ENGINEERING TRIPOS   PART IA

Monday 10 June 2002   9 to 12

Paper 1

MECHANICAL ENGINEERING

Answer not more than ***eight*** *questions, of which not more than* ***four*** *may be taken from section A and not more than* ***four*** *from section B.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

*Answer not more than **four** questions from this section.*

1 A tank is separated into two sections which are connected through a gap at the bottom of a vertical partition wall. It contains regions of water of density $\rho_w = 10^3$ kg m^{-3}, oil of density $\rho_o = 700$ kg m^{-3} and air as shown in Fig. 1. The dimension of the tank in the direction perpendicular to the view of Fig. 1 is 1 m. The width of the left part is $L_1 = 0.5$ m while the width of the right part is $L_2 = 1$ m. The same volumes of oil and water are stored, each being 0.5 m^3. The air on both sides of the tank is at atmospheric pressure $p_{atm} = 1$ bar.

(a) For both sides of the tank, sketch the profiles of the pressure distribution from the bottom to the top of the tank. [5]

(b) Calculate the level z_1 of water on the left side of the tank, the level z_2 of oil on the right side and the level z_3 of the water-oil interface, all measured from the bottom of the tank. [Hint: first express the volumes of oil and water in terms of z_1, z_2 and z_3.] [9]

(c) Calculate the net force due to fluid pressure acting on the vertical partition wall. [6]

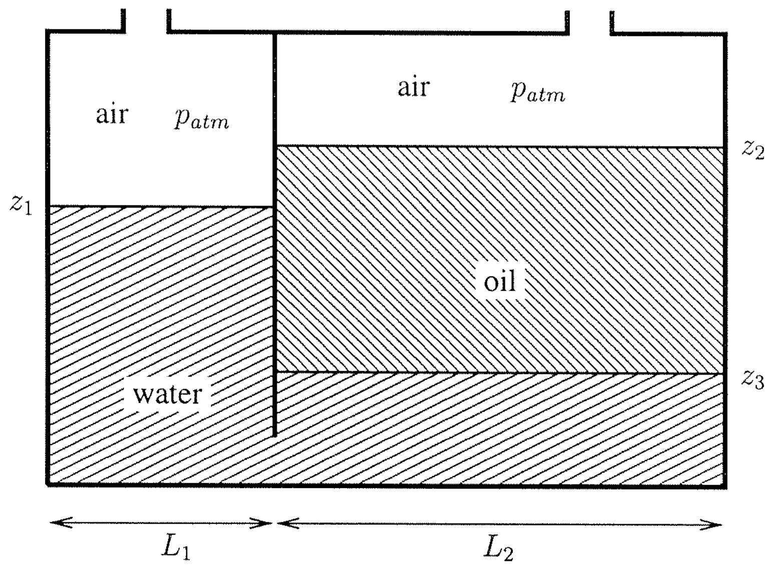

Fig. 1

2 Figure 2 shows an underwater vertical chimney of area $A = 2\times10^{-3}$ m^2 fitted with a heating element at its base providing heat at a rate $\dot{Q}$ = 100 kW. The top of the chimney is flush with the water surface and the heating element is H = 2 m below. The surface friction and heat transfer through the chimney wall are negligible. Water density and temperature are ρ_0 = 1000 kg m^{-3} and T_0 = 10 °C in the reservoir.

(a) Of what shape is the pressure distribution across the jet at the exit of the chimney? [4]

(b) By considering the water entering the chimney, find an expression for the pressure p_1 just before the heating element in terms of the velocity V_1 at the same position. [5]

(c) The specific heat capacity of water is c_p = 4180 J kg^{-1}K^{-1}. Obtain a relationship between the water temperature T after the heating element and the mass flow rate $\dot{m}$. [4]

(d) The density of water in the chimney is related to temperature through the equation $\rho = \rho_0 - 0.4\,(T - T_0)$ where ρ and T are expressed in kg m^{-3} and °C. Obtain another expression for p_1 (which is the same on both sides of the heating element) using Bernoulli in the chimney. By comparing this with (b), find the value of $\dot{m}$. [7]

