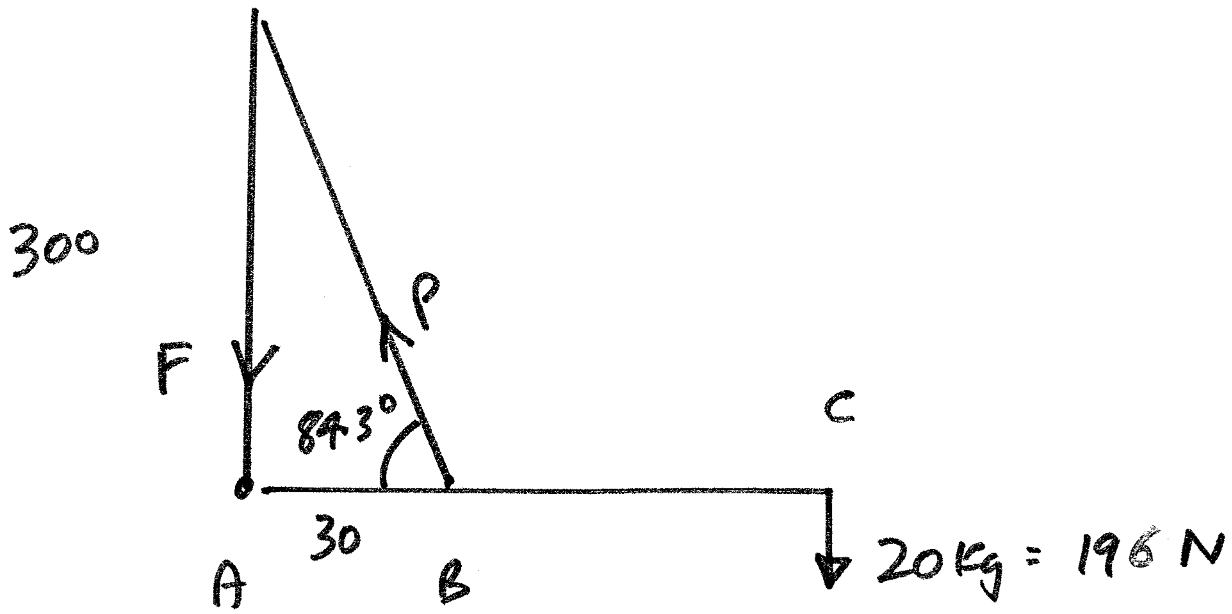
# 1A. 2003 STRUCTURES CRIB

PAPER 2

1.(a)




Take moments about A for ulna.

$$P \cdot \sin 84.3 \cdot 30 = 196 \cdot 300$$

$$\Rightarrow \underline{\underline{P = 1972 \text{ N}}}$$

Resolve vertically

[5 marks]

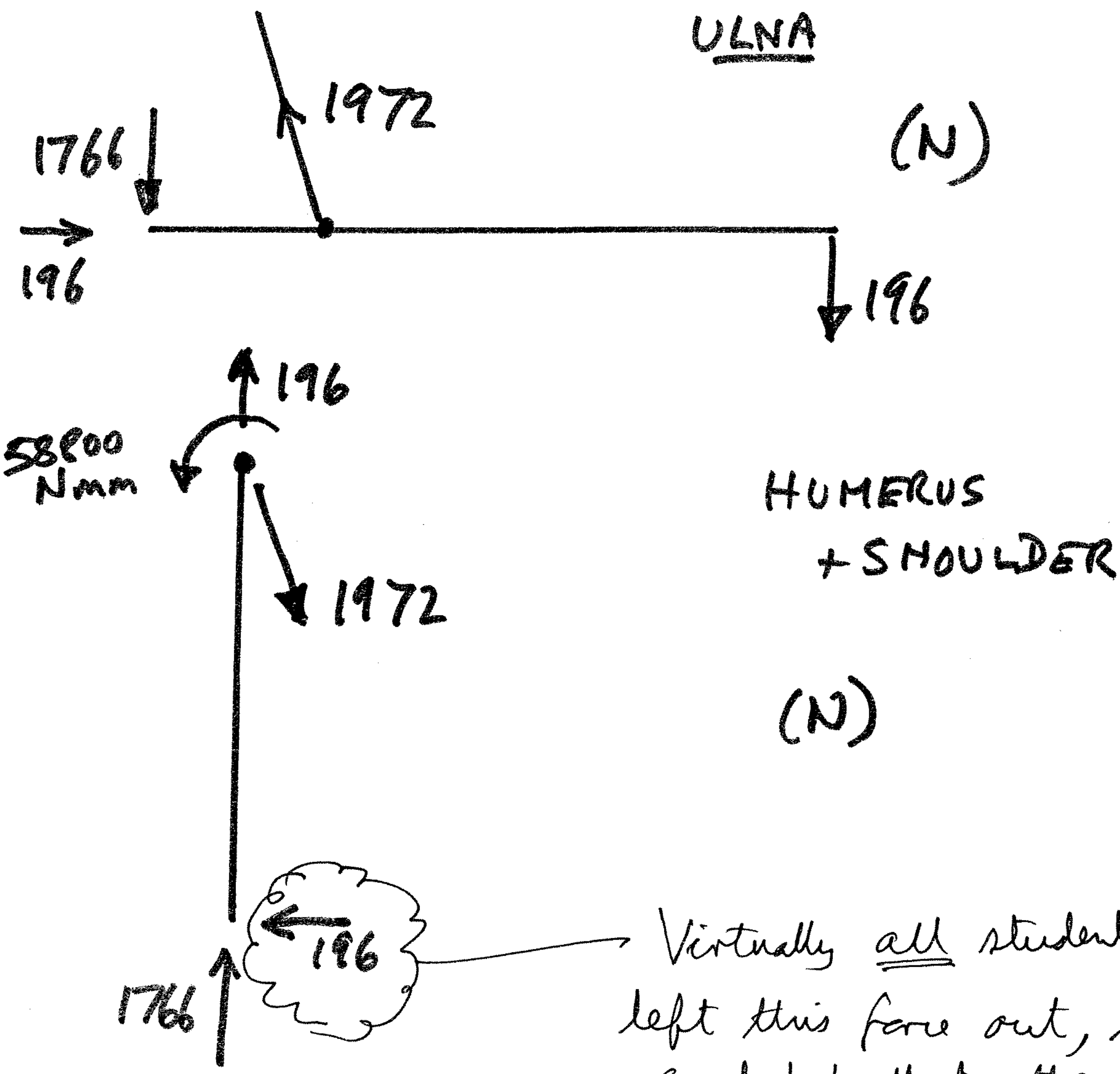
$$F + 196.2 = P \cdot \sin 84.3$$

$$\Rightarrow \underline{\underline{F = 1766 \text{ N}}}$$

Humerus is in <u>compression</u>.

