ENGINEERING TRIPOS PART IA

Monday 10 June 2002 1.30 to 4.30

Paper 2

STRUCTURES AND MATERIALS

Answer not more than **eight** *questions, of which not more than* **four** *may be taken from Section A, and not more than* **four** *from Section B.*

All questions carry the same number of marks.

The **approximate** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

*Answer not more than **four** questions from this section.*

1 A crane (as shown diagrammatically in Fig. 1) has a frame CBD, rigidly jointed at B. The only connection to the supporting structure is immediately below B. The jib AB is pinned to the frame at B, and is supported by a cable ACD that passes over a roller at C. The load of 500 kN is supported by a cable that passes over pulleys at A and B to a winch at D.

(a) Draw a polygon of forces for the joint at A including all the cable forces and any other forces that are relevant. [4]

(b) Draw the bending moment diagram for the frame CBD. What reactions must the supporting structure supply at B? [8]

(c) If the jib AB is to be made from steel with a yield stress of 250 N/mm^2, and there is to be a factor of 2 reserve of strength, calculate the required cross-sectional area of the jib. [4]

(d) With the same reserve of strength, calculate the required second moment of area of the jib to prevent elastic buckling. You may assume that the jib is prevented from moving out of the plane of the figure. [4]

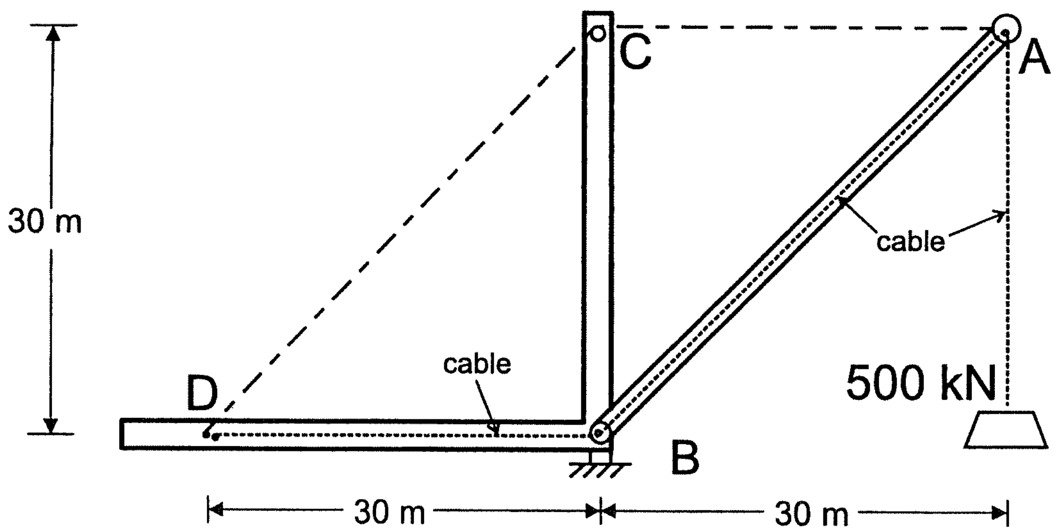

Fig. 1

2 A medieval masonry tower is leaning because of ground settlement (Fig. 2). The tower is 25 m high, with a square section of 3 m on each side. The wall thickness is 0.5 m and is constant through the full height of the tower. The masonry has a weight of 25 kN/m^3. The masonry cannot resist tensile stress but has very a high strength in compression.

(a) Calculate the total weight of the tower. [3]

(b) If the angle of inclination (ϕ) of the tower is 2 degrees, determine the stress variation across the base section A-B. [6]

(c) Find the angle of inclination when the stress at A reduces to zero. [2]

(d) How does the stress distribution alter if the angle of inclination increases above the value calculated in (c)? Illustrate your answer by determining the angle of inclination that gives zero stress at the mid-point of A-B. [7]

(e) At what angle would the tower fall over? [2]

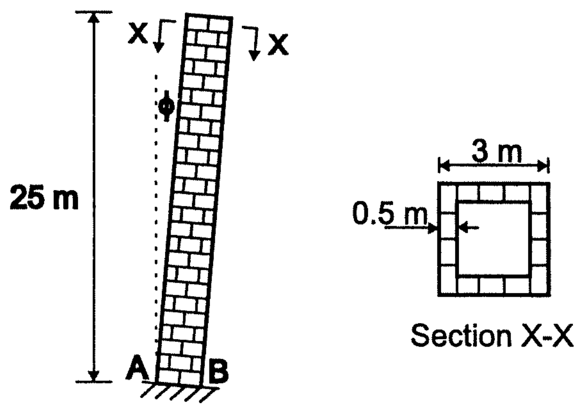

Fig. 2

3 A cable, of weight w per metre of horizontal length, is attached to two supports at equal height, a distance L apart. Under its own weight the cable sags by δ at its mid point.

