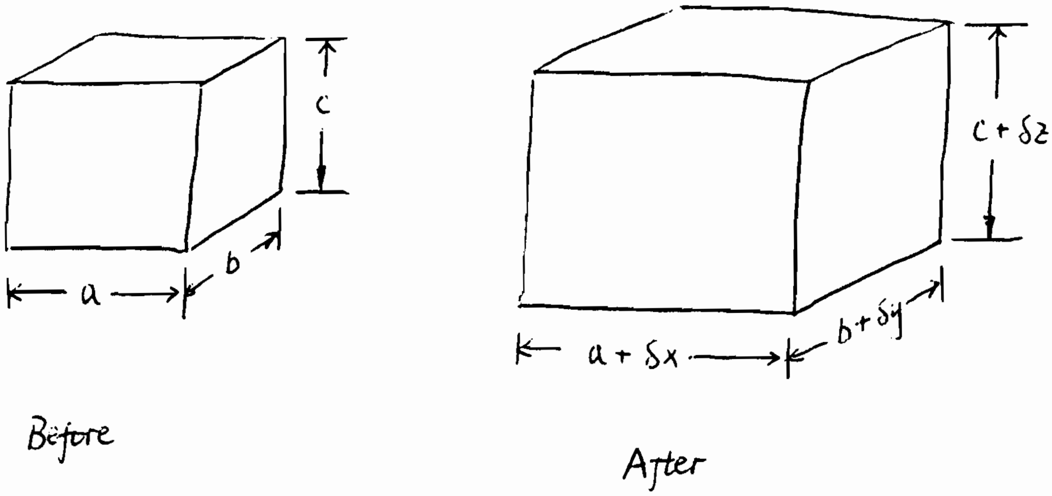
2. The Leaning Tower of Pisa, or at least a simplified form of it. The least popular question and the worst done, by a large margin. Several did not recognise it as a beam bending question, despite there being an examples sheet question which treats a chimney under wind load in the same way. Most managed to calculate the weight correctly, although a few only took account of a single thickness of masonry. A number put the centre of gravity at the top of the tower or calculated the stresses at the base on the assumption that the tower was on two rigid supports. Most were able to say when the tower would topple, and many made intelligent comments about the possibility of earlier material failure. Very few tackled the part that asked what happened when the masonry went into tension, but those that did got it right.

3. Cables with Dimensional Analysis. Most do not know the difference between mass and weight, which meant they left out g, without which they had only one variable with units of time (E). There was a vicarious pleasure in watching them try to wriggle out of that – one candidate resorted to the inclusion of natural frequency as a variable, which might have worked, but not in a way of which the mechanics group would have approved. For some inexplicable reason, very few thought that the length of the cable was an important variable when determining their non-dimensional groups. They knew that $E=\sigma/\varepsilon$; they had just derived a formula relating cable tension to the weight of the cable, and they knew that weight was related to density, but they could not join up the dots.

4. Virtual Work applied to a truss similar, but not identical, to the Statics Lab that they have all done. Most of the errors were failures to get the angles right, or simple slips in the algebra. Almost none drew polygons of forces which would have eliminated most errors although one who did still managed to get tension and compression mixed up. Several tried to apply symmetry to a blatantly non-symmetrical problem, especially those who tried to work out bar forces by starting at the load rather than at the supports.

5. Aluminium beam with CFRP plate glued on. A variation on a very standard theme. About half could not calculate the bending moment at the centre of a simply supported beam under uniformly distributed load. There were also serious problems with units in this question. The overall beam length was in m, and the load in kN, but cross-section dimensions were given in mm, and in the data book the values were quoted in $mm^2 \times 10^2$ or $mm^4 \times 10^4$. Almost none of the candidates correctly allowed for the changes in units or the powers of 10, and most made significant errors in the dimensionality of their units ("stresses" of N, "moments" of kN/m etc), if they showed any units at all. I think this comes from a reliance on their calculators to sort out powers of 10 for them, which may work when the units they start with are consistent but should not be relied on. Several ignored the statement that the beam was made of Aluminium and used the Modulus for steel instead. A fair number confused centre of gravity and centroid; "*the centroid of the section does not move since the weight of the carbon fibre is negligible*". Oh yes it does! A significant number were confused by whether to increase the width of the CFRP when converting into Aluminium units, or to decrease it. Some increased the thickness instead! The correct figures were plugged into the shear formula by most. No one pointed out that gluing CFRP onto Al is pointless since it makes very little difference to the stress in the Al, primarily because their numerical results were wrong by the end.