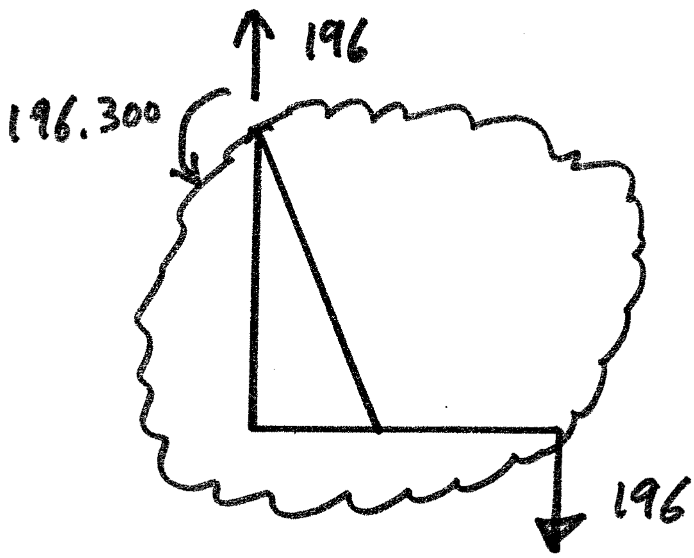
Biceps must exert a horizontal force at B of $P \cos 84.3 = 196$ N

This must be balanced by a force at the elbow




Virtually all students left this force out, so concluded that the shoulder must apply a horizontal reaction and no moment.

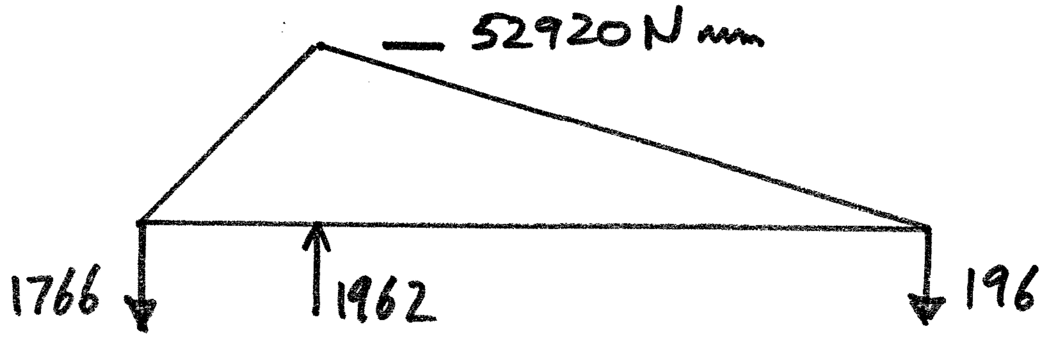
Shoulder reactions can also be found from global equilibrium




Free body.

[3 marks]

(b)




[4 marks]

(c) Need to split the loading into components. Several valid ways of doing this.

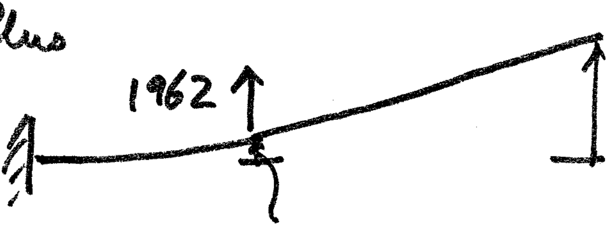
<u>Method 1.</u> Assume clamp at elbow and then apply rigid body rotation later


$\downarrow \delta_{C1}$

$$\delta_{B1} = \frac{196(30)^3}{3EI} + \frac{196.270.30^2}{2EI} = \frac{1.76\ 10^6}{EI} + \frac{23.8.10^6}{EI}$$

$$= \frac{196.(300)^3}{3EI} = \frac{1.76\ 10^9}{EI}$$

Plus




$$\delta_{C2} = -\frac{270.1962.(30)^2}{2.EI} = -\frac{238\ 10^6}{EI}$$

$$\delta_{B1} = \frac{-1962(30)^3}{3.EI}$$

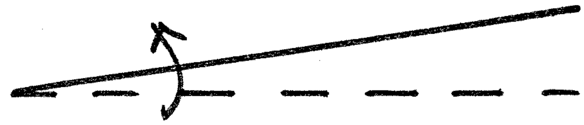
$$\frac{17.7\ .10^6}{EI}$$

Total downward deflection at C is

$$\frac{(1760 - 238)10^6}{EI} = \frac{1522.10^6}{EI}$$

Correct this with rigid body rotation

of $\dfrac{\dfrac{1522 \quad 10^6}{EI}}{300}$



$= \dfrac{5.07\, 10^6}{EI}$ (units rads, N, mm)

$\therefore$ Upward movement at B

$= \underbrace{\dfrac{5.07.10^6}{EI}.30}_{\text{(due to rotation)}} + \underbrace{\dfrac{17.7.10^6}{EI}}_{\text{(due to bricks)}} - \underbrace{\dfrac{1.76\,10^6}{EI} - \dfrac{23.8.10^6}{EI}}_{\text{(due to load)}}$

$= \dfrac{144.10^6}{EI}$ (units N, mm)

<u>Method 2</u>


Assume support at A & B

$\delta = 196.\dfrac{270.30}{3EI}.270 + \dfrac{196.(270)^3}{3EI}$

$= \dfrac{1430.10^6}{EI}$

b

Rotate anticlockwise about A to eliminate δ, as a rigid body

$$\text{Rotation} = \frac{1430.10^6}{EI} \Big/ 300$$

$$\therefore \text{Upwards movement of } B = \frac{1430.10^6}{EI} \cdot \frac{30}{300}$$

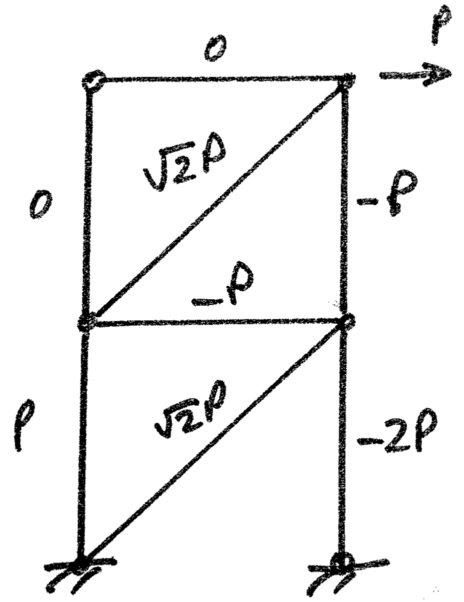
$$= \frac{143.10^6}{EI} \cdot \quad (N, mm)$$

which is the same as method 1 (to rounding error) as expected.