(a) What is the horizontal component, H, of the reaction at each support? [4]

(b) Use Dimensional Analysis to find appropriate non-dimensional groups linking the sag ratio under self weight, δ / L, the diameter of the cable, d, its density, ρ, its Young's modulus, E, and other relevant quantities. [4]

(c) Cables are generally analysed on the assumption that they are inextensible, but this may not always be the case. If the tension in the cable is everywhere assumed to be the same as the horizontal support reaction, H , calculated in (a), find an expression for the strain of the cable, ε, in terms of your non-dimensional groups. [5]

(d) A designer has the choice of using either steel wire or aramid fibres (ρ = 1440 kg/m^3; E = 126 kN/mm^2), for a particular application. If the cable has to have the same sag in both cases, which will stretch more? [4]

(e) Comment briefly on other assumptions that are made in the analysis of cables. [3]

4 A non-equilateral truss PQRST carries a load W at S, as shown in Fig. 3. Members PS, ST and TR are made from a material with Young's modulus $4E$, and cross-sectional area $A/2$. All other members have area A and Young's modulus E.

Use Virtual Work to calculate the vertical deflection at S. [20]

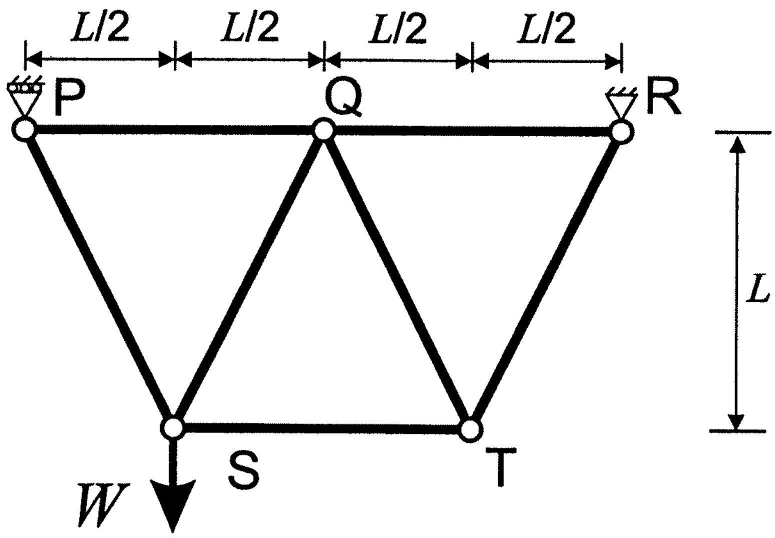

Fig. 3

5 An aluminium alloy I-beam 100×50 (Structures Data book, page 20) is simply supported over a horizontal span of 3 m, and carries a uniformly distributed total load of 25 kN. It is to be strengthened by gluing a plate of Carbon Fibre Reinforced Plastic (CFRP) onto the outside of one flange. The plate is 40 mm wide and 2 mm thick and extends the full length of the beam. The self weight is negligible.

(a) Calculate the maximum bending stress in the unstrengthened beam. [4]

(b) If the CFRP has a Young's modulus of 140 GPa, calculate the elastic section modulus of the strengthened beam. [5]

(c) Calculate the maximum bending stresses in both the aluminium and the CFRP in the strengthened beam. [6]

(d) Calculate the maximum shear stress in the adhesive used to join the two components. [5]

SECTION B

Answer not more than ***four*** *questions from this section.*

6 (a) An isotropic metallic test specimen deforms elastically under a tensile stress. With ε_x, ε_y and ε_z denoting the strains along the principal axes of the specimen, show that for small strains the dilatation Δ of the specimen is [4]

$$\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

(b) As the magnitude of the tensile stress is increased, the metallic specimen experiences extensive yielding. Comment on the validity of assuming negligible dilatation in the specimen. [2]

(c) The specimen in (a) is now subjected to hydrostatic pressure p, with its dilatation given by $\Delta = -p/K$, where K is the bulk modulus.

(i) Show that

$$K = \frac{E}{3(1-2\nu)}$$

where E is the Young's modulus and ν is Poisson's ratio. State clearly any assumptions you make. [5]

(ii) Assuming the bulk and Young's modulus are numerically equal, determine the value of ν. Is this a reasonable value? [2]

(iii) Is $\nu = 0.5$ possible? If so, name one such material. [2]

(d) Describe how you would measure E and ν. Comment on the experimental accuracy. [5]

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7 (a) Derive equations for the plastic flow of metals that relate true stress σ_T to nominal stress σ_N and true strain ε_T to nominal strain ε_N. Comment on the validity of these relationships following necking. [4]

(b) For a ductile metal such as steel, sketch the σ_T versus ε_T and σ_N versus ε_N curves. Indicate on both curves the salient features of the material. [4]

(c) A metallic specimen is subjected to uniaxial tension.

(i) Show that the assumption of constant volume during plastic deformation leads to $\sigma_T \geq \sigma_N$ and $\varepsilon_T \leq \varepsilon_N$. [6]

(ii) With elastic deformation ignored, it is found that the true stress and true strain in the specimen are related to each other by

$$\sigma_T = A(\varepsilon_T)^n$$

where A and n are material constants. Show that, at necking, $\varepsilon_T = n$. Hence discuss the physical significance of n. [6]

8 (a) The Weibull survival probability, $P_s(V_0)$, of a ceramic sample of volume V_0 subjected to a uniform tensile stress σ is given by

$$P_s(V_0) = \exp\left[-(\sigma / \sigma_0)^m\right]$$

where σ_0 and m are constants.

