Engineering Tripos Part IA 2003 Paper 1

THERMO FLUID MECHANICS.

Question 1.

(a) Show that the pressure in the pipe calculated from the tank and from the reservoir are the same

from the tank $P = P_{atm} - \rho g (h_1 - H) = P_{atm} - 0.1\rho g$

from the reservoir $P = P_{atm} - \rho g h_2 = P_{atm} - 0.1 \rho g$

same value

$P - P_{atm} \simeq -981$ Pa

(b) The level in the pipe must reach -0.1 m on the reservoir side, hence pressure in the pipe must be

$$P - P_{atm} = +981 \text{ Pa}$$

Moreover, to conserve the volume of air, h_1 must have gone up by 0.2 m. hence: $h_1 = 0.4$ m

Since pressure in the pipe is above atmospheric, H must be higher than h_1

$$P - P_{atm} = \rho g (H - h_1)$$

So $H = h_1 + 0.1 = 0.5$ m

(c) Water will flow when $h_1 = L$, and pressure in the pipe will remain at the level calculated in (b). Hence H will still be 0.1 higher than h_1:

$$H = 0.6 \text{ m}$$

(d) Water is poured slowly so we expect a constant temperature. Between the initial condition and the moment where bubbles appear, the pressure difference is $\Delta p = 2 \times 981$ Pa

$$\frac{\Delta \rho}{\rho} = + \frac{\Delta P}{P} \simeq \frac{2 \times 981}{101325} \simeq 2\%$$

Hence the length of the air column is reduced by 2%

Δ length $= 0.02 \times (0.3 + 0.3 + 0.4) = 2$ cm

Hence H is 2cm above (b): $H = 0.52$ m

Question 2

(a) Mass conservation $W_1 h_1 V_1 = W_2 h_2 V_2$

$W_1 = 13\,m$ width upstream
$W_2 = 8\,m$ width at the bridge

two unknowns h_2 and V_2

Other equation: since the flow is inviscid, Bernoulli applies between 1 and 2, for example at the surface

$$\rho g h_1 + \frac{1}{2}\rho V_1^2 = \rho g h_2 + \frac{1}{2}\rho V_2^2$$

This is another equation relating h_2 and V_2. Substituting $h_2 = \frac{W_1 h_1 V_1}{W_2 V_2}$

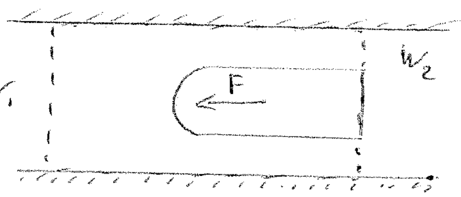
$$g h_1 + \frac{V_1^2}{2} = g \frac{W_1 h_1 V_1}{W_2 V_2} + \frac{V_2^2}{2}$$) equation for V_2

Check $V_2 = 5\,m.s^{-1}$

$10 \times 5 + \frac{2.5^2}{2} = 53.125$ and $10 \frac{13 \times \cancel{5} \times 2.5}{8 \times \cancel{5}} + \frac{5^2}{2} = 53.125$

Hence $h_2 = \frac{W_1 h_1 V_1}{W_2 V_2} = \frac{13 \times 5 \times 2.5}{8 \times 5} = 4.0625\ m$

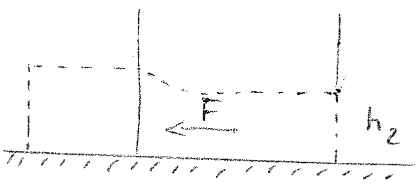
(b) Control Volume



$\dot{m}_1 = \rho W_1 h_1 V_1 = 10^3 \times 13 \times 5 \times 2.5 = 162500$

$\dot{m}_2 = \rho W_2 h_2 V_2 = 10^3 \times 8 \times 4.0625 \times 5 =$ 〃

Pressure forces: hydrostatic



$P_1 A_1 = \rho g \frac{h_1^2}{2} W_1 = 10^4 \times \frac{5^2}{2} \times 13 = 1625000$