Most students tried a version of Method 1, but as can be seen here it is much more long-winded, and most attempts made some simplifying assumptions, or simply didn't express the logic properly.

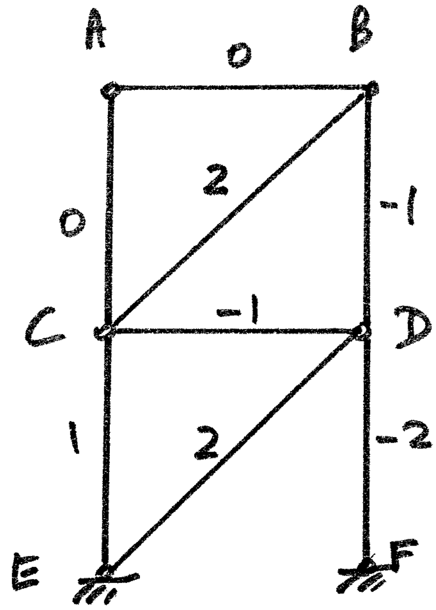
Since EI has dimensions, which are not specified, it is important to note what units are being used in the calculation. Many candidates lost marks for lack of care with units.

2 (a) FORCES




[ 6 marks ]

EXTENSIONS




all $\times \frac{PL}{AE}$

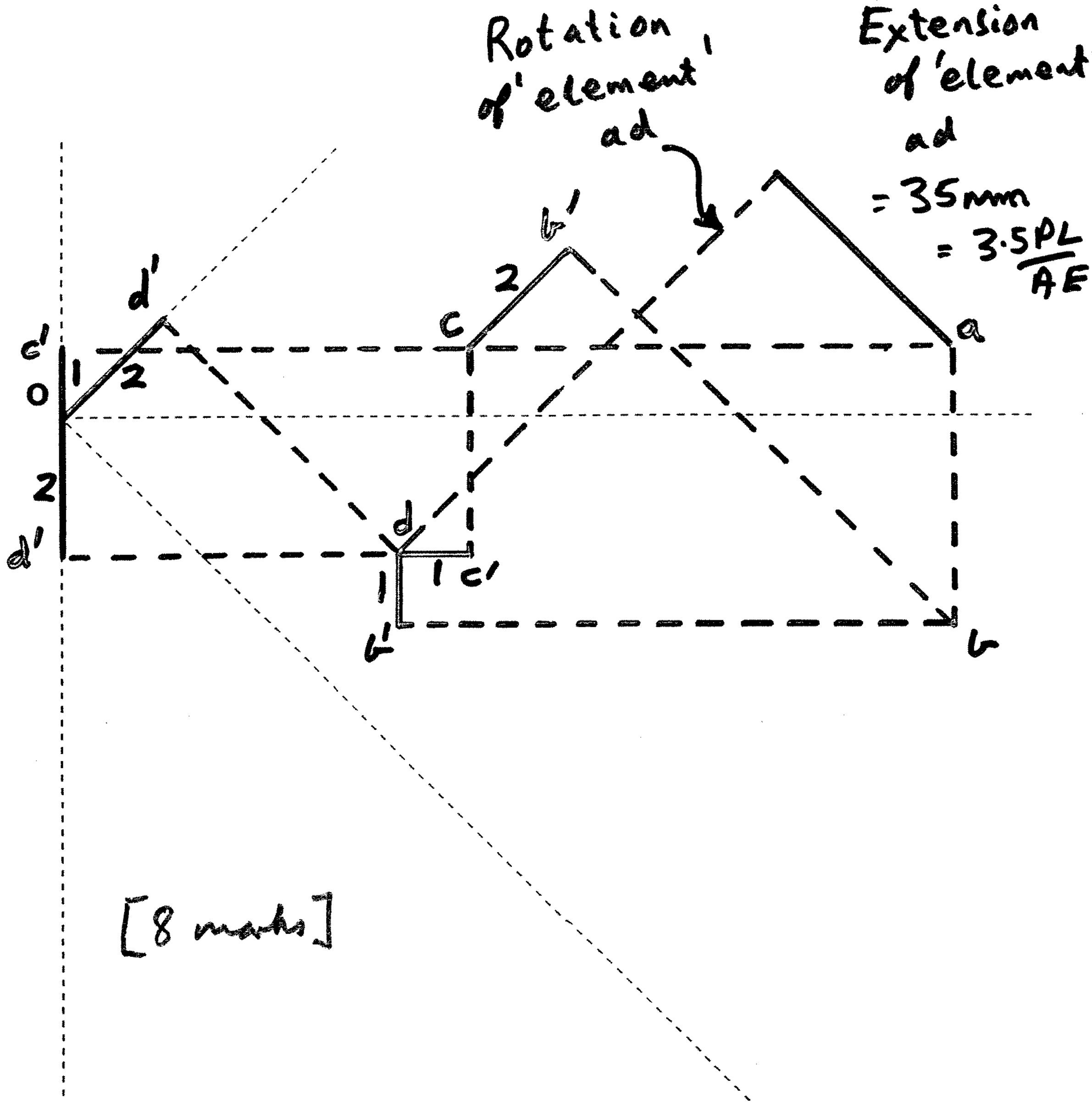
Displacement diagram on next sheet

<u>Common errors.</u> Forces wrong. Must start analysis with point where only two unknown forces, since only 2 equil. equations. So order should be A, B, C, D. Many students forgot that length of CB & ED is $\sqrt{2}L$.

To find change in length A → D, imagine there is a member AD. Then plot on diagram the extension and rotation of this imagined element. The extension is the part that is wanted, <u>not</u> the total movement ad.

Many candidates got the direction of the extensions wrong on the displacement diagram, especially c as found from d.

