Engineering Tripos Part IA 2002

Paper 3, Electrical and Information Engineering

Section A - Numerical Answers

1. (b) $R_o = 2304\Omega$; $X_o = 758\ \Omega$; $R_t = 1.6\ \Omega$; $X_t = 8.9\ \Omega$

(c) (ii) $\theta_{TOTAL} = 530$ VAR ; $C = 29.3\mu F$

2. (b) (ii) $V_{oc} = 25$ V
(iii) $V_{meas} = 22.2$ V
(iv) $R_{in} \geq 24.9$ kΩ

(c) 11.2 % error

3. (c) (i) $Z_{in} = \dfrac{R_1R_2}{R_1+R_2}$

(ii) $Z_{out} = \dfrac{R_sr_d}{R_s+r_d+g_mR_sr_d}$

(iii) $\dfrac{v_{out}}{v_{in}} = \dfrac{g_m}{g_m + \left(\dfrac{R_s+r_d}{R_sr_d}\right)}$

(d) [e.g.] $R_1 = R_2 = 2$ MΩ ; $R_S = 1978\ \Omega$

4. (c) gain $= \dfrac{-R_2}{R_1+jwL}$

(d) $L = 15.9$ mH

(e) $A = 1.99$; $B = -1.47$ [or equivalent]

RJM/vg

15/8/02

PART IA, PAPER 3, SECTION A CRIB, 2002

1/ (a) (i) Ideal? Cu loss ③

(ii) X_2? Leakage induction ②

Total ⑦ (iii) Magnetising X_0, R_0 ②

(b) ① $R_0 = V^2/P = 240^2/25 = 2304\,\Omega$

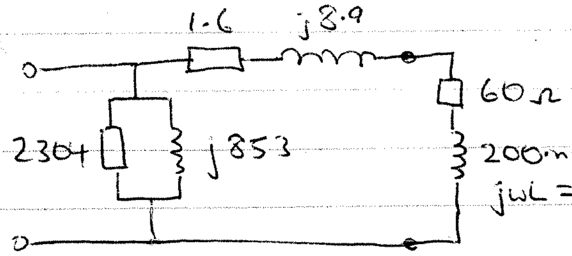
Total ⑥ ② $X_0 = V^2/Q = 240^2/\sqrt{(VI)^2 - P^2} = 240^2/\sqrt{72^2-25^2} = 853\,\Omega$

(N.B. At 50Hz assume) Turns ratio = 2 = N_1/N_2

① $R_L = P/I^2 = 1.6\,\Omega$

② $X_L = Q/I^2 = \sqrt{(VI)^2 - P^2}/I^2 = 8.86\,\Omega$

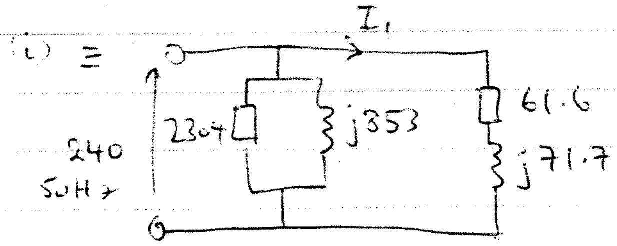
(c) (i)



③ ie. $(N_1/N_2)^2 R_L$

∴ Total L = j71.7

(ii)



$|I_1| = \frac{240}{\sqrt{61.6^2 + 71.7^2}}$

$= 2.539$

$\angle I_1 = -49.3°$

$\cos\theta = 0.652$

$P_{LOAD} = VI_1\cos\theta$

$= 397$ Watts

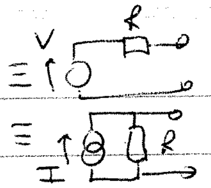
$Q_{LOAD} = 462$ Vars

② ANS $\boxed{Q_{TOTAL} = 529.5\text{ VAR}}$

② $\therefore V^2\omega C = Q_{TOT} \Rightarrow \boxed{C = 29.3\,\mu F}$

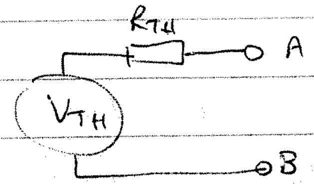
2/

(a) Thevenin $\equiv$
Norton $\equiv$



} ②

(b) (i) Th.



$V_{TH} = 25V$
$R_{TH} = 75 + 100//100$
$= 125\Omega$ } ④

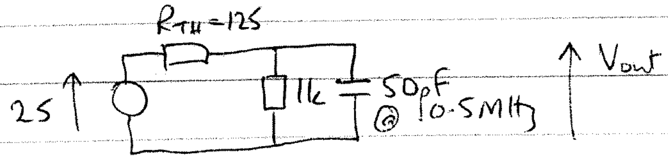
(ii) $V_{OC} = 25V$ ②

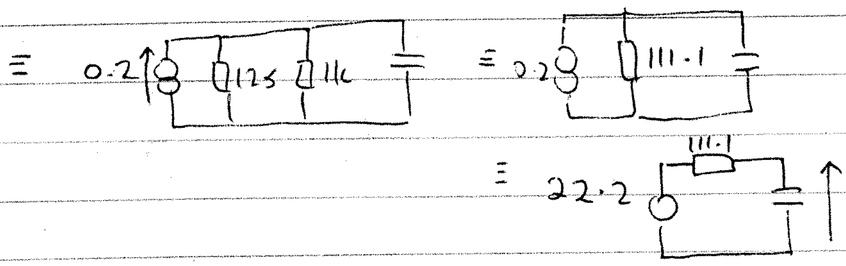
(iii) $V_{meas} = \left(\frac{1k}{1k+125}\right) \cdot 25 = 22.2V$ ②