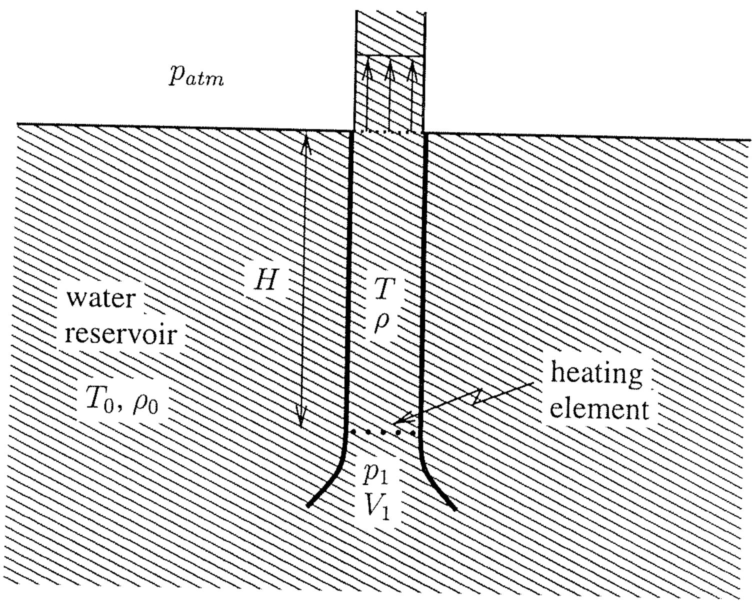

Fig. 2

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3 As a model of a spark ignition engine, a cycle is represented on the p-V diagram shown in Fig. 3. The compression and expansion phases are modelled by the polytropic equation pV^n = constant where n = 1.3 for compression and n = 1.5 for expansion. The maximum volume of a cylinder is $V_{max} = 0.4 \times 10^{-3}$ m^3 and the volumetric compression ratio is $r_V = 8$. Air is admitted at a temperature T_1 = 293 K and atmospheric pressure. Fresh air and combustion products should be treated as a perfect gas with the same properties as air. Combustion provides 600 J per cycle for one cylinder.

(a) Show that the work done by the gas during compression is

$$W_c = \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

and calculate its numerical value. [7]

(b) Calculate the temperature T_2 at the end of the compression. [4]

(c) During the short combustion phase, heat transfer through the cylinder is negligible. Determine the temperature T_3 at the beginning of the expansion. [5]

(d) Determine the work done by the gas during expansion, the net work output and the thermal efficiency of the cycle. [4]

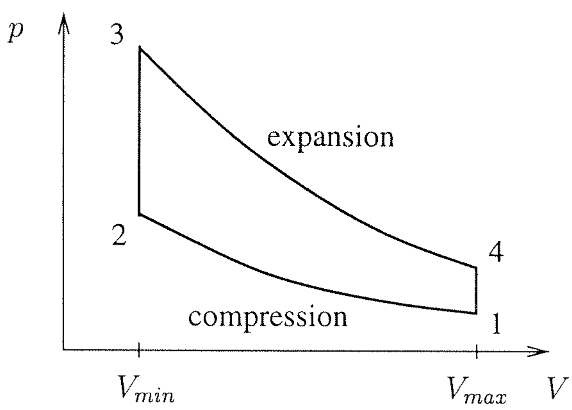

Fig. 3

4 The cross-section of a square electrical connector is shown in Fig. 4. The central element, of side length s = 10 mm, is carrying an electric current generating heat at a steady rate $\dot{q}$ = 1 kW per metre length. The outer element, of side length S = 40 mm and of thickness e = 1 mm, is made of ceramic of thermal conductivity $\lambda = 0.5$ W m^{-1}K^{-1}. There is a vacuum between the two elements. All bodies should be treated as black bodies. The Stefan-Boltzmann constant of radiation is $\sigma = 5.67 \times 10^{-8}$ W m^{-2}K^{-4}. The convection heat transfer coefficient to the atmosphere of temperature T_0 = 20 °C is h = 15 W m^{-2}K^{-1}.