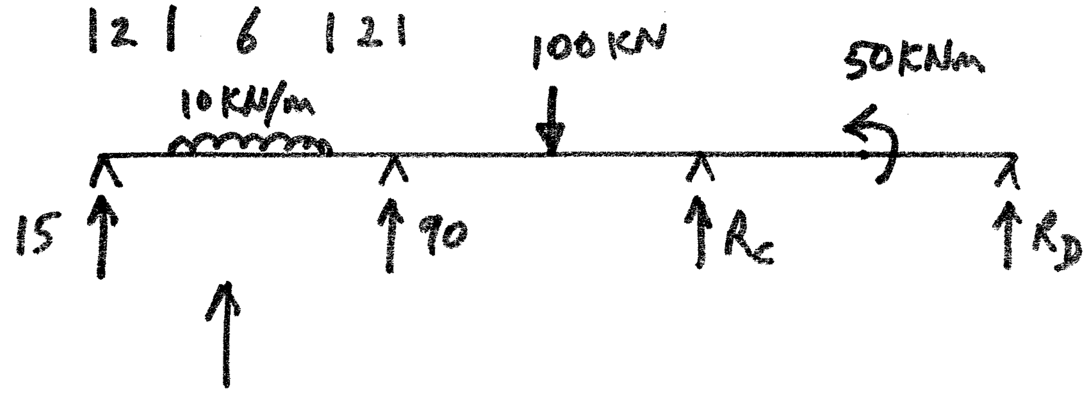
[6 marks]




[8 marks]

Scale $\frac{PL}{AE}$ = 10 mm.

3 (a)




Total load = 10 KN/m × 6 m = 60 KN

More than half of the candidates made this 80 KN!!

A beam on 4 supports would normally be statically indeterminate, but two of the support reactions have been given.

∴ $R_C$ + $R_D$ can be found by simple equilibrium

Resolve vertically

$$15 + 90 + R_C + R_D = 60 + 100$$

$$\Rightarrow R_C + R_D = 55 \text{ KN}$$

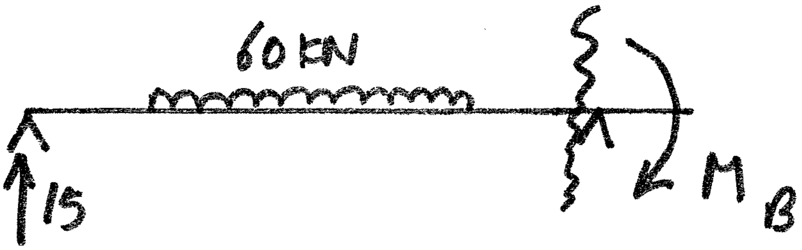
Take moments about D (to eliminate one of the unknowns)

$$15.30 + 90.20 + R_C.10 = 60.25 + 100.15 + 50$$

$\Rightarrow$ $R_C = 80$ KN ↑ $R_D = -25$ KN (downwards)

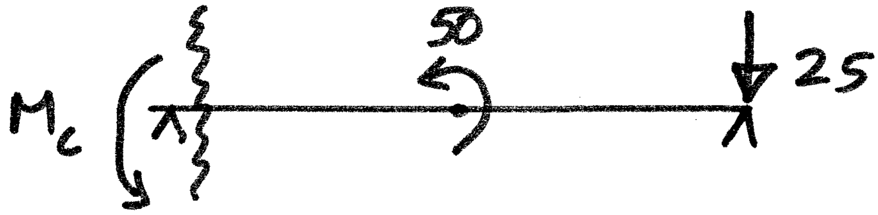
[5 marks]

(b) Take a free body cut at B




$$M_B = 60 \times 5 - 15 \cdot 10 = \underline{\underline{150 \text{ kNm}}}$$

Take a free body cut at C

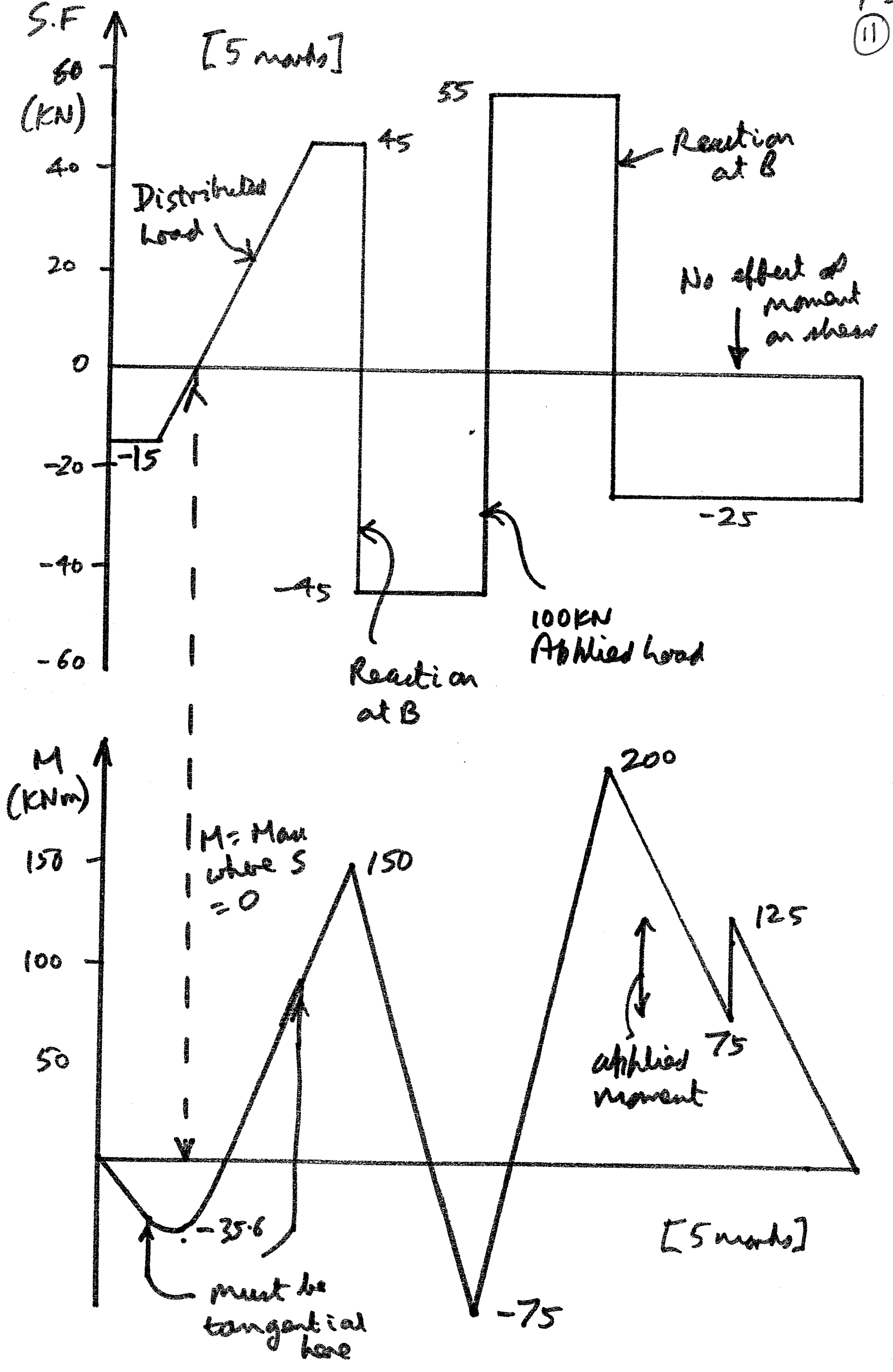



$$M_C = 25 \cdot 10 - 50 = \underline{\underline{200 \text{ kNm}}}$$

[5 marks]

To draw B.M & S.F diagrams, take free body cuts at appropriate places.