(iv) $V_{meas} = 0.995 \times 25$ when $V_{meas} = \left(\frac{R_{in}}{R_{in}+125}\right)25$

$\therefore R_{in}(0.005) = 0.995 . 125 \Rightarrow R_{in} \geq 24.9k\Omega$ } ⑤

(c)





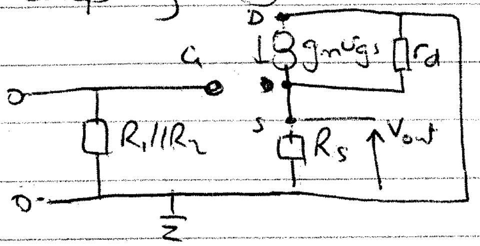
$$V_{out} = \frac{1/j\omega C}{111.1 + 1/j\omega C} \times 22.2$$

$$\frac{V_{out}}{25} = \frac{-j\,6366}{\sqrt{111.1^2 + 6366^2}} \cdot \frac{22.2}{25} = \boxed{11.2\%\ \text{error}}$$ ⑤

3/

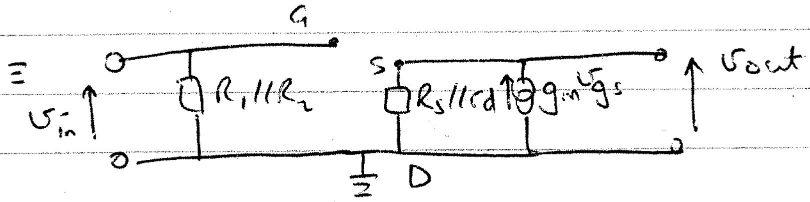
(a) decoupling ②

(b)



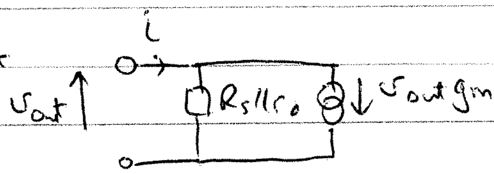
④

v_{in}/v_{out} label = ①
$R_1//R_2$ = ①
$R_s // r_d$
$g_m v_{gs}$



(c) (i) $Z_{in} = \dfrac{R_1 R_2}{R_1 + R_2}$ ①

(ii) Z_{out}



$Z_{out} = v_{out}/i$

$i = \dfrac{v_{out}}{R_s//r_d} + v_{out} g_m$

$= \left[\dfrac{R_s + r_d}{R_s r_d} + g_m\right] v_{out}$

② $\boxed{Z_{out} = \dfrac{R_s r_d}{R_s + r_d + g_m R_s r_d}}$

(iii) $v_{in} = v_{out} + v_{gs}$

$v_{out} = g_m v_{gs} (R_s // r_d) = g_m (v_{in} - v_{out})(R_s // r_d)$

$v_{out}\left[1 + g_m (R_s // r_d)\right] = g_m (R_s // r_d)\, v_{in}$

③ $\text{Gain} = \dfrac{v_{out}}{v_{in}} = \dfrac{g_m \, R_s r_d / R_s + r_d}{1 + g_m \, R_s r_d / R_s + r_d} = \boxed{\dfrac{g_m}{\left[\dfrac{R_s + r_d}{R_s r_d}\right] + g_m}}$

(d) $Z_{in} = 1M\Omega$ eg $\boxed{R_1 = 2M\Omega \;;\; R_2 = 2M\Omega}$ [NB ambiguity]

$Z_{out} = \dfrac{1}{1/R_{11} + g_m} = 180 \Rightarrow 1/R_{11} (180) = 1 - 180 g_m =$ ⑥

$R_{11} = 1800 = \dfrac{R_s r_d}{R_s + r_d}$

$R_s(20k - 1800) = 1800 \times 20k$ $\Rightarrow \boxed{R_s = 1978\ \Omega}$

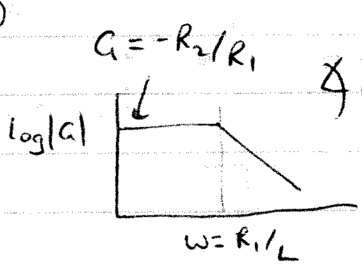
(e) Source follower : impedance converter ②

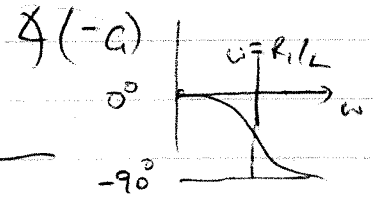
4/ (a) ideal $R_{in} \to \infty$ $R_{out} \to 0$ $A \to \infty$ $B \to \infty$ (3)