6. (a)



Volume change $\Delta V = (a+\delta x)(b+\delta y)(c+\delta z) - abc$

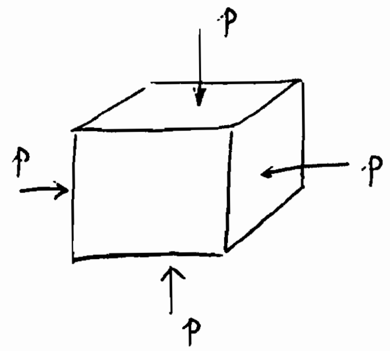
$$= \delta x \cdot bc + \delta y \cdot ac + \delta z \cdot ab + \cdots$$

$$\Rightarrow \quad \Delta = \frac{\Delta V}{abc} = \frac{\delta x}{a} + \frac{\delta y}{b} + \frac{\delta z}{c} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

(b) For ductile materials experiencing extensive yielding, elastic deformation is negligible. Because yielding does not cause volume change (for metals at least), the assumption of $\Delta = 0$ is valid.

(c) (i) Assuming deformation is elastic with $\Delta \neq 0$.

$\sigma_x = \sigma_y = \sigma_z = -p$ (hydrostatic pressure)



$$\Rightarrow \Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= 3 \times \left[\frac{-p}{E} - \nu(-p-p) \right]$$

$$= -\frac{3(1-2\nu)}{E} p$$

But $K = -\dfrac{p}{\Delta} \quad \Rightarrow \quad K = \dfrac{E}{3(1-2\nu)}$

(ii) $K = \dfrac{E}{3(1-2\nu)} = E \quad \Rightarrow \quad 3(1-2\nu) = 1$ i.e. $\nu = 1/3$

This is close to experimental measurement for many materials.

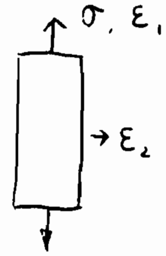
$K \approx E$ is a valid assumption.

(iii) When $\nu = 0.5$, $K \to \infty$ and hence $\Delta \to 0$, i.e. material incompressible even in elastic range. Rubber is such a material.

6 (d)

Compared with other measurement methods, measuring E with ultrasound technique is the most accurate (measure the velocity of sound in the material).

For measurement of ν, apply σ in the axial direction and measure ε_1 & ε_2



$\Rightarrow \quad \nu = \varepsilon_2 / \varepsilon_1$

Both strains can be small for materials with high modulus, which can pose difficulties in measurement.

7 (a) Define A = current area, A_0 = initial area
l = current length, l_0 = initial length

