IA
ANSWERS
2003

Engineering Tripos Part IA, June 2003, Paper 1: Mechanical Engineering

Section A: Thermofluid

Answers

1 (a) -981 Pa

(b) $H = 0.5$ m

(c) $H = 0.6$ m

(d) $H = 0.52$ m

2 (a) Use mass flow rate conservation and Bernoulli: $h_2 = 4.063$ m

(b) 146 kN

3 (a) –

(b) (i) 40 m s^{-1}

(ii) 22 m s^{-1}

(c) (i) 3.2 N

(ii) 0.96 N

(d) 443.7 J

4 (a) P_{H_2} = 1240 Pa, P_{O_2} = 4960 Pa, P = 6200 Pa

(b) O_2: 1.75 $\times$ 10^{-3} kmol s^{-1}, H_2O: 5 $\times$ 10^{-4} kmol s^{-1}

(c) –

5 (a) 692 K, 40.6 MN

(b) –

(c) 688 K, 40.5 MN

Engineering Tripos Part IA 2003
Paper 1 (Mechanical Engineering)
Section B (Mechanics/Vibrations)
Answers to Questions

6.
(a) 55 mm s^{-1}. (b)(i) 550 N mm. (b)(ii) 92 N.

7.
(a) 89.2 kg. (b)(i) 9.80 m. (b)(ii) 1.98 kN.

8.
(a) $t = \sqrt{(2h/g)}$. (b) $t = \sqrt{(3h/g)}$.

9.
(a) $(dv/dt) + (\lambda/m)v = 0$. (b) $v = u\exp[-(\lambda/m)t]$, $T = m/\lambda$. (c) $L = um/\lambda$. (d) An infinite time.

10.
(b) 13159 N m^{-1}. (c) 0.3.

1A Exam 2003

Structures Paper

Answers

1	(a)	1766 N; 1972 N; compression
	(c)	$144.10^6/EI$ (units N, mm)
2		3.5 PL/AE
3	(a)	80 kN; -25 kN
	(b)	150 kNm; 200 kNm
4	(a)	$0.666.10^8$ mm^4; $0.746.10^8$ mm^4
	(b)	7.01 MPa; 68.6 MPa
	(c)	0.133 MPa
5	(d)	0.01248L; 0.0225PL

ENGINEERING TRIPOS PART IA **JUNE 2003**
SECTION B: MATERIALS

Answers

6. (a) $\sigma(h) = \rho g(H-h)$ and $\varepsilon(h) = \frac{\rho g}{E}(H-h)$

$\frac{\Delta H}{H} = \frac{\rho g H}{2E}$, H = 92.7 m (for change on length of 0.001%)

$H \approx 37$ km (for failure at base)

(b) $E \propto f^2$

(c) atomic packing factor = 0.74
theoretical (and actual) density = 2700 kg/m^3

7. (b) Internal work rate $= 6\,k\,L\,v$; Hardness $= 3\sigma_y$

8. (b) (i) $\sigma_t = 423$ MPa (not needed, but $\sigma_o = 523$ MPa if less direct method used)

(ii) $\sigma(y) = \sigma_{max}\frac{y}{4}$; $\sigma_{max} = 607$ MPa

9. (c) $\mu \approx 0.5$ (assuming normal stress at asperity contacts $\approx \sigma_y$)

(or $\mu \approx 1/6$, if normal stress at asperity contacts $\approx$ hardness, $3\sigma_y$)

10. (a) maximise $E^{1/2}/\rho$

ENGINEERING TRIPOS PART IA 2003

Paper 3 Electrical and Information Engineering

Questions 1 – 8

No answers have been received from the Examiner for these questions

Part IA --- Paper 3 – Electrical and Information Engineering

Section C – 2003 Answers

9

a) $C = 2\pi\varepsilon_0 \Big/ \ln\frac{(2d-r)}{r}$ Farads/per unit length

b) $B = \frac{\mu_0 I}{2\pi R}$

c) $F = \frac{\mu_0 I^2}{4\pi d}$ N/unit length (attractive)

10.

a) $B = 0.126$ Tesla/Amp

b) I= 10.98 A (note B = 1.37 Tesla which is sensible)

c) $v = 0.063 ms^{-1}$

11.

a) $I = 0.149A$; Peak voltage = 49.3 V

b) loss per cycle = 0.0468 J/cycle (At 100 Hz this is equal to 4.68 Watts)

c) Speaker power = 38 Watt ; Magnetic loss = 4.68 Watt ; Efficiency = 88 %

ENGINEERING TRIPOS Part IA 2003

Paper 4 – Mathematical Methods

Answers:

1. a) $h = a\sqrt{2}$

   b) (i) $\mathbf{r} = (0, 0, a\sqrt{2}) + \lambda_1(-a, a, a\sqrt{2}) + \lambda_2(-a, -a, a\sqrt{2})$ (ii) $a\sqrt{(2/3)}$ (iii) 70.5°

   c) $(\pm a\sqrt{2}, 0, 0)$ $(0, \pm a\sqrt{2}, 0)$ $(0, 0, \pm a\sqrt{2})$

2 a) (i) 3/2 (ii) 1/2

   b) (i) $z = \sqrt{2}\, e^{i\pi/4 + in\pi/2}$ (4 solutions) (ii) $z = e^{i(\pm\pi/9 + 6n\pi/9)}$ (6 solutions)

3 a) $y(x) = (-x - 1)e^{-x} + 1$ b) $y(x) = (1+x)e^{x} + (1/6)x^3e^{x}$

4 a) $\det A = -0.25 \neq 1$

   b) $\lambda = 1, \pm 0.5$ $\mathbf{x}_1 = [1\ 0\ 0]^T$ $\mathbf{x}_2 = [0\ 1/\sqrt{2}\ 1/\sqrt{2}]^T$ $\mathbf{x}_3 = [1\ 1/\sqrt{2}\ -1/\sqrt{2}]^T$

   c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5 c) (i) $y(t) = \frac{1}{2}\left(e^{-t} - e^{-3t}\right)$ $t \geq 0$ $y(t) = 0$ $t < 0$

   (ii) $y(t) = \frac{1}{2}\left(e^{2T} - 1\right)e^{-3t}$ $t \geq T$ $y(t) = \frac{1}{2}\left(e^{-t} - e^{-3t}\right)$ $T \geq t \geq 0$ $y(t) = 0$ $t < 0$

6 c) $a_n = \frac{4}{n^2\pi^2}\left(1 - (-1)^n\right)$ $n > 0$

7 c) P(A wins) = $6/11$, P(B wins) = $5/11$

   d) P(A wins) = $\frac{1}{1+5p}$, P(B wins) = $\frac{5p}{1+5p}$

   e) $p = 1/5$

8 $y(t) = \frac{1}{2}e^{-2t} + \frac{1}{2}e^{-2t}\cos t + \frac{1}{2}e^{-2t}\sin t$

9 a) $\phi = \frac{x^2}{y} + 2xy^2 + 3x + \sin y + 1 - \sin 1$

   b) (0,0) (0,2) (–2,0) are saddle points, $\left(-\frac{2}{3}, \frac{2}{3}\right)$ is a minimum

10 d) Quicksort is O(n log (n))

11 c) Estimate stops increasing at 3183 terms