(b) $\boxed{\text{Gain} = \frac{-R_2}{R_1 + j\omega L}}$ (2)

$\omega \to 0; G \to -R_2/R_1$

$\omega \to \infty$





(3)

(c) At low freq $R_{in} = \boxed{R_1 = 1k\Omega}$ (2)

At low freq $|G| = R_2/R_1 = 10 \Rightarrow \boxed{R_2 = 10k\Omega}$ (2)

(d) $\omega_{3dB} = 2\pi 10^4 = R_1/L \Rightarrow L = \frac{10^3}{2\pi 10^4} = \boxed{15.9\,mH}$ (4)

(e) Linear cct ∴ $2\pi f t = 2\pi 10^5 t \Rightarrow f = 10^5$ Hz ~~(4)~~

$A = +2|G(10^5 Hz)|$

$G = \frac{10k}{1k + j10k} = 0.995$

∴ $\boxed{A = +1.99}$ (or −1.99 if keep −ve) (2)

$B = \angle G(10^5 Hz) = -1.47$ ($= -84.2°$) (or $-264.2°$) (2)

Engineering IA 2002

Paper 3, Electrical and Information Engineering

Numerical answers

5) (c) 26 FA,

(d) (i) The program will add three numbers. The result is 2A,

(d)(ii) 32.5 μs.

6) (b) NAND gate

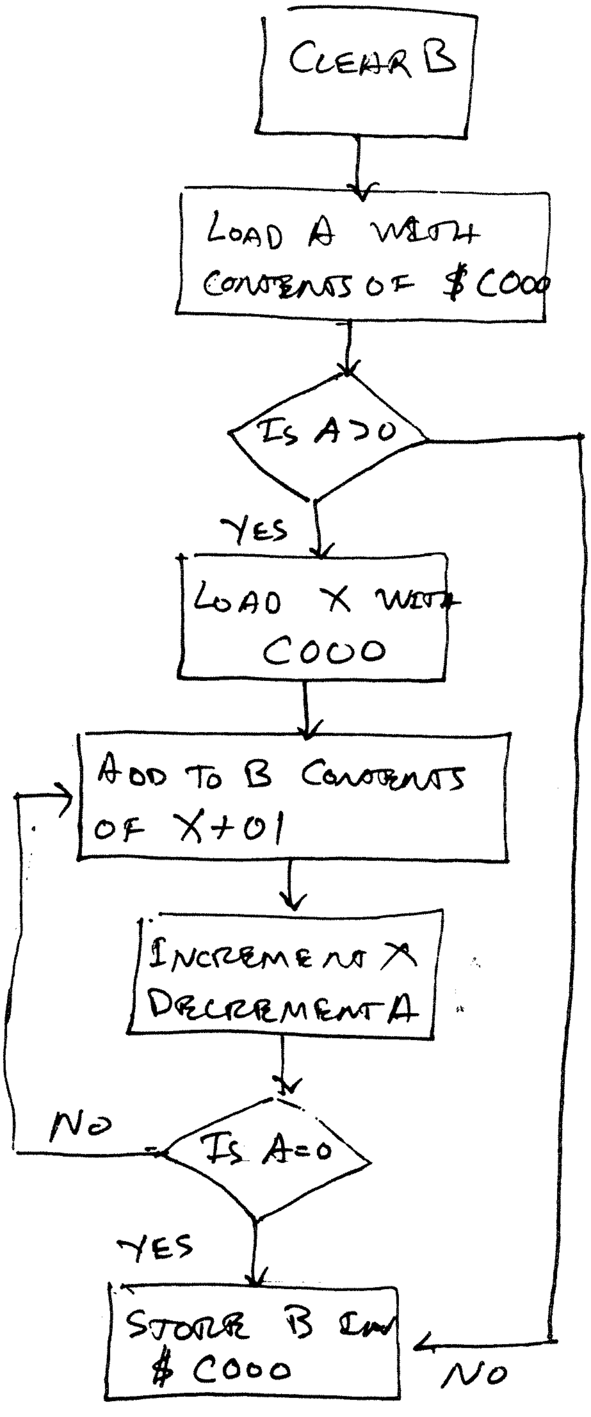
(c) (iv) $F = \overline{B} + \overline{C}\,\overline{D} + C\,D$, $I = C\,\overline{D} + \overline{B}\,\overline{D}$

7) (b) OR gate

8) (c) $J_A = BM$, $K_A = B + \overline{M}$, $J_B = \overline{A}\,P$, $K_B = A + M + \overline{P}$

QUESTION 5:

(a)



(b) ADD TO B THE CONTENTS OF MEMORY LOCATION AFTER THE MEMORY LOCATION STORED IN INDEX REGISTER X. USES INDEXED ADDRESSING

(c) BNE → 26. HAVE TO BRANCH BACK OVER ADDB (2 BYTES), INX (1 BYTE), DECA (1 BYTE), BNE (2 BYTES) TOTAL −6 BYTES, FA IN 2'S COMPLEMENT, ∴ $\boxed{26\ FA}$

(d)(i) THE PROGRAM WILL ADD 3 NUMBERS

$05 + 0A + 1B = \boxed{2A_H}$

(ii) CLOCK CYCLES

$2 + 4 + 2 + 4 + 3 + \underbrace{5 + 4 + 2 + 4}_{\times 3} + 5$

TOTAL 65 CYCLES $\times$ 0.5 μs/CYCLE = $\boxed{32.5\ \mu s}$

MOST STUDENTS WERE ABLE TO FIGURE OUT WHAT THE PROGRAM IS SUPPOSED TO DO & DRAW A ROUGH FLOW CHART TO REPRESENT ITS OPERATION. RELATIVELY FEW SUCCEEDED IN GETTING ALL THE DETAILS OF THE FLOW CHART RIGHT. THE MAJORITY OF STUDENTS MADE MISTAKES IN READING FIGURES FROM THE DATA BOOK (e.g. PART d(ii)) AND IN THE CALCULATIONS (d(ii) AND ESPECIALLY (c)).