$P_2 A_2 = \rho g \frac{h_2^2}{2} W_1 = 10^4 \times \frac{4.0625^2}{2} \times 13 = 1072754$

<u>SFME</u>: $\dot{m}_2 V_2 - \dot{m}_1 V_1 = F + P_1 A_1 - P_2 A_2$

$F = 162500(5 - 2.5) - 1625000 + 1072754$

$= -145996\ N$ (to the left)

(2)

Question 3 [see page 11 for air properties in databook]

(a) Heat pulse in the air system $Q = m\,c_v\,\Delta T$

Mass of air $PV = mRT$ $\qquad m = \dfrac{10^5 \times 10^{-2}}{287 \times 300} \simeq 0.0011614$ kg

Hence $\Delta T = \dfrac{2 \cdot 10^3}{0.0011614 \times 718} \simeq 2398$ K

$T = 2398 + 300 = 2698$ K $\qquad \dfrac{P}{P_{initial}} = \dfrac{T}{T_{initial}}$ $\qquad P = \dfrac{10^5\ 2698}{300} \simeq 9$ bar

(b) Initial water jet velocity : Bernoulli $\quad P = P_{atm} + \rho\dfrac{v_i^2}{2}$

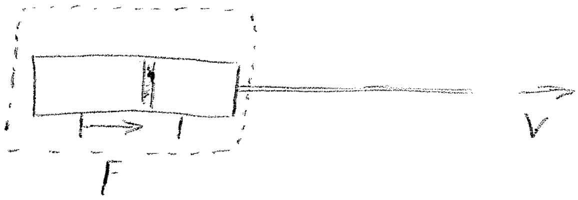
$v_i = \sqrt{\dfrac{2 \times 8 \times 10^5}{10^3}} = 40\ \text{m.s}^{-1}$

Inside pressure at the end of the air expansion (adiabatic)
$PV^\gamma = c^{ste}$

Final pressure $\quad P_f = P_i\left(\dfrac{V_i}{V_f}\right)^\gamma = 9 \times 10^5 \left(\dfrac{1}{2}\right)^{1.4} \simeq 3.41$ bar

Hence $v_f = \sqrt{\dfrac{2 \times 2.41 \times 10^5}{10^3}} = 21.95\ \text{m.s}^{-1}$

(c) Thrust. Control Volume



$\dot{m} v = F$ (no pressure force)

$F = \rho A v^2 = 10^3 \times 2 \times 10^{-6} \times 40^2 = 3.2$ N initially

$= 10^3 \times 2 \times 10^{-6} \times 21.95^2 = 0.964$ N at the end

(d) Between initial and final time : energy conservation

change in internal energy of air = water kinetic energy + work against atmosphere
$m_{air}\,c_v\,\Delta T$

final air temperature : $\quad T_f = \dfrac{P_f V_f}{m R} = \dfrac{3.41 \times 10^5 \times 2 \times 10^{-3}}{0.0011614 \times 287} = 2046$ K

Water kinetic energy $= 0.0011614 \times 718\,(2698 - 2046) - 10^5 \times 10^{-3}$

$= 543.7 - 100 = 443.7$ J

<u>NB</u>: this is 1 kg of water at 29.79 m.s^{-1}

(3)

Question 4

(a) Hydrogen $P_{H_2} \dot{V} = \dot{n} \bar{R} T \longrightarrow P_{H_2} = 5 \times 10^{-4} \times 8.3143 \times 10^{3} \times 298.15$