(a) Indicate the mechanisms of heat transfer from the inner element to the atmosphere. [3]

(b) Explain why the view factor from the outer surface of the outer element to the atmosphere is unity. What is the view factor from the central element to the inner surface of the outer element? [4]

(c) By finding an expression for the heat transfer towards the atmosphere, show that the temperature of the outer surface of the outer element is close to 500 K. [5]

(d) Determine the temperature of the inner surface of the outer element. [3]

(e) Calculate the temperature of the surface of the inner element. [5]

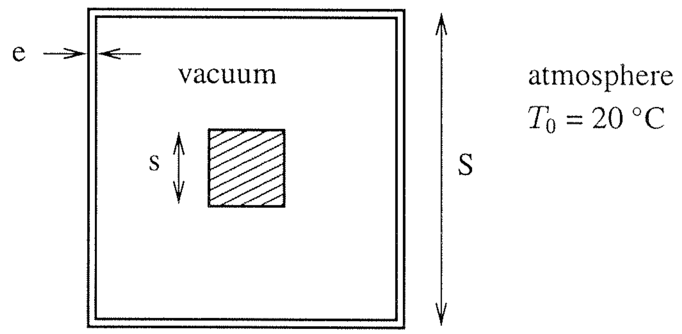

Fig. 4

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5 Air in a large pipe of radius R = 0.3 m is pumped by injection of air through a smaller pipe of radius r = 0.1 m as shown in Fig. 5. At the section 1, the absolute pressure is p_1 = 0.2 bar and the temperatures of both streams are T_1 = 300 K. The velocity of the injected air is V_{1a} = 1417 $\mathrm{m\,s^{-1}}$ and the main stream velocity is V_{1b} = 600 $\mathrm{m\,s^{-1}}$. Downstream, in section 2 where the streams are fully mixed, the pressure is p_2 = 1 bar. Between section 1 and section 2, there may be heat input.

(a) Find the total mass flow rate in section 1 and the contributions from the main pipe and the injection pipe. [4]

(b) Using the S.F.M.E. and (a), find the velocity V_2 in section 2. Hence find the density ρ_2 and the temperature T_2. [8]

(c) Apply the S.F.E.E. to calculate the rate of heat transferred to the air in the pipe between section 1 and section 2. Comment on the result. [8]

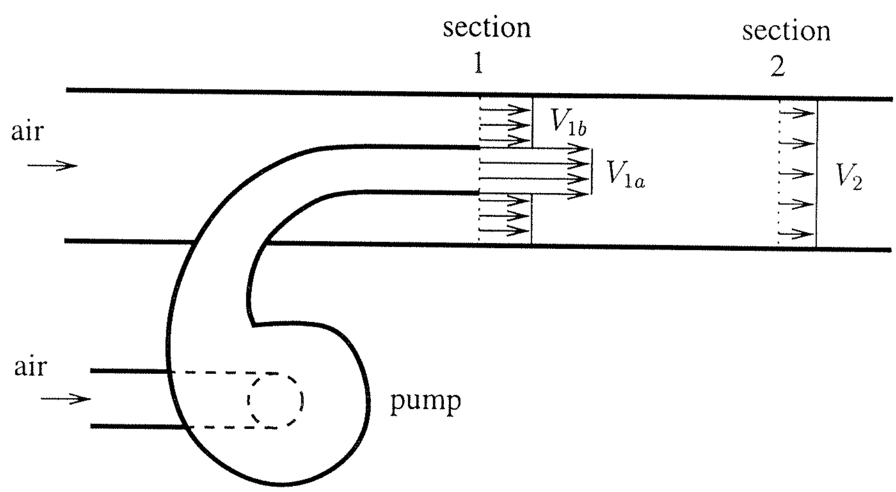

Fig. 5

SECTION B

*Answer not more than **four** questions from this section.*

6 (a) Under what circumstance is moment of momentum about an axis conserved? [2]

(b) Figure 6 shows a frictionless horizontal plane upon which a particle A of mass m slides. This mass is connected to a number of discs at B, of total mass $8m$, via a light inextensible string which passes through a small frictionless hole. The discs hanging down at B move in a vertical line at all times.