QUESTION 6:

(a) RAM: RANDOM ACCESS MEMORY, CAN BE READ FROM/WRITTEN TO BY MICROPROCESSOR. VOLATILE, CONTENTS LOST IF POWER IS REMOVED.

ROM: READ ONLY MEMORY. CONTENTS CAN NOT BE CHANGED BY MICROPROCESSOR. PERMANENT, DATA REMAINS EVEN WITHOUT POWER.

EPROM: ERASABLE, PROGRAMABLE ROM. SIMILAR FUNCTION TO ROM. CAN BE ERASED USING SPECIAL HIGH VOLTAGE PROGRAMMER DEVICE (OR SOMETIMES UV LIGHT) AND REPROGRAMMED USING HIGH VOLTAGES.

HARD DRIVE: UNLIKE THE REST USES MAGNETIC DISK FOR LONG TERM STORAGE. PERMANENT. HIGH CAPACITY, LOW SPEED. READ/WRITE.

(b) $Y = \overline{AB + \overline{\overline{AB}+\overline{BC}}}$

$= \overline{AB + AB\cdot BC}$ (DE MORGAN)

$= \overline{AB}$ (ABSORPTION)

(c)(i) TRIVIAL

(ii) 5 → 1011011 7 → 1110000 (7 ALSO O.K.)

(iii)

F:

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	0	1	0
11	X	X	X	X
10	1	1	X	X

I:

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	1
11	X	X	X	X
10	1	0	X	X

$F = \overline{B} + \overline{C}\cdot\overline{D} + C\cdot D$

$I = C\cdot\overline{D} + \overline{B}\cdot\overline{D}$

OTHER COMBINATION WITH SAME No OF TERMS POSSIBLE.

MOST STUDENTS PRODUCED REASONABLE ANSWERS TO (a) AND (b). THE MANIPULATIONS OF THE FORMULAS IN (b) WAS UNNECESSARILY COMPLICATED IN SOME CASES, OFTEN LEADING TO MISTAKES & WASTING TIME. PART (c)(i) – (iii) WERE ALSO DONE WELL. MANY STUDENTS FAILED TO SPOT THE SIMPLEST FORMULAS FOR (c)(iv).

QUESTION 7:

(a) NMOS : COMPACT, SLOW, CHEAP, RELATIVELY HIGH POWER CONSUMPTION.

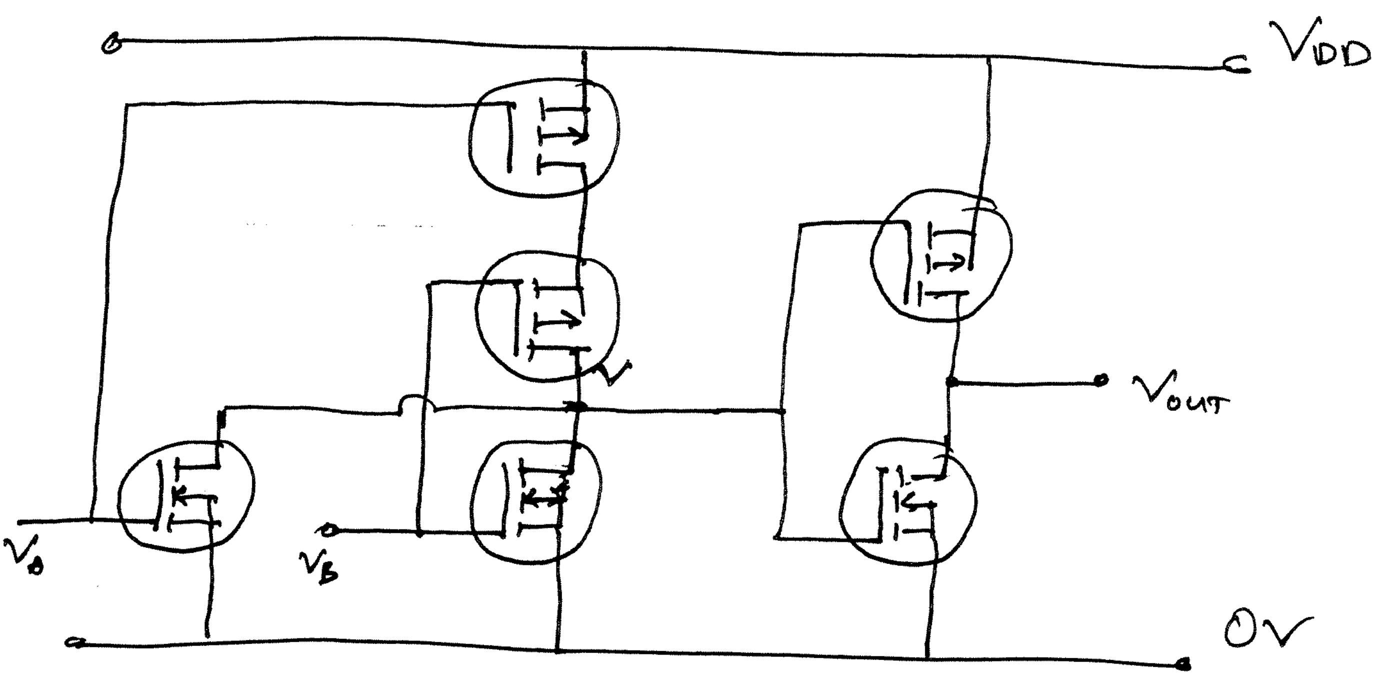
CMOS : MAX CLOCK FREQUENCY 12-40 MHz. LOW POWER CONSUMPTION (POWER CONSUMED ONLY DURING STATE TRANSITIONS)