$P_{H_2} = 1239.45 \ Pa$

Oxygen $P_{O_2} = 2 \cdot 10^{-3} \times 8.3143 \times 10^{3} \times 298.15$

$= 4957.8 \ Pa$

Hence $P = P_{O_2} + P_{H_2} = 4957.8 + 1239.4 = 6197.2 \ Pa$

(b) Combustion with excess oxygen

$$n\, H_2 + m\, O_2 \longrightarrow n\, H_2O + (m - \tfrac{n}{2})\, O_2$$

$n = 5 \times 10^{-4}$ kmol.s^{-1} of water H_2O

$m - \frac{n}{2} = 2 \times 10^{-3} - \frac{5}{2} \times 10^{-4} = 1.75 \times 10^{-3}$ kmol.s^{-1} of oxygen O_2

(c) S.F.E.E with lower enthalpy of reaction $120.000 \ MJ.kg^{-1}$

$= 240 \ MJ.kmol^{-1}$ of H_2

$$n\, h_{H_2O}(T) + (m - \tfrac{n}{2})\, h_{O_2}(T) = n\, h_{H_2}(T_0) + m\, h_{O_2}(T_0)$$

$$n\left(h_{H_2O}(T) - h_{H_2O}(T_0)\right) + (m - \tfrac{n}{2})\left(h_{O_2}(T) - h_{O_2}(T_0)\right) = n\left[h_{H_2}(T_0) - h_{H_2O}(T_0) + \tfrac{1}{2} h_{O_2}(T_0)\right]$$

$$= n \times 240 \ MJ.kmol^{-1}$$

$$h_{H_2O}(T) - h_{H_2O}(T_0) + 3.5\left(h_{O_2}(T) - h_{O_2}(T_0)\right) = 240 \ MJ.kmol^{-1}$$

Table page 12

$T = 1700 \ K \quad 67.65 - 9.9 + 3.5(56.63 - 8.66) = 225.645$

$T = 1800 \ K \quad 72.58 - 9.9 + 3.5(60.35 - 8.66) = 243.595$

$$T = 1800 - 100 \frac{243.595 - 240}{243.595 - 225.645} \simeq 1780 \ K$$

Pressure being different from 1.01324 bar plays no rôle since enthalpies do not depend on pressure for perfect or semi perfect gases.

4

Question 5

(a) Adiabatic & fully resisted (reversible) $\therefore$ isentropic

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 1500 \left(\frac{1}{15}\right)^{\frac{0.4}{1.4}} \simeq 691.93\ K$$

$$\text{S.F.E.E.}: \cancel{\dot{Q}} - \dot{W}_x = \dot{m}(h_2 - h_1) = \dot{m} c_p (T_2 - T_1)$$

$$\dot{W}_x = 50 \times 1005 \times (1500 - 691.93) \simeq 40.61\ MW$$

(b)
$$du = dQ + dW = 0.01\, du - p\, dv$$

$$0.99\, du = -d(pv) + v\, dp$$

$$0.99\, c_v\, dT = -R\, dT + \frac{RT}{p} dp$$

$$(0.99\, c_v + R)\frac{dT}{T} = R \frac{dp}{p}$$

$$\frac{dT}{T} = \frac{R}{0.99 c_v + R} \frac{dp}{p}$$

$$= \frac{c_p - c_v}{c_p - 0.01\, c_v} \frac{dp}{p} = \frac{\gamma - 1}{\gamma - 0.01}$$

$$T \big/ p^{\frac{\gamma-1}{\gamma-0.01}} = c^{ste}$$

(c) New exit temperature $T = 1500 \left(\frac{1}{15}\right)^{\frac{0.4}{1.39}} \simeq 688.09\ K$

$$\text{S.F.E.E.} \quad \underset{\substack{\uparrow \\ 0.01\, \dot{m} c_v (T_2 - T_1)}}{\dot{Q}} - \dot{W}_x = \dot{m} c_p (T_2 - T_1)$$

hence $W_x = 50 \times 1005 (1500 - 688.09) - 0.01 \times 50 \times 718 (1500 - 688.09)$

$$W_x \simeq 40.507\ MW$$

Very small change in W_x, but the isentropic efficiency $\frac{W_x}{\dot{m} c_p (T_2 - T_1)} \simeq 0.993$ drops by nearly 1%