(i) Initially particle A travels in a steady circular path of radius R. Show that the speed V of the particle equals $\sqrt{8gR}$. [2]

(ii) A number of the discs, of total mass $7m$, are quickly removed from B, without disturbing the remaining mass m which still hangs down from the string. Show that the maximum radius from the hole of the subsequent path of A (the 'apogee') is equal to $R(2 + 2\sqrt{2})$. [10]

(iii) What further mass must be removed from B when A is at the apogee to put A into a circular orbit at this radius? [3]

(iv) By how much has the mechanical energy of particle A changed between the initial circular orbit of radius R and the final circular orbit established in part (iii)? Where has this energy come from or gone to? [3]

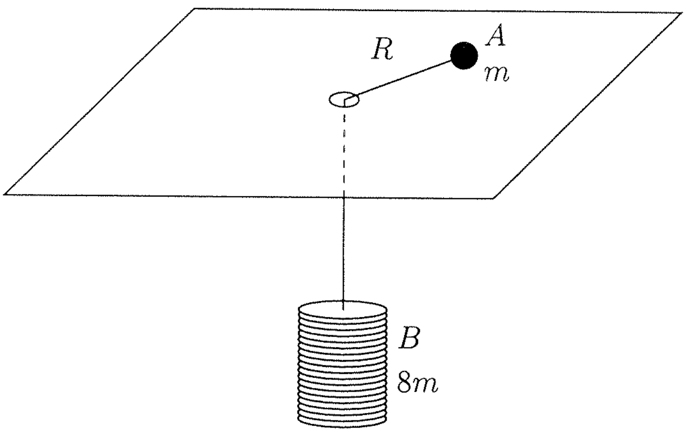

Fig. 6

7 Figure 7 shows schematically a planar representation of a deck chair that is collapsing under its self weight. Point A is pivoted to the ground while point B, which is at the same height as A, is free to slide horizontally. Bars AE and BD are connected via a freely rotating joint at C. Bar FG is attached to BD via a frictionless hinge at F, while point G slides down bar AE. Member lengths are as shown on Fig. 7. A separate version of Fig. 7, drawn to scale, is supplied for your working and should be handed in with your answer.

(a) At the position shown in Fig. 7, point B is moving at a speed of 10 mm s^{-1}. Draw a velocity diagram for the mechanism in the position shown. Use a scale of 50 mm to represent a velocity of 10 mm s^{-1}. [9]

(b) Find the angular velocities of AE and BD, the vertical component of the velocity of F and the sliding speed between bars GF and AE at G. [6]

(c) Bars AE and BD each have mass m, assumed to be concentrated at C and F, respectively, while bar FG has mass $m/2$ assumed to be concentrated midway along FG. Find the magnitude of the frictional force acting on the member FG at G, in the direction along AE, which is needed to stop the deck chair collapsing. Ignore inertial effects and the weight of the joints. [5]

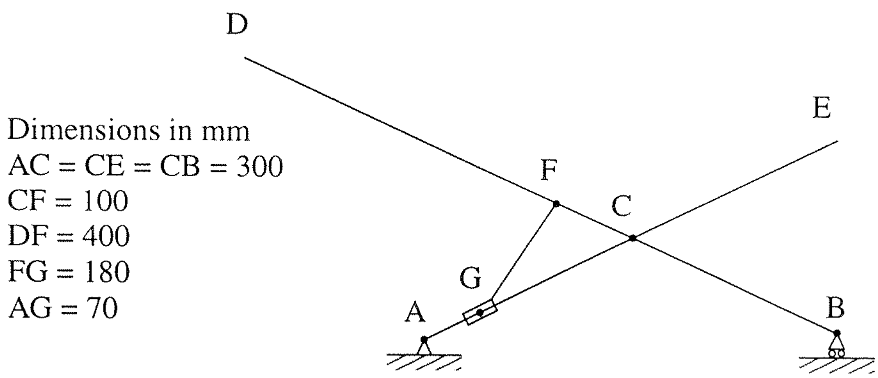

Fig. 7

(Not to scale)

(A separate version of this figure, which is to scale, is supplied for your working.)