TTL : FAST (35-200 MHz), HIGH POWER. BASED ON BIPOLAR TRANSISTORS

ECL : VERY HIGH SPEED, HIGH POWER. ALSO BIPOLAR.

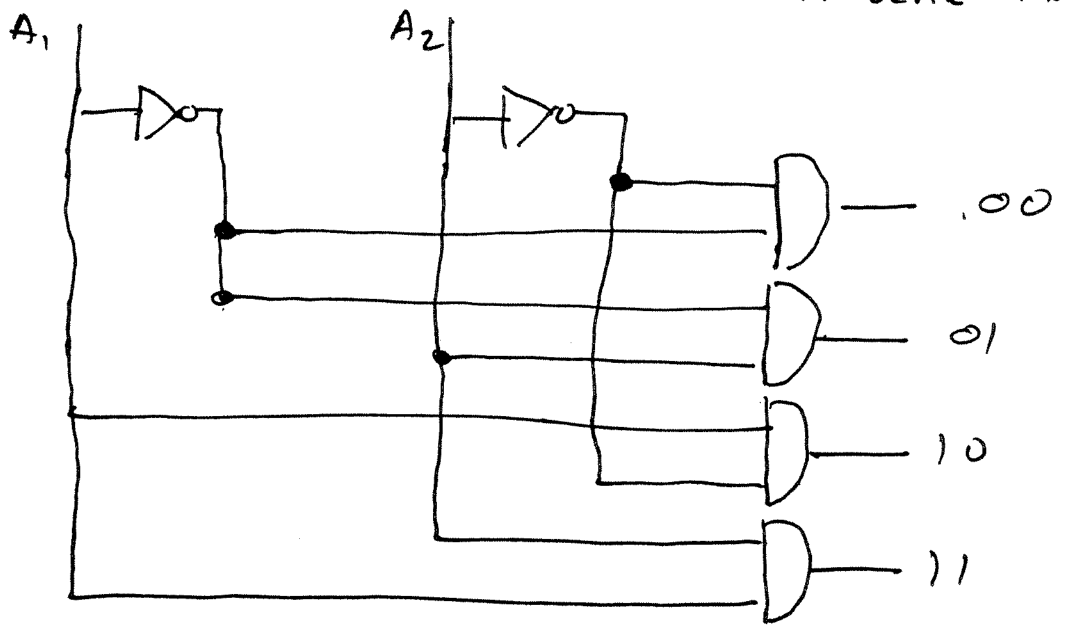
(b) THE CIRCUIT IS AN OR GATE. WHEN VA AND/OR VB ARE HIGH, T_1 AND/OR T_2 ARE ON, THEREFORE V IS LOW, THEREFORE T_3 IS OFF, AND V_{OUT} IS HIGH. WHEN VA AND VB ARE BOTH LOW, T_1 AND T_2 ARE OFF, HENCE V IS HIGH, T_3 IS ON AND V_{OUT} IS LOW.

CMOS CIRCUIT



(C) THE ONE OUT OF 2^n ADDRESS DECODER HAS n INPUT WIRES (CONNECTED TO THE ADDRESS BUS) AND 2^n OUTPUT WIRES (CONNECTED TO MEMORY ELEMENTS).
AN ADDRESS IS ENCODED AS A BINARY NUMBER IN THE n INPUT BITS. THE DECODER MAKES THE (ONE-OUT-OF-2^n) OUTPUT WIRE CORRESPONDING TO THE ADDRESS HIGH, AND THE REMAINING $2^n - 1$ OUTPUT WIRES LOW.

A 1-OUT-OF-2^2 DECODER MAY LOOK LIKE THIS



(d) A IS THE OUTPUT OF AN ADDRESS DECODER. IT IS HIGH WHENEVER WE ARE READING/WRITING TO THE MEMORY LOCATION CONTAINING THE ELEMENT, AND LOW OTHERWISE.