Common errors. Making it far too complicated. Not taking free body cuts carefully when constructing M + S diagrams. Question is not difficult but many students tried to do it "by inspection" which does not work!


S.F
(KN)
60
40
20
0
-20
-40
-60
[5 marks]
55
45
Reaction at B
Distributed load
No effect of moment on shear
-15
-25
-45
Reaction at B
100KN Applied load
M
(KNm)
150
100
50
200
M = Max where S = 0
150
125
applied moment
75
-35.6
must be tangential here
-75
[5 marks]

4(a) $I_{timber} = \dfrac{100 \cdot 200^3}{12} = \underline{\underline{0{\cdot}666 \cdot 10^8 \text{ mm}^4}}$

[2 marks]

Area of CFRP $= 100 \text{ mm}^2$

Timber removed and replaced by CFRP

$\therefore$ Effective area of CFRP as timber $= \left(\dfrac{108}{9} - 1\right) \cdot 100$

$= 1100 \text{ mm}^2$

Find height above soffit of new centroid $= \bar{y}$

$$\bar{y} = \frac{100 \cdot 200 \cdot 100 + 1100 \cdot 12{\cdot}5}{100 \cdot 200 + 1100} = \underline{\underline{95{\cdot}4 \text{ mm}}}$$

New $I =$ $0{\cdot}666 \cdot 10^8$ (old value)

$+\ 100 \cdot 200\,(100 - 95{\cdot}4)^2$ (about new axis)

$+\ \dfrac{110 \cdot 5^3}{12}$. (CFRP about own axis – negligible)

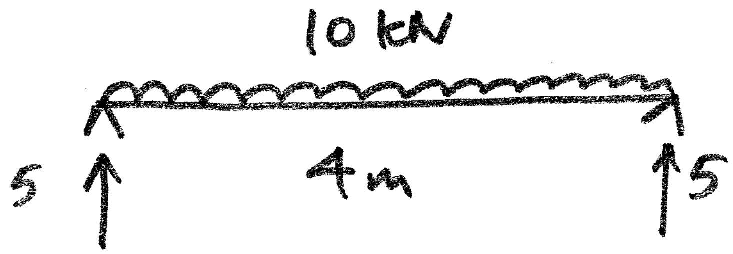
$+\ 1100\,(95{\cdot}4 - 12{\cdot}5)^2$ (CFRP about $\bar{y}$)

$= \underline{\underline{0{\cdot}746 \cdot 10^8 \text{ mm}^4}}$ (timber units)

[6 marks]

This part done well by most students.

(b)




[2 marks]

$$M \text{ at centre} = \frac{10}{4}\,2.1 - 5.2 = -5\,\text{kNm}$$

More than 50% of candidates got the moment wrong!

Maximum bending stress in timber without repair (not asked for but a useful comparison)

$$= \frac{My}{I} = \frac{5.10^6 . 100}{0.666 . 10^8} = 7.5 \text{ N/mm}^2$$

Maximum bending stress in timber after repair ($y = 104.6$ mm)

$$= \frac{5.10^6 . 104.6}{0.746 . 10^8} = 7.01 \text{ N/mm}^2$$

[2 marks]

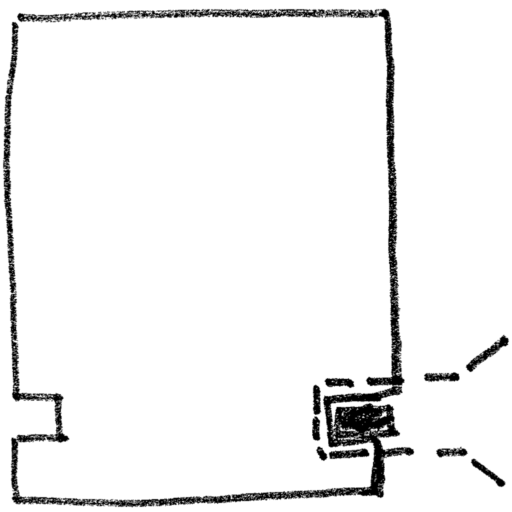
Maximum bending stress in CFRP ($y = 95.4 - 10 = 85.4$ mm)

$$= \frac{5.10^6 . 85.4}{0.746 . 10^8} . \frac{108}{9} = 68.6 \text{ N/mm}^2$$

[2 marks]

Many students left this term out.

(c)



This was not recognised by most students. They took a shear perimeter across whole width of beam

This is the perimeter where shear failure could occur if the CFRP would break away

$$S.F = \frac{10\,kN}{2} \text{ at ends } = \underline{5\,kN}$$

Many students got this wrong!

$$\tau . t = \frac{S(A\bar{y})}{I}$$

$S(A\bar{y})$: $50 \times \frac{108}{9} . (95.4 - 12.5)$ (i.e. in timber units)

$t$: 25 mm $(2 \times 10 + 5)$

$I$: $0.746 . 10^8\ mm^4$ (timber units)

Important that these are both in the same material units

$$\therefore \tau = \frac{5.10^3}{25} . 50 . \frac{108}{9} \frac{(95.4 - 12.5)}{0.746 . 10^8}$$

$$= \underline{\underline{0.133\ N/mm^2}}$$

[8 marks]

Area being considered is $50 . \frac{108}{9} = 600\ mm^2$

because it is the actual CFRP that would break away, not leaving behind a piece of timber.

5(a) Because the misalignment causes an increase in moment due to $Pe$, which then causes an additional deflection $\Delta$ which in turn causes an increased moment. This is the basic cause of buckling.

When dealing with flexural loads, the lever arm is not affected by small deflections of the beam, so there is no vicious circle with moments being made worse.