8 Figure 8 shows schematically a coating process, in which an object of mass m is impacted by a jet of material of density ρ and cross-sectional area A, which travels at a constant absolute velocity of u. All the material impacting the object sticks to it. The object, which is initially held at rest, is released at time $t = 0$, when it is free to move along the path of the jet at a time dependent speed v under the action of the force from the jet. The mass of the object when it is released is m_0. There are no other forces acting on the object and gravity may be neglected.

(a) Write down an expression for the force on the object when it is at rest in terms of ρ, A and u. [3]

(b) Show that the rate of increase of mass of the object is given by

$$\frac{dm}{dt} = \rho A(u - v)$$ [2]

(c) Derive a differential equation governing the variation of speed v with time t for the object after it is released and show that this can be expressed in the form

$$-m\frac{d^2m}{dt^2} = \left(\frac{dm}{dt}\right)^2$$ [10]

(d) Hence show that the variation of mass with time has the form $m^2 = \alpha t + \beta$, and find the constants α and β in terms of m_0, ρ, A and u. [5]

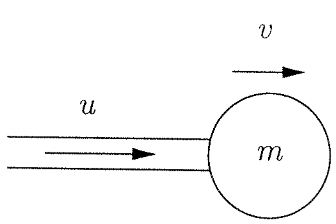

Fig. 8

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9 (a) A linear system with input x and output y is governed by the differential equation

$$T\dot{y} + y = x$$

where T is a constant.

(i) Show that, if the response of the system to input $x_1(t)$ is $y_1(t)$, then the response of the system to $\dot{x}_1(t)$ is $\dot{y}_1(t)$. [2]

(ii) Show that the step response of a linear system is the integral of the impulse response. [2]

(b) Figure 9 illustrates a mass m at position y connected to a fixed wall by a viscous dashpot of rate λ and acted on by a force $f(t)$.

(i) The mass, which is initially at rest, is struck with an impulsive force Q at time $t = 0$. Show that the velocity of the mass immediately after the impulse is Q/m. [2]

(ii) Derive an expression for the subsequent position of the mass, in terms of m, λ, Q and t. Sketch the response. [6]

(iii) The mass, initially at rest for $t < 0$, has a force applied to it which has the following variation with time,

$$f(t) = 0 \quad \text{for} \quad t < 0$$
$$f(t) = R \quad \text{for} \quad t \geq 0$$

Derive an expression for the variation with time of the position of the mass and sketch the response. [8]

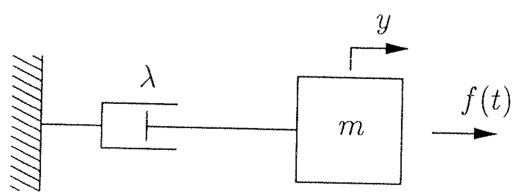

Fig. 9

10 Figure 10 illustrates a schematic model of the mounting to a fixed rigid base of a microscope stage of mass m, consisting of a spring of stiffness k in parallel with a viscous damper of rate λ. The microscope stage suffers a step input in force as a small sample of mass Δm is placed on the stage.

(a) Derive a differential equation of motion for the microscope stage, giving expressions for the undamped natural frequency and the damping factor ζ, in terms of m, k and λ. [5]

(b) For the case m = 15 kg, k = 4000 N m^{-1}, Δm = 0.02 kg, find the values of λ which will ensure that the displacement from the final equilibrium position does not exceed 5 μm at any time later than 0.3 seconds after the sample has been placed on the stage. Sketch typical responses under these conditions. [8]

(c) In practice a low-damping mounting is supplied with the stage to reduce problems associated with harmonic inputs. Repeat the question in part (b), but for the case where, 60 seconds after the sample is placed on the stage, the maximum displacement must have fallen to 5 μm. [5]

(d) What practical steps could be taken to reduce the time taken for the vibration to die away? [2]

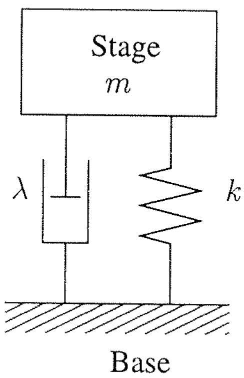

Fig. 10

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ENGINEERING TRIPOS PART IA

Paper 1, Mechanical Engineering

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Loose-leaf copy of Figure 7, drawn to scale.