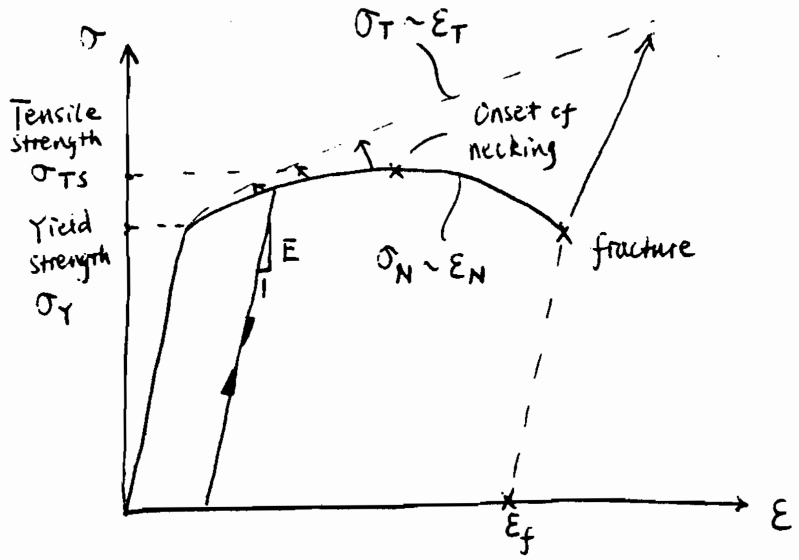
$$\sigma_T = \frac{F}{A} = \frac{F}{A_0}\,\frac{A_0}{A} = \sigma_N \frac{A_0}{A}$$

$$\varepsilon_T = \ln\left(\frac{l}{l_0}\right) = \ln(1+\varepsilon_N)$$

Following necking, these relations are no longer valid. The true stress and true strain are now given by

$$\sigma_T = \frac{F}{A_{neck}}, \qquad \varepsilon_T = \ln\left(\frac{A_0}{A_{neck}}\right)$$

(b)



(c) (i)

Assumption of constant volume $\Rightarrow A_0 l_0 = A l$

$\Rightarrow \sigma_T = \sigma_N \dfrac{l}{l_0}$, But $\varepsilon_N = \dfrac{l}{l_0} - 1$

$\Rightarrow \quad \sigma_T = \sigma_N(1+\varepsilon_N) \geq \sigma_N$ as $\varepsilon_N \geq 0$

On the other hand

$\varepsilon_T = \ln(1+\varepsilon_N) \leq \varepsilon_N$ as $\ln(1+x) \leq x$ when $|x| \leq 1$

7 (c) (ii)

At necking $$\frac{d\sigma_N}{d\varepsilon_N} = 0$$

$\sigma_T = A(\varepsilon_T)^n$ becomes

$$\sigma_N(1+\varepsilon_N) = A\left[\ln(1+\varepsilon_N)\right]^n$$

or

$$\sigma_N = A\,\frac{\left[\ln(1+\varepsilon_N)\right]^n}{1+\varepsilon_N}$$

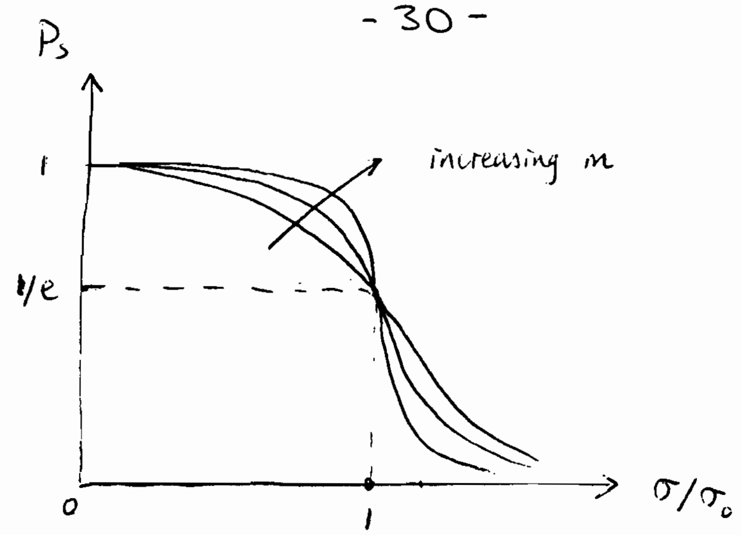
Differentiating w.r.t. ε_N gives

$$A.\frac{n\{\ln(1+\varepsilon_N)\}^{n-1}\cdot\frac{1+\varepsilon_N}{1+\varepsilon_N} - \{\ln(1+\varepsilon_N)\}^n}{(1+\varepsilon_N)^2} = 0$$

$\Rightarrow$ $$n = \ln(1+\varepsilon_N) = \varepsilon_T \quad \text{at necking}$$

Consequently, n can be considered as the material's resistance to necking.