[4 marks]

(b)


$$M = -EI\frac{d^2v}{dx^2} = P(e+v)$$

so $$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{Pe}{EI}$$ [5 marks]

(c) Solution is $v = A\sin\alpha x + B\cos\alpha x - e$

where $\alpha^2 = P/EI$

Apply boundary conditions

$x=0,\ v=0 \quad \therefore\ 0 = B.1 - e \Rightarrow B = e$

$x = L,\ v = 0 \qquad 0 = A\sin\alpha L + e(\cos\alpha L - 1)$

$$\Rightarrow A = e\frac{(1-\cos\alpha L)}{\sin\alpha L}$$

So solution is

$$\frac{v}{e} = \frac{(1-\cos\alpha L)}{\sin\alpha L}.\sin\alpha x + \cos\alpha x - 1$$

[7 marks]

Many students made no attempt to apply boundary conditions, or invented totally false ones, like $\frac{dv}{dx} = 0$ at $x = 0$!

(d) ½ of Euler load $\therefore P = \frac{\pi^2 EI}{2L^2}$

$\Rightarrow \alpha L = \frac{\pi}{\sqrt{2}} = 2.22$ rads

$\cos\alpha L = -0.604 \qquad \cos\alpha L/2 = 0.445$

$\sin\alpha L = 0.797 \qquad \sin\alpha L/2 = 0.896$

$$\therefore \left(\frac{v}{e}\right)_{\text{mid point}} = \frac{(1+0.604)}{0.797}.0.896 + 0.445 - 1$$

$$= 1.248$$

If $e = \frac{L}{100}$ additional deflection $= 0.01248L$.

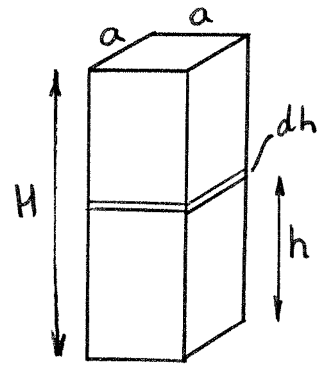
Additional moment at centre [4 marks]

$= P(e+v) = \underline{0.0225\,PL}$

Many students did not try to apply results of (c) to this part and got ridiculous answers.

C J Burgoyne

**ENGINEERING TRIPOS PART IA** **JUNE 2003**
**SECTION B: MATERIALS**




6. (a) $m = \rho V = \rho a^2 (H - h)$ above a typical section

$$\sigma(h) = \frac{mg}{a^2} = \rho g (H - h) \qquad \text{(compression)}$$

$$\text{Local } \varepsilon(h) = \frac{\sigma(h)}{E} = \frac{\rho g}{E}(H - h) \qquad \text{(compressive)}$$

Length change of a typical element of length $dh$ is $\varepsilon\, dh$.

$$\text{Total length change} \quad \Delta H = \int_0^H \varepsilon\, dh = \frac{\rho g}{E}\int_0^H (H - h)\, dh = \frac{\rho g H^2}{2E}$$

$$\text{Fractional change in length} = \frac{\Delta H}{H} = \frac{\rho g H}{2E}$$

(strictly $\dfrac{\Delta H}{H - \Delta H}$ , negligible difference for small $\dfrac{\Delta H}{H}$)

For $\dfrac{\Delta H}{H} = 0.001\%$, $\rho = 2750$ kg/m$^3$, $E = 123\times10^9$ N/m$^2$, $g = 9.81$m$^2$/s

$$\Rightarrow H = 92.7\,\text{m}$$

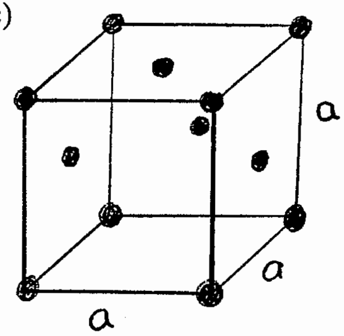
$\sigma_{base} = \rho g H = 1000\,\text{MPa} \quad \Rightarrow H \approx 37$ km , i.e. stone is very strong.
(Note: the strength value in the question is on the high side, and is strictly for small samples of high quality stone; values for bulk masonry could be around 50 times lower, but the conclusion is essentially the same – tall stone structures are not close to their compressive strength, but are limited by bending).

(b) $f = f(EI, l, M_o)$, i.e. assume $f \propto (EI)^{\alpha}\, l^{\beta}\, M_o^{\gamma}$
Dimensions M, L, T: $\mathrm{T}^{-1} \propto (\mathrm{M L^3 T^{-2}})^{\alpha}\, (\mathrm{L})^{\beta}\, (\mathrm{M})^{\gamma}$

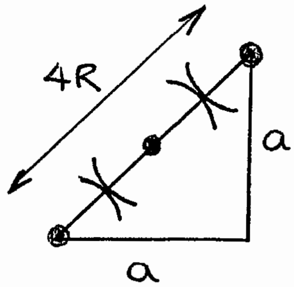
Considering only T: $-1 = -2\alpha$, $\alpha = 1/2$ so $f \propto \sqrt{E}$, $E \propto f^2$

(Not needed, but $\alpha = -\gamma$, $\gamma = -1/2$ and $3\alpha = -\beta$, $\beta = -3/2$; so $(EI) \propto M_o l^3 f^2$

(c)




Atoms touch on diagonals of the faces.




$$2a^2 = (4R)^2$$

$$a = 2\sqrt{2}\,R$$

Cube volume $= a^3 = (2\sqrt{2}\,R)^3$

Number of atoms/cube $= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

Volume of atoms/cube $= 4 \times \frac{4}{3} \pi R^3$

Hence packing factor $= \dfrac{\left(\frac{16\pi}{3}\right) R^3}{(2\sqrt{2})R^3} = 0.74$

Mass of Al atom $= \dfrac{\text{atomic mass}}{\text{Avogadro's constant}} = \dfrac{26.9815}{6.022 \times 10^{26}} = 4.48 \times 10^{-26}$ kg

Hence $\rho = \dfrac{4 \times 4.48 \times 10^{-26}}{(2\sqrt{2} \times 1.432 \times 10^{-10})^3} = 2700$ kg/m$^3$; same as databook value.

<u>Examiner's comments</u>

Popular question, done by most candidates, with above average marks.

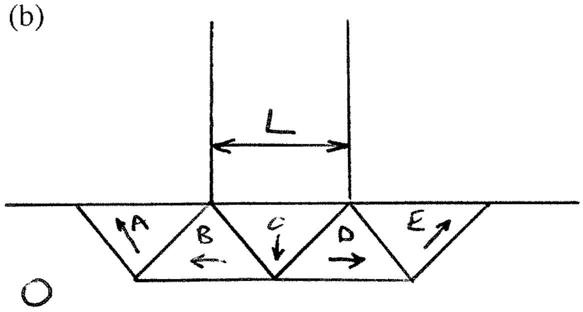
Part (a) was very close to an Examples problem, but many still failed to appreciate that stress and strain varied with depth. The purpose of the question is to recognise that the change in length required integration, but many simply used the stress at the base to find a strain and apply this to the whole length. Significant numbers omitted "g" from their expression for weight.