5

T. Alboussière

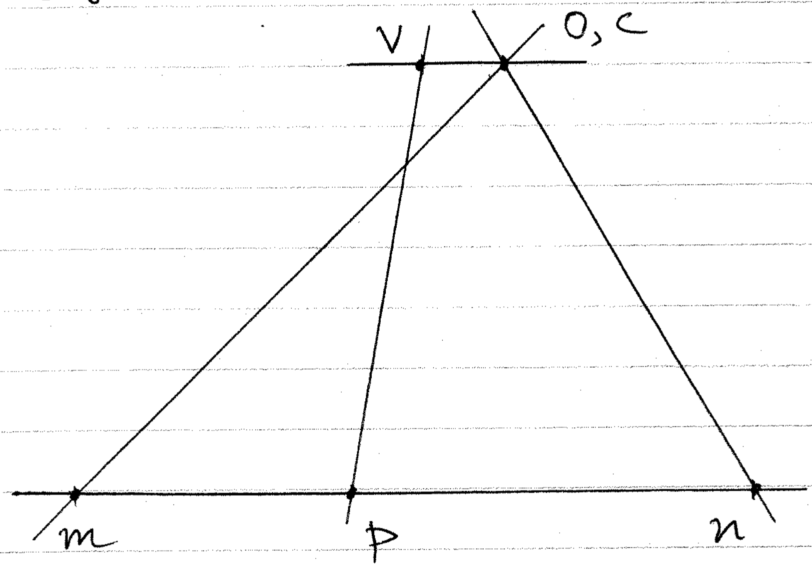
Q.6 PAPER I : MECHANICS

(a) Velocity of M :-

$V = r\omega = 20\,mm \times 20\,rad\,s^{-1} = 400\,mm\,s^{-1}$

$\rightarrow$ 80 mm on diagram

o, c / m = 80



Note that $\frac{pm}{nm} = \frac{PM}{NM} = \frac{40}{100} = \frac{pm}{90\text{ mm diagram}}$

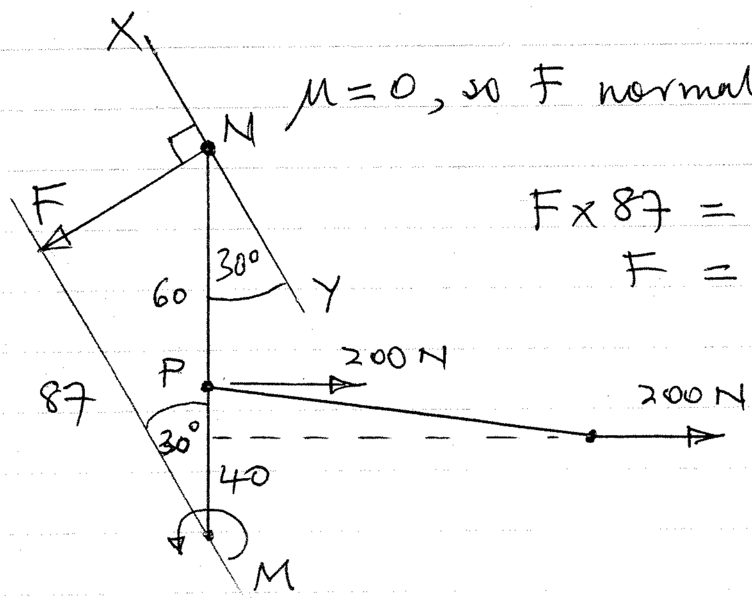
$\therefore pm = \frac{40}{100} \times 90\,mm = 36$ mm diagram

Velocity OV = 10 mm diagram $\rightarrow$ 50 $mm\,s^{-1}$

(b) $F \times 400\,mm\,s^{-1} = 200\,N \times 50\,mm\,s^{-1}$

$F = 25\,N$

$25\,N \times 20\,mm = 500\,N\,mm$



$M = 0$, so F normal to XY

$F \times 87 = 200\,N \times 40$

$F = 92\,N$

Q.7.