8 (a) (i)



When $\sigma = \sigma_0$, $P_s(V_0) = 1/e = 0.37$ is independent of m.

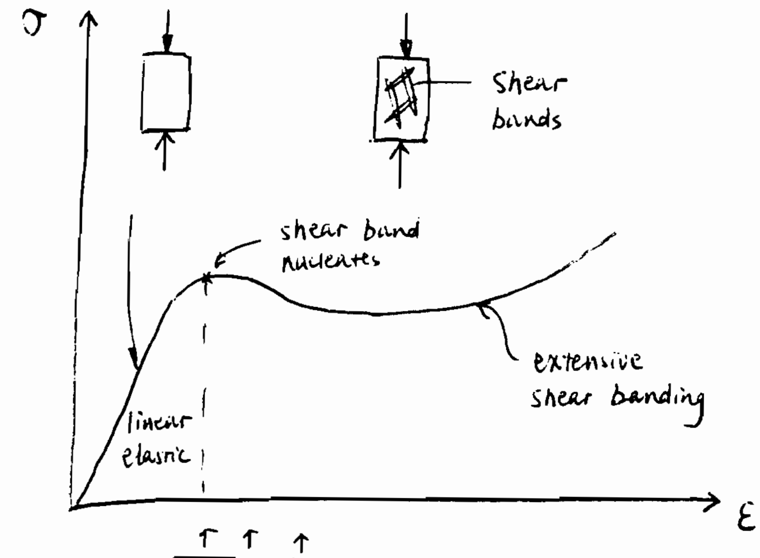
(ii) When $\sigma < \sigma_0$, P_s increases with increasing m.
When $\sigma > \sigma_0$, P_s decreases with increasing m.

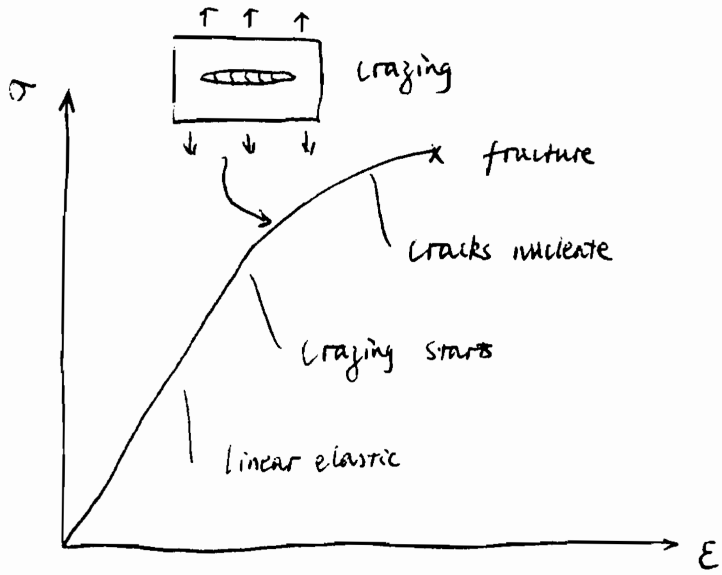
When $\sigma = \sigma_0$, $P_s = 0.37$, i.e., σ_0 is the tensile stress that allows 37% of the samples to survive.

(iii) For materials with low values of m, the variability in strength is high. Hence there will be a small but finite probability of failure even at low stress levels. To use such materials for critical applications would require very high safety factors (for safe design), and very good risk analysis.

(iv) Test a batch of samples of volume V_0 at stress σ_1 and determine the number that survive. Test another bauth at stress σ_2, and so on. Plot the survival probability as a function of stress. The stress corresponding to $P_s = 0.37$ is σ_0. The coefficient m can be estimated by curve-fitting the graph.