Part (b) was a simple dimensional analysis problem, perhaps not expected by many on the Materials paper. Many tried to relate E to (f, EI, $l$ and $M_o$) rather than f to (EI, $l$ and $M_o$). More worrying though were: an inability to find the dimensions of E or EI; treating force "F" as a dimension (expressing mass as $FT^2/L$); and turning "m" (metres) directly into dimension "M" (mass).

7. (a) Yield stress is the load per unit area when the limit of elasticity is reached, usually measured under uniaxial loading in a tensile test. Hardness is the load (either in kg or N) applied to an indenter pressed into a flat surface, divided by the projected area of the indentation made. Ductility is the total plastic strain to failure in a tensile test after fracture.

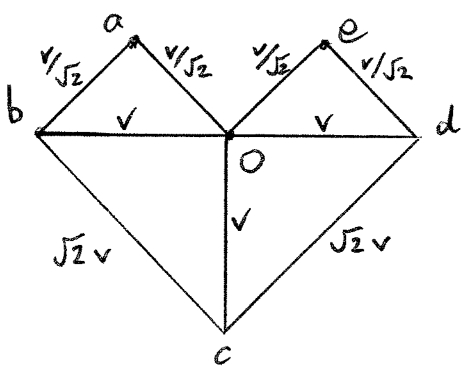
Yield stress and hardness do not depend on specimen dimensions. Ductility includes both the uniform straining prior to necking in a tensile test, and the localised extension in the necked region – hence its value depends on the length and cross-section area of the test piece (i.e. not a material property).

(b)




| INTERFACE | LENGTH |
|---|---|
| OB, OD | $L$ |
| OA, AB, BC, CD, DE, OE | $L/\sqrt{2}$ |

Velocity (or displacement) diagram:




| INTERFACE | LENGTH $L_i$ | RELATIVE VELOCITY $V_i$ | $L_i V_i$ |
|---|---|---|---|
| OA | $L/\sqrt{2}$ | $v/\sqrt{2}$ | $Lv/2$ |
| AB | $L/\sqrt{2}$ | $v/\sqrt{2}$ | $Lv/2$ |
| BC | $L/\sqrt{2}$ | $\sqrt{2}\,v$ | $Lv$ |
| CD | $L/\sqrt{2}$ | $\sqrt{2}\,v$ | $Lv$ |
| DE | $L/\sqrt{2}$ | $v/\sqrt{2}$ | $Lv/2$ |
| OE | $L/\sqrt{2}$ | $v/\sqrt{2}$ | $Lv/2$ |
| OB | $L$ | $v$ | $Lv$ |
| OD | $L$ | $v$ | $Lv$ |

$\sum L_i V_i = 6Lv$

External work rate $= F\,v$

Internal work rate $= 6\,k\,L\,v$

Hence $F = 6\,k\,L = 3\sigma_y\,L$

Hardness $= F/L$ (as unit depth) $= 3\sigma_y$

(c) For all hardening mechanisms, the contribution to strengthening is of the order Gb/l, where G is the shear modulus, b is the Burgers vector, and l is the obstacle spacing. The microstructural parameters controlling obstacle spacing are:

- work hardening: dislocation density
- solid solution hardening: concentration of solute
- precipitation hardening: volume fraction and size of precipitates

Examiner's comments

Very unpopular question (attempted by 40% of candidates), below average marks
Descriptive parts mostly done well, but the upper bound problem proved too difficult – though some students got full marks. In retrospect it would have been better to have provided the hodograph in outline, and asked for this to be annotated and analysed.

8 (a) Tensile failure: worst defect (in terms of size and orientation to tensile stress) governs failure (when $K = K_{IC}$).
Compressive failure: many cracks propagate stably, growing parallel to the applied stress; final failure by crushing and an unstable shear band forming.
Typically compressive strength is 10-15 times greater than tensile strength.

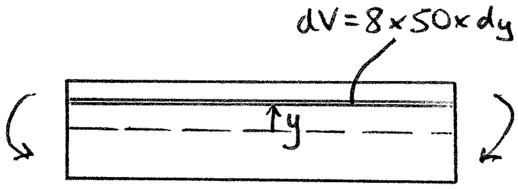
(b) (i) $P_s(V) = exp\left\{-\left(\frac{\sigma}{\sigma_o}\right)^m\left(\frac{V}{V_o}\right)\right\}$ as stress uniform; $\sigma_o$, $V_o$, m: constants

For same probability of failure: $\sigma_1^m\, V_1 = \sigma_2^m\, V_2$

$$\left(\frac{\sigma_t}{500}\right)^m = \left(\frac{V_{cyl}}{V_{sq}}\right) = \frac{30\times\pi\times3^2}{50\times8\times8} \quad \text{and as } m = 8, \quad \sigma_t = 423 \text{ MPa}$$

(Check: larger volume, thus lower failure stress).

(Note: no need to find $\sigma_o$, but if $V_o$ is taken to be the volume of the square section specimen, can solve for $\sigma_o$ by substituting $P_s = 0.5$ into the Weibull equation, with $V = V_o$. This gives $\sigma_o = 523$ MPa, which is then substituted into the equation with the new volume).




(ii) By inspection: $\sigma(y) = \sigma_{max}\frac{y}{4}$

For same failure probability: $\sigma_t^m\, V = \int_V [\sigma(y)]^m\, dV$

$$\sigma_t^m\,(8\times8\times50) = \int_0^4\left[\sigma_{max}\frac{y}{4}\right]^m 8\times50\;dy \qquad \text{(top half only: tensile region)}$$

$$\sigma_t^m\times8 = \sigma_{max}^m\left[\frac{y^{m+1}}{4^m(m+1)}\right]_0^4 = \sigma_{max}^m\frac{4}{(m+1)}$$

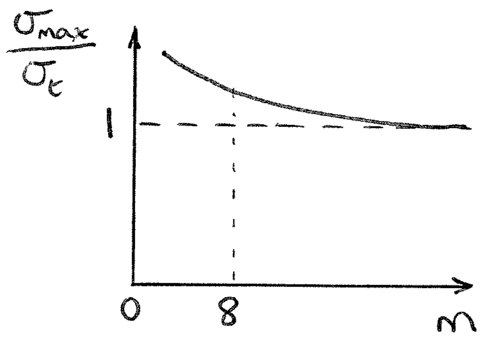
$$\frac{\sigma_{max}}{\sigma_t} = (2(m+1))^{1/m} = 1.435 \qquad \text{hence } \sigma_{max} = 607 \text{ MPa}$$

(iii) Tension locates the worst flaw in the whole volume; bending loads only half in tension, with the stress varying from zero to $\sigma_{max}$, giving a lower probability of large flaws seeing a high tensile stress (or alternatively, requiring a higher $\sigma_{max}$ to give the same probability).

$$\frac{\sigma_{max}}{\sigma_t} = (2(m+1))^{1/m}$$

Hence as m $\rightarrow$ large value, $\frac{\sigma_{max}}{\sigma_t} \rightarrow 1$ (i.e. failure stress is deterministic, like a yield stress).