L IS THE DATA FROM THE INTERFACE LOGIC. IT MAY GET STORED IN THE ELEMENT IF A AND W ARE HIGH

W IS HIGH IF WE ARE WRITING TO THE MEMORY AND LOW OTHERWISE

H IS THE DATA STORED IN MEMORY ELEMENTS WITH HIGHER ADDRESSES.

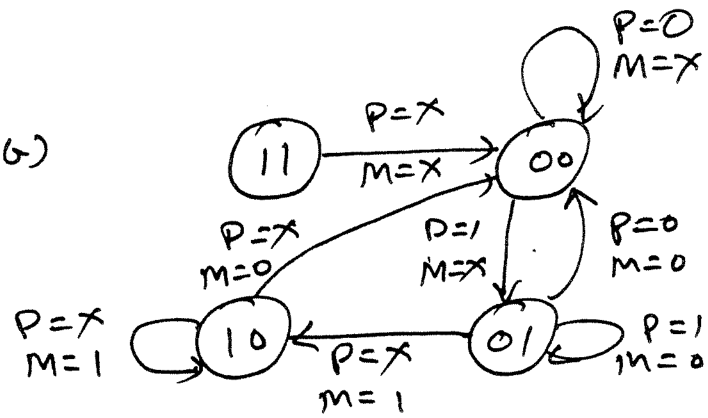
THE AVERAGE MARK WAS FAIRLY LOW FOR THIS QUESTION. MOST STUDENTS DID OK IN PARTS (a) AND (c). RELATIVELY FEW REMEMBERED THE ANSWER TO (d). VERY FEW WERE ABLE TO DETERMINE THAT THE CIRCUIT IN (b) IS AN "OR" GATE. VIRTUALLY NONE WAS ABLE TO INTRODUCE THE PMOS TRANSISTOR CORRECTLY.

QUESTION 8:

(a) 1 UNUSED STATE (11)

STATE	A	B
I	0	0
W	0	1
D	1	0
U	1	1

(b)



(c)

INPUT P	INPUT M	STATE A	STATE B	NEXT A	NEXT B	J_A	K_A	J_B	K_B
0	X	0	0	0	0	0	X	0	X
1	X	0	0	0	1	0	X	1	X
0	0	0	1	0	0	0	X	X	1
1	0	0	1	0	1	0	X	X	0
X	1	0	1	1	0	1	X	X	1
X	0	1	0	0	0	X	1	0	X
X	1	1	0	1	0	X	0	0	X
X	X	1	1	0	0	X	1	X	1

J_A

PM \ AB	00	01	11	10
00	0	0	X	X
01	0	1	X	X
11	0	1	X	X
10	0	0	X	X

K_A

PM \ AB	00	01	11	10
00	X	X	1	1
01	X	X	1	0
11	X	X	1	0
10	X	X	1	1

$J_A = B \cdot M$

$K_A = B + \overline{M}$

PM \ AB	00	01	11	10
00	0	X	X	0
01	0	X	X	0
11	1	X	X	0
10	1	X	X	0

J_B

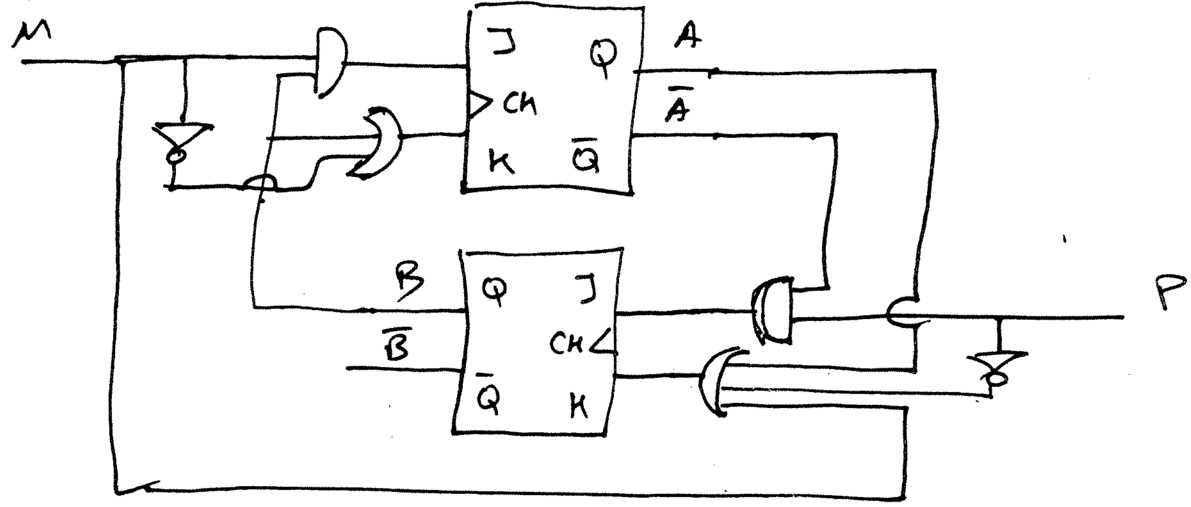
$J_B = \bar{A} \cdot P$

K_B

PM \ AB	00	01	11	10
00	X	1	1	X
01	X	1	1	X
11	X	1	1	X
10	X	0	1	X

$K_B = A + M + \bar{P}$

(d)



PARTS (a) AND (b) WERE MORE OR LESS TRIVIAL. MANY PEOPLE WASTED EASY MARKS BY NEGLECTING TO DRAW STATE DIAGRAMS, ALLOCATION TABLES, ETC. MOST STUDENTS DID OK IN PART (C). SOME DID NOT MAKE USE OF "WILD CARDS" LEADING TO SMALL MISTAKES AND/OR UNNECESSARILY COMPLICATED CIRCUITS IN PART (d).