(a) $mgh = U_{el}$, $m = U_{el}/gh$

$$m = \frac{(1000\,N \times 15\,m + \frac{1}{2} \times 1500\,N \times 15\,m)}{9.81 \times 30} = \underline{89.2\ kg}$$

(b) $mg(l+y) = U_{el}(y)$

$$60 \times 9.81 \times 15 + 60 \times 9.81\,y = 1000y + \frac{1}{2} \cdot \frac{y}{15} \cdot 1500y$$

$$50y^2 + 411y - 8829 = 0$$

$$y = \frac{-411 \pm \sqrt{411^2 + 4 \times 50 \times 8829}}{100} = \underline{9.80\,m}$$

(the root $-18.02\,m$ is discarded, since $18.02 > 15$)

$$F = 1000\,N + \frac{9.80}{15} \times 1500 = \underline{1.98\ kN}$$

(c) Taking the maximum extension, in one cycle of loading / unloading, $500\,N \times 15\,m = 7500\,J$ is dissipated as friction in the rope, compared to a U_{el} of $26250\,J$, i.e., 29% of the energy input is lost. This results in the jumper rebounding to a position which is well below the bridge deck, which is obviously essential for a safe jump.

(d) For $m = m_{max}$, $3.0\,kN \geq F \geq 2.5\,kN$. This can impose a very large force on the person, which could cause injury or detachment of the person from the end of the rope. For $m > m_{max}$, the rope will snap.

Q.8.

(a) $s = ut + \frac{1}{2}at^2$,

$$h = \frac{1}{2}gt^2, \quad t = \sqrt{\frac{2h}{g}}$$

(b) $F = ma$

$$mg - T = m\frac{d^2h}{dt^2}$$

$$C = I\dot{\omega}$$

$$a\omega = \frac{dh}{dt}, \quad \frac{d^2h}{dt^2} = a\dot{\omega}$$

$$C = Ta = \frac{I}{a}\frac{d^2h}{dt^2}, \quad T = \frac{I}{a^2}\frac{d^2h}{dt^2}$$

$$mg - \frac{I}{a^2}\frac{d^2h}{dt^2} = m\frac{d^2h}{dt^2}$$

$$mg - \frac{ma^2}{2a^2}\frac{d^2h}{dt^2} = m\frac{d^2h}{dt^2} \quad \text{(I value from 'MDB')}$$

$$\frac{d^2h}{dt^2} = \frac{2g}{3}, \quad \int_0^{dh/dt} d\left(\frac{dh}{dt}\right) = \int_0^t \frac{2g}{3}\,dt,$$

$$\frac{dh}{dt} = \frac{2g}{3}t, \quad \int_0^h dh = \int_0^t \frac{2g}{3}t\,dt, \quad h = \frac{2gt^2}{6}$$

$$t = \sqrt{\frac{3h}{g}}$$

[Alternative energy-based method:—

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\frac{Iv^2}{a^2} = \frac{v^2}{2}(m + I/a^2)$$

$$mgh = \frac{v^2}{2}(m + m/2) \quad \text{(I value from MDB)}$$

$$= 3mv^2/4$$

$$v^2 = \frac{4gh}{3}, \quad v = \sqrt{\frac{4gh}{3}}$$

$$\frac{dh}{dt} = \sqrt{\frac{4g}{3}}\, h^{1/2}$$

$$\int_0^h \frac{dh}{h^{1/2}} = \int_0^t \sqrt{\frac{4g}{3}}\, dt, \quad 2h^{1/2} = \sqrt{\frac{4g}{3}}\, t,$$

$$t = 2\sqrt{h}\sqrt{\frac{3}{4g}} = \underline{\sqrt{\frac{3h}{g}}}$$

]

(c) Simple disc, $mgh \rightarrow \frac{1}{2}mv^2$.
Yo-yo, $mgh \rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.
Since mgh is the same in both cases, the yo-yo falls more slowly than the simple disc, so the time is greater.