8 (b)





(c) (i)

No. At $T \sim T_g$, polymer chains will be free to slide so that it is difficult to open up a craze.

(ii) Necking in nylon is stable (due to molecules alignment), whereas necking in metal leads to failure.

9 (a) G is the energy released per unit crack advance,

K is the measure of stress singularity at a crack tip.

Both increase in value as load is increased. $G = K^2/\bar{E}$

G_{Ic} and K_{Ic} are the critical values of G and K at failure loads, and they are material constants. $G_{Ic} = K_{Ic}^2/\bar{E}$

Fracture criteria: $G = G_{Ic}$ or $K = K_{Ic}$

Yielding relaxes stresses near crack tip. In comparison with a brittle crack with negligible yielding, the length of a crack with yielding is effectively increased at the same load level.

(b) $$K_{Ic} = \underbrace{Y}_{\text{geometry dependent parameter}} \sigma_F \sqrt{\pi c} \qquad (*)$$

(c) Design philosophy 1: Material given, hence K_{Ic} fixed. In addition, a relatively large stable crack of size c is present (which can be readily detected and repaired). The use of $(*)$ determines the design stress must satisfy

$$\sigma < \frac{K_{Ic}}{Y\sqrt{\pi c}}$$

Design philosophy 2: Material given $\rightarrow$ K_{Ic} fixed. Design stress σ is also given (like in the case of an aircraft wing where stress level must be high to increase the payload). Then the allowable crack size is

$$c < \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2.$$

(d) $\sigma_Y = 1000 \times 10^6\ N/m^2$, $K_{Ic} = 50 \times 10^6\ N\sqrt{m}/m^2$, $c = 10^{-2}\ m$

$$K_{Ic} = \frac{1.1\sigma\sqrt{\pi c}}{1.4 - 0.2(\sigma/\sigma_Y)^2}$$

$$\Rightarrow \quad 50\left[1.4 - 0.2(\sigma/1000)^2\right] = 1.1\sigma\sqrt{\pi \times 10^{-2}}$$

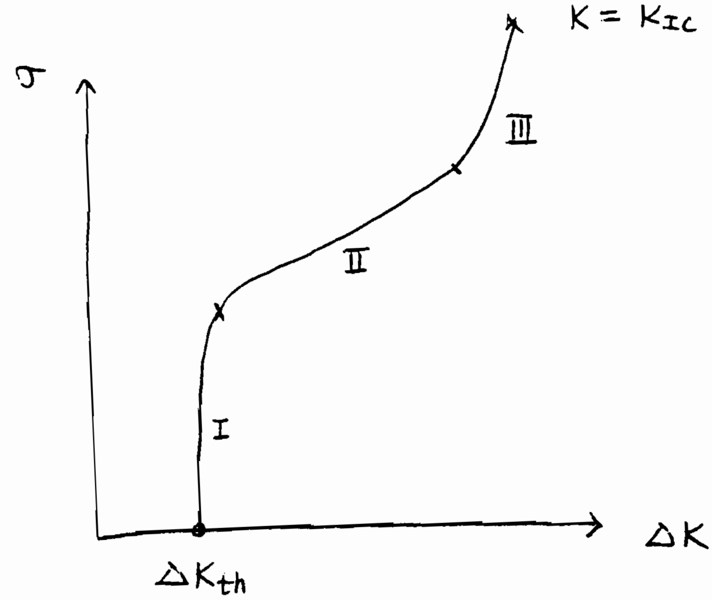
9 (d) continued

$$\Rightarrow \quad 2\times10^{-4}\,\sigma^2 + 3.9\sigma - 1400 = 0$$

$$\sigma = \frac{-3.9 \pm \sqrt{3.9^2 + 4\times2\times10^{-4}\times1400}}{2\times2\times10^{-4}} = \frac{-3.9 \pm 4.041}{4}\times10^4 \text{ MPa}$$