(i) Sketch $P_s(V_0)$ as a function of σ / σ_0 for several values of the distribution coefficient m. At what stress level is the survival probability independent of m? [5]

(ii) Discuss the physical significance of σ_0. [2]

(iii) The coefficient m can be taken as a measure of the variability of material strength. Explain why statistical failure analysis is dangerous when materials with low values of m are used in critical situations, such as when the loss of human life might result from component failure. [4]

(iv) Propose an experimental method to determine σ_0 and m. [1]

(b) Plot separately the uniaxial stress versus strain curve for crazing and shear yielding for polymers. Indicate on each curve the salient features associated with each mechanism. [4]

(c) (i) Will a nylon specimen subjected to a uniaxial tensile stress craze at temperatures around its glass transition temperature T_g? [2]

(ii) How does the necking observed in a nylon specimen differ from the necking of a metallic specimen? [2]

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9 (a) Explain the difference between energy release rate G and stress intensity factor K, and between fracture toughness G_{Ic} and critical stress intensity factor K_{Ic}. Which of these parameters are material properties? Give two fracture criteria for fast crack growth. Comment on the effect of localised material yielding in the vicinity of a crack tip on fast fracture. [6]

(b) How is K_{Ic} related to fracture stress σ_F and critical crack size c? [2]

(c) Describe carefully the procedures of using K and K_{Ic} in design against fracture. [3]

(d) A pressure vessel is made of a metal having a yield strength σ_y of 1000 MPa and K_{Ic} of 50 MPa$\sqrt{\text{m}}$. In a routine safety test, it fails at a stress level σ. Post failure investigation revealed that an elliptical surface crack of depth c = 10 mm and surface length $2a$ = 40 mm was present prior to fracture (Fig. 4). For such an elliptical surface crack, the stress intensity factor is given by

$$K = \frac{1.1\sigma\sqrt{\pi c}}{1.4 - 0.2(\sigma / \sigma_y)^2}$$

where the denominator is introduced to account for the increase in effective crack length due to material yielding near the crack tip. Assuming that fast fracture from this flaw is responsible for the failure, determine the magnitude of σ when failure occurs. [5]

(cont.

(e) For a pre-cracked specimen subjected to fatigue loading, sketch and describe its crack growth rate as a function of the cyclic stress intensity factor ΔK. Explain the significance of fatigue threshold ΔK_{th} and critical stress intensity factor K_{Ic} in studying the fatigue behaviour of the crack. [4]

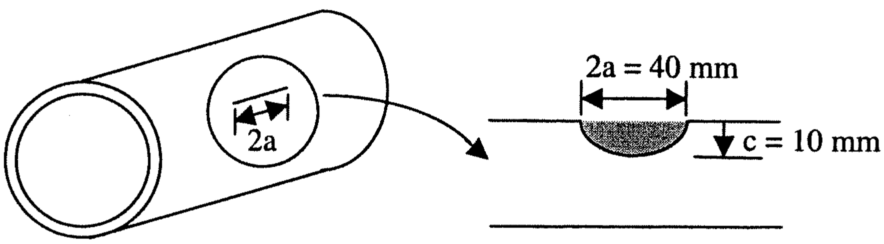

Fig. 4

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10 A uniform cantilever beam of given length l and square cross-section $t \times t$ is subjected to a given transverse end load F. The cantilever material has a fracture toughness K_{Ic}, tensile strength σ_f, and density ρ.

(a) Show that the maximum stress in the cantilever is given by $\sigma_{\max} = 6Fl/t^3$, and hence determine the material merit index for designing the lightest cantilever with adequate strength. [5]

(b) By using the data of Table 1, select the best material according to the merit index derived in (a). [2]

(c) Assuming brittle fracture, find the expression for the largest flaw size the cantilever can tolerate without instigating failure. Comment on the influence of l and t on this allowable flaw size. [3]

(d) Due to manufacturing difficulties, a distribution of small elliptical cracks is found to exist on the surface of the cantilever. The largest such surface flaw has a given depth c. For minimum weight design against fast fracture and a fixed maximum flaw size c applicable to all materials, propose a merit index of the cantilever material in terms of K_{Ic} and ρ. [4]

(e) By using the data of Table 1, select the best material according to the merit index you derived in (d). [2]

(f) Discuss carefully how you would select the material if the lightest cantilever is to be designed against both strength and brittle fracture. Further comment on how other issues would affect your answer. [4]

Material	ρ (Mg/m^3)	σ_f (MN/m^2)	K_{Ic} (MN/m$^{3/2}$)
Stainless steel	7.6	800	140
PMMA	1.2	110	1.0
Alumina	3.9	300	4.0

Table 1

END OF PAPER