Q.9.

(a) $$m\frac{dv}{dt} = -\lambda v$$

$$m\frac{dv}{dt} + \lambda v = 0$$

$$\underline{\frac{dv}{dt} + \left(\frac{\lambda}{m}\right)v = 0}$$

(b) $$\int \frac{dv}{v} = \int -\left(\frac{\lambda}{m}\right) dt$$

$$\ln v = -\left(\frac{\lambda}{m}\right)t + c$$

$$v = ce^{-(\lambda/m)t}$$

$v = u$ at $t = 0$ gives $c = u$

$$\underline{v = ue^{-(\lambda/m)t}}$$

$$\underline{T = m/\lambda}$$

(c) $$v = \frac{dx}{dt} = ue^{-t/T}$$

$$\int dx = \int ue^{-t/T}\,dt$$

$$x = -uTe^{-t/T} + D$$

$x = 0$ at $t = 0$ gives $D = uT$

$$x = uT\left(1 - e^{-t/T}\right)$$

$x = L$ at $t = \infty$ gives

$$\underline{L} = uT = \underline{um/\lambda}$$

10

(d) $\underline{t = \infty}$. Friction in rain and at train wheels / bearings will result in a finite t.

(e) The characteristics of the buffer-stops are bad, because they result in maximum force at the moment of collision, and rapidly decreasing force as the train slows. Damage / injury may result on first impact. Ideally, want constant force. Use active or passive control of oil valves to obtain this. Or, use a frictional device (see King's Cross).

Q.10.

(a) $m\ddot{y} = -k(y-x) - \lambda(\dot{y}-\dot{x})$

$$m\ddot{y} + \lambda\dot{y} + ky = \lambda\dot{x} + kx$$

$$\frac{m}{k}\ddot{y} + \frac{\lambda}{k}\dot{y} + y = \frac{\lambda}{k}\dot{x} + x$$

This is Case (c), since $\omega_n = \sqrt{\frac{k}{m}}$, $\zeta = \frac{\lambda}{2\sqrt{km}}$

$$\frac{m}{k} = \frac{1}{\omega_n^2}, \quad \frac{\lambda}{k} = \frac{1}{k}2\zeta\sqrt{km}$$

$$= 2\zeta\sqrt{\frac{m}{k}} = \frac{2\zeta}{\omega_n}$$

$\therefore$ DE is

$$\frac{\ddot{y}}{\omega_n^2} + \frac{2\zeta}{\omega_n}\dot{y} + y = \frac{2\zeta}{\omega_n}\dot{x} + x$$

(b) For $\zeta = 0$, MDB gives

$$\left|\frac{Y}{X}\right| = \frac{1}{1-(\omega/\omega_n)^2} = \frac{10\,\mu m}{50\,\mu m} = \frac{1}{5}$$

$$1-(\omega/\omega_n)^2 = 5, \quad \omega/\omega_n = \sqrt{5+1} = 2.45$$

$$k = m\omega_n^2 = 20\left\{2\pi \times \frac{10}{2.45}\right\}^2 = \underline{13159 \text{ N/m}}$$

(c) $\left|\frac{Y}{X}\right| \leq \frac{100\,\mu m}{50\,\mu m} = 2$. From the chart on

p13, a peak response of 2 occurs for $\underline{\zeta = 0.3}$

12

(d) From the chart on p13, for $\omega/\omega_n = 2.45$, adding the damping increases $|Y/X|$ from approximately 0.2 to approximately 0.4. In other words, the amplitude of the machine increases from 10 μm to approximately 20 μm.

13

D. Jones