Part 1A Electrical and Information Engineering Examination - Section C
June 2002

Numerical answers

9. a) $\underline{C = \frac{A\varepsilon_0\varepsilon_r}{x}}$

b) $F = \frac{1}{2}V^2\frac{dC}{dx}$

c) 0.22mN (no edge or fringing effects)

d) 334 kV

10

b) 1.33 Tesla

c) 3.54 A

d) 140 N

11

c) $\frac{\sqrt{2}\mu_0 I}{\pi}\frac{n}{b-a}\ln\left(\frac{b}{a}\right)$ --- assuming n is large

Exam crib

9. a)

Gauss's Law:

$\oint_S \underline{D}.\underline{dS} = Q$ --- The total electric flux emerging from any surface is equal to the charge enclosed by that surface. Colloquially electric flux lines begin and end on charges.

From Gauss's law we get (assuming that there are no fringing fields and consequently that the electric field lines pass perpendicularly between the plates and that D can therefore be treated as a scalar quanitity)

$D * A = Q$ where A is the area of the plates

But

$D = \varepsilon_0 \varepsilon_r E$ $\quad \rightarrow E = \dfrac{q}{A\varepsilon_0\varepsilon_r}$

$V = \int E dx$

$= \dfrac{qx}{A\varepsilon_0\varepsilon_r}$ Where x is the separation of the plates

Since $q = CV$ $\quad \rightarrow \underline{C = \dfrac{A\varepsilon_0\varepsilon_r}{x}}$

b) NB: the following analysis assumes constant charge q

from work

$-F\delta x = \frac{1}{2} q \delta V$

Hence in the limit:

$F = -\frac{1}{2} q \frac{dV}{dx}$

Also: $V = q/C$ $\quad \rightarrow \dfrac{dV}{dx} = -\dfrac{q}{C^2}\dfrac{dC}{dx}$

And therefore $F = \frac{1}{2} V^2 \frac{dC}{dx}$

c) Using the expression for capacitance derived in part a) we obtain:

$$\frac{dC}{dx} = -\frac{A\varepsilon_0\varepsilon_r}{x^2}$$

➔ Force $= \dfrac{(\frac{1}{2}*100*0.1*5*8.854*10^{-12})}{10^{-6}}$

$= 0.22\text{mN}$

As stated above there are no edge or fringing effects

d) Starting from Young's Modulus

E = Stress / Strain = Force/Area / (1/200)

We get

$$F = \frac{EA}{200}$$

We also know that :

$$F = -\frac{1}{2}V^2 \frac{A\varepsilon_0\varepsilon_r}{x^2}$$

➔ $V^2 = \dfrac{2E}{200\varepsilon_0\varepsilon_r}x^2 = (100*10^9*0.995*10^{-6}*2/200*1000*8.854*10^{-12})^{1/2}$

= 334 kV --- This figure is actually rather high and in practice would probably not be achieved due to breakdown of the dielectric

Note:

This question is a fairly straightforward application of Gauss's law which was very well answered by most students despite the inclusion of Young's modulus in the final part ! Some students chose to derive the expression for force by assuming constant voltage rather than constant charge.

10

a) $\oint_c \underline{H}.\underline{dl} = \int_s (\underline{J} + \dot{\underline{D}})\underline{ds}$: For the purposes of first year work we ignore the term in $\dot{\underline{D}}$ (Maxwell's displacement current) and then the interpretation of Ampere's law is that if you draw any closed loop and integrate H around that loop the answer you arrive at will equal the total current (current density J * area) passing through that loop. It is especially useful in tasks such as tranformer design where the right hand side is usually the total current expressed in terms of ampere turns i.e. the current through the coil times the number of loops in the coil.

b) For the magnetic circuit we can write down the following, since there is initially no current in the coil.

$$H_{iron}d_{iron} + H_{magnet}d_{magnet} = 0$$

We treat the iron as a linear material and so can say:

$B_{iron} = \mu_0 \mu_r H_{iron}$

In addition we can apply conservation of flux and observing that the cross-sectional area is constant around the circuit we end up with:

$B_{iron} = B_{magnet}$

Substituting in:

$$\frac{B_{magnet}}{\mu_0 \mu_r} d_{iron} + H_{magnet}d_{magnet} = 0$$

Hence

$$B_{magnet} = -H_{magnet}\mu_0\mu_r\left(\frac{d_{magnet}}{d_{iron}}\right)$$

$$= -4\pi * 10^{-7} * 1000 * \frac{30}{370} H_{magnet}$$

$$= -10.18 * 10^{-5} H_{magnet}$$

From the graph for Columax in the Data Book we obtain

$B_{magnet} = 1.33$ Tesla

c) For this part we add in the effect of an electric current in the coil so Ampere's law reduces to:

$$H_{iron}d_{iron} + H_{magnet}d_{magnet} = nI$$

In this case B in the iron and the magnet is zero as the question states. We are treating Iron as a linear material so H in the iron is also zero so the above reduces to:

$$H_{magnet}d_{magnet} = nI$$

From the graph H for columax when B is equal to nought is -5.9×10^4 A/m

Hence

$I = 5.9 \times 10^4 \times 30 \times 10^{-3}\ / 500$

$I = 3.54$ A

d) The force developed at the jaws of the clamp is given by:

$$F = \frac{B^2 A}{2\mu_0} = \frac{1.33^2 * 100 * 10^{-6}}{2 * 4\pi 10^{-7}}$$
$$= 70N$$