$$\Rightarrow \quad \sigma = 353 \text{ MPa}$$

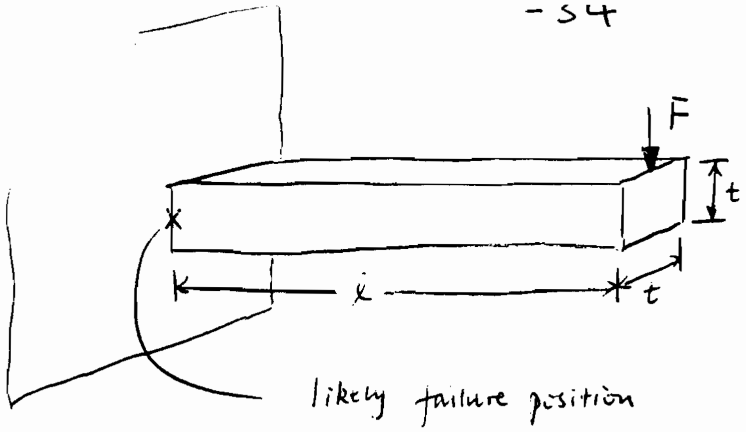
(e)



If $\Delta K < \Delta K_{th}$, no crack growth is detected.

When the crack has grown sufficiently long such that $K = K_{Ic}$, fast fracture occurs.

10.



(a) $M = F\ell$, $I = \frac{1}{12} t^4$

$$\sigma_{max} = \frac{y_{max} M}{I} = \frac{t}{2} \cdot \frac{F\ell}{t^4/12} = 6F\ell/t^3$$

Let $\sigma_{max} = \sigma_f \Rightarrow t = (6F\ell/\sigma_f)^{1/3}$

Mass $m = \rho \ell t^2 = \rho \ell \left(\frac{6F\ell}{\sigma_f}\right)^{2/3} = \ell (6F\ell)^{2/3} \frac{\rho}{\sigma_f^{2/3}}$

$\Rightarrow$ Maximise $\boxed{I_1 = \sigma_f^{2/3}/\rho}$ to minimise m.

(b)

	$I_1 = \sigma_f^{2/3}/\rho$	$I_2 = K_{Ic}^{2/3}/\rho$
Stainless steel	11.34	3.55
PMMA	19.	0.83
Alumina	11.50	0.65

PMMA is best according to ranking based on merit index I_1.

(c) $K_{Ic} = \sigma_{max}\sqrt{\pi c} = \frac{6F\ell}{t^3}\sqrt{\pi c}$

$\Rightarrow$ $c = \frac{1}{\pi}\left(\frac{K_{Ic}\, t^3}{6F\ell}\right)^2$ is the largest tolerable flaw size

Note that $c \sim t^6$, $c \sim \ell^{-2}$

$\Rightarrow$ c more sensitive to changes in t.

10. (d) $$K_{IC} = \sigma_{max}\sqrt{\pi c} = \frac{6Fl}{t^3}\sqrt{\pi c}$$

$$\Rightarrow \quad t^3 = \frac{6Fl\sqrt{\pi c}}{K_{IC}}, \qquad t = \left\{ \frac{6Fl\sqrt{\pi c}}{K_{IC}} \right\}^{1/3}$$

$$\Rightarrow \quad m = \rho l t^2 = l\,(6Fl\sqrt{\pi c})^{2/3} \cdot \frac{\rho}{K_{IC}^{2/3}}$$

$$\Rightarrow \quad \text{Material index } \boxed{I_2 = K_{IC}^{2/3}/\rho}$$

(e) The values of I_2 for 3 materials are included in (b). Stainless steel is now the best choice.

(f) To design against both strength and brittle fracture, stainless steel would be the best, as it has the most balanced properties. PMMA and Al_2O_3 are both good in terms of strength, but they are brittle and prone to fast fracture.

In practice, need also to worry about cost, attachment, durability etc.