However the load is applied at the midpoint of the hinge and the clamps jaws so the force developed is actually twice this

Force = 140 N

NOTES: This question was generally answered well although part d calculating the force developed was often missed out. This could be because the students assumed it was harder than it actually was. A common mistake with part c was to use the value for H in the magnet which was calculated in part b.

11

a) Biot-Savart Law (in scalar notation)

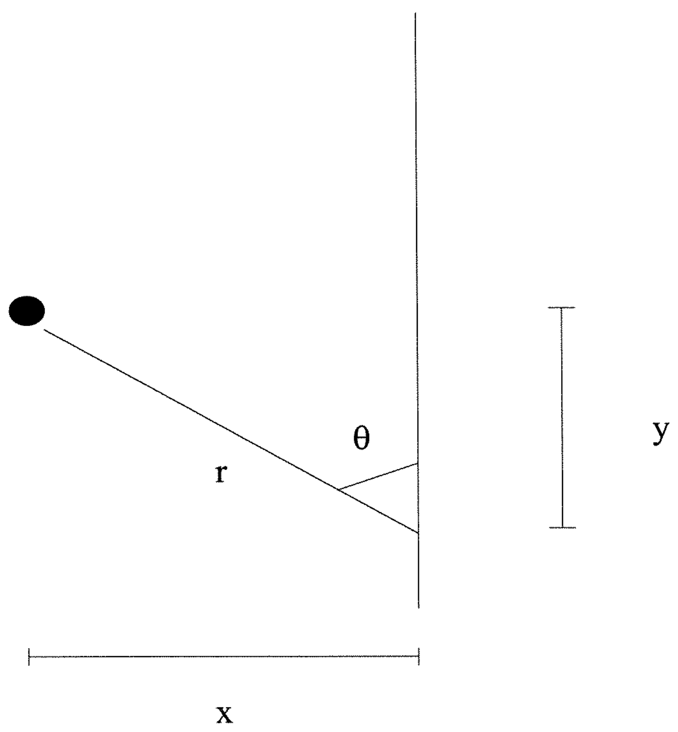
$dH = \dfrac{Idl\sin\theta}{4\pi r^2}$ The Biot-Savart law enables you to calculate the magnetic field at any point by giving the contribution of each current element I dl. Integrating the expression over all current elements gives the total field at a particular point in space. In the equation r refers to the distance between the element and the selected point and theta is the angle between the current element and the line joining it to the selected point.

In free space we can rewrite the expression using B instead of H

$$dB = \frac{\mu_0 Idl\sin\theta}{4\pi r^2}$$

Note the expression is given in scalar notation as this is sufficient for first year work but B and for that matter H are both vectors and it is worth noting that the direction of B (or H) is perpendicular to the plane containing the current element dl and the selected point.

b)



$$B = \int_{-L/2}^{L/2} \frac{\mu_0 I}{4\pi r^2}\frac{r\sin\theta}{r}dl \quad \text{and} \quad y = l; x = r\sin\theta; r^2 = \left(x^2 + y^2\right)$$

$$\Rightarrow \quad B = \int_{-L/2}^{L/2} \frac{\mu_0 I x}{4\pi(x^2+y^2)^{3/2}} dy = \frac{\mu_0 L I}{2\pi x(4x^2+L^2)^{1/2}}$$

b) B at the centre of a square coil with a single loop is therefore given by:

$$B = \frac{2\mu_0 L I}{\pi x(4x^2+L^2)^{1/2}}$$

Where x is the 'radius' of the coil, and, importantly, the side length of the coil is 2x

So the total B is given by

$$B = \sum^{n} \frac{2\mu_0 L_N I}{\pi x_N (4x_N^{\ 2} + L_N^{\ 2})^{1/2}} \quad and\ L_N = 2x_N$$

$$\Longrightarrow B = \sum^{n} \frac{\sqrt{2}\mu_0 I}{\pi x_N}$$

Where

$$x_N = \frac{a}{2} + (N-1)\frac{\left(\frac{b}{2} - \frac{a}{2}\right)}{n};$$

Assume n is large and then integrate with respect to N we get:

We get

$$\frac{\sqrt{2}\mu_0 I}{\pi} \int_1^n \frac{1}{a + \frac{N-1}{n}(b-a)} dN = \frac{\sqrt{2}\mu_0 I}{\pi} \frac{n}{b-a} \ln\left(\frac{b}{a}\right)$$

Notes:

This question was attempted by very few students very few of whom managed to solve it. It is more a test of maths than engineering as the Biot-Savart law is quoted in the data book.