As m $\rightarrow$ smaller value (e.g. m = 3), $\frac{\sigma_{max}}{\sigma_t} > 1$, i.e. a greater spread in tensile failure stress (low m) means that the difference between bending and tensile strength increases.

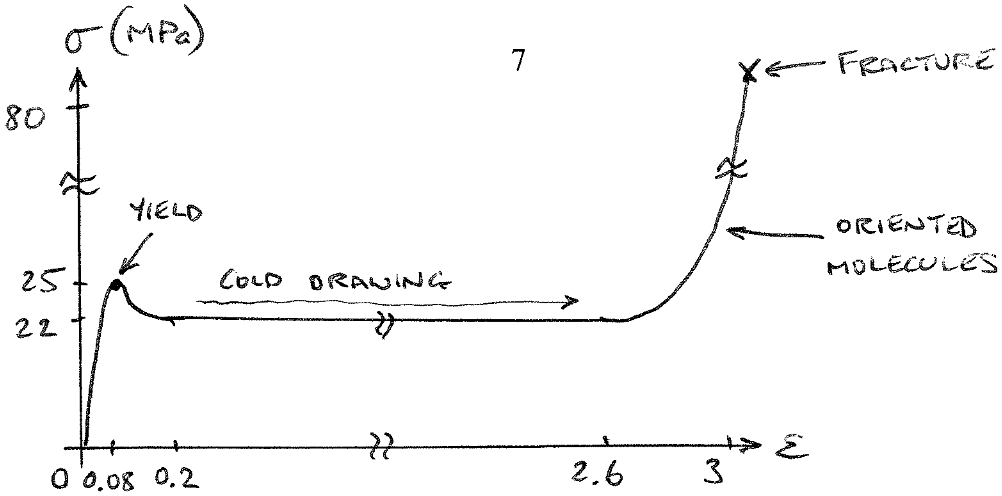



Examiner's comments

Popular question, below average marks.

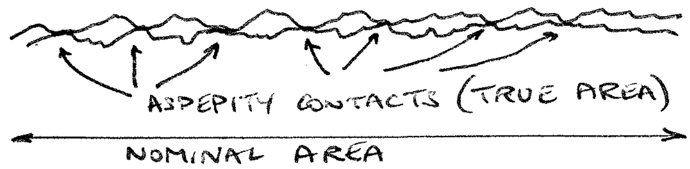
This was the first time Weibull analysis had appeared in the IA exam, having moved from IB the previous year. A large proportion did not understand the meaning of the reference stress $\sigma_o$, taking this to be the value given (for probability ½, rather than 1/e). Very few recognised they didn't need it anyway, and could use simple scaling of stress and volume. The integration in (b,ii) was done badly – many struggled to relate $\sigma(y)$ to $\sigma_{max}$, thinking they needed to know the moment, and there were few complete correct integrals. This probably reflected removal of the (harder) 3-point bending problem from the Examples, replacing it with a rather obscure 1D problem.

9 (a)




(b) Add fibres (e.g. glass) or particles (glass, silica, rubber) which either promote multiple cracking, or which bridge cracks, both leading to greater energy dissipation as the material fractures.

(c) (i) Nominal area: projected area of component over which contact is made
True area: sum of area of microscopic contacts at tips of surface asperities




True contact area: $\propto$ load, and $\propto$ 1/Hardness.

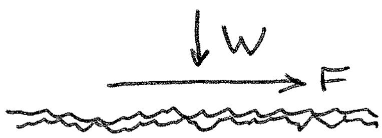
At each contact, assume load/area $\approx \sigma_y$, $a_{true} \approx \dfrac{W}{\sigma_y} \approx \dfrac{3W}{H}$

(Alternatively, treat each contact like a hardness indentation, so $a_{true} \approx \dfrac{W}{H}$)

True contact area independent of nominal contact area (for small fractions, typical of metals).

For elastomers, the low modulus allows the surfaces to conform elastically, so true area approaches nominal area and contacts do not yield (so doesn't depend on hardness).

(ii)




True contact area carries both F and W.

Normal stress $= \dfrac{W}{a} \approx \sigma_y$; shear stress $= \dfrac{F}{a} \approx k$ (for dry sticking contact)

As $k = \sigma_y / 2$, $\dfrac{F}{W} = \mu \approx \dfrac{\sigma_y / 2}{\sigma_y} \approx 0.5$

(Alternatively, if normal stress $= \dfrac{W}{a} \approx$ hardness, $3\sigma_y$, then $\mu \approx 1/6$).

| (d) Aqueous corrosion examples: (only 2 required) | Prevention: |
|---|---|
| Rusting of iron due to exposure to water + oxygen in the atmosphere (e.g. bicycles) | Paint (or other coatings) |
| Rusting of steel sheet (corrugated roof, car body) | Galvanise with Zn, which corrodes preferentially |
| Corrosion of steel ships, pipes | Galvanic protection – attach sacrificial anode which is more electronegative (e.g. Mg) |
| Rusting in central heating system | Use closed system, so oxygen is used up; repair leaks to prevent fresh oxygen supply. |

Examiner's comments

Popular question, with above average marks.
Answers to (a,d) on polymers and corrosion were excellent, and the discussion of friction was OK apart from the estimate of $\mu$, which was full of attempts to equate forces and stresses.
Part (b) on polymers produced greatest confusion – lots of detailed explanations of how to toughen a polymer, using the techniques applied to strengthen metals (such as work hardening to raise the dislocation density). Many thought it a good idea to raise the temperature – not the most practical solution in a design context.

10. (a) Mass, $m \propto L^a\,(\delta/F)^b\,\rho^c\,E^d$ with $a = 5/2$

Dimensions M, L, T: $M^1 \propto L^{5/2}\,(M^{-1}T^2)^b\,(ML^{-3})^c\,(ML^{-1}T^{-2})^d$

M: $1 = -b + c + d$ (1)
L: $0 = 5/2 - 3c - d$ (2)
T: $0 = 2b - 2d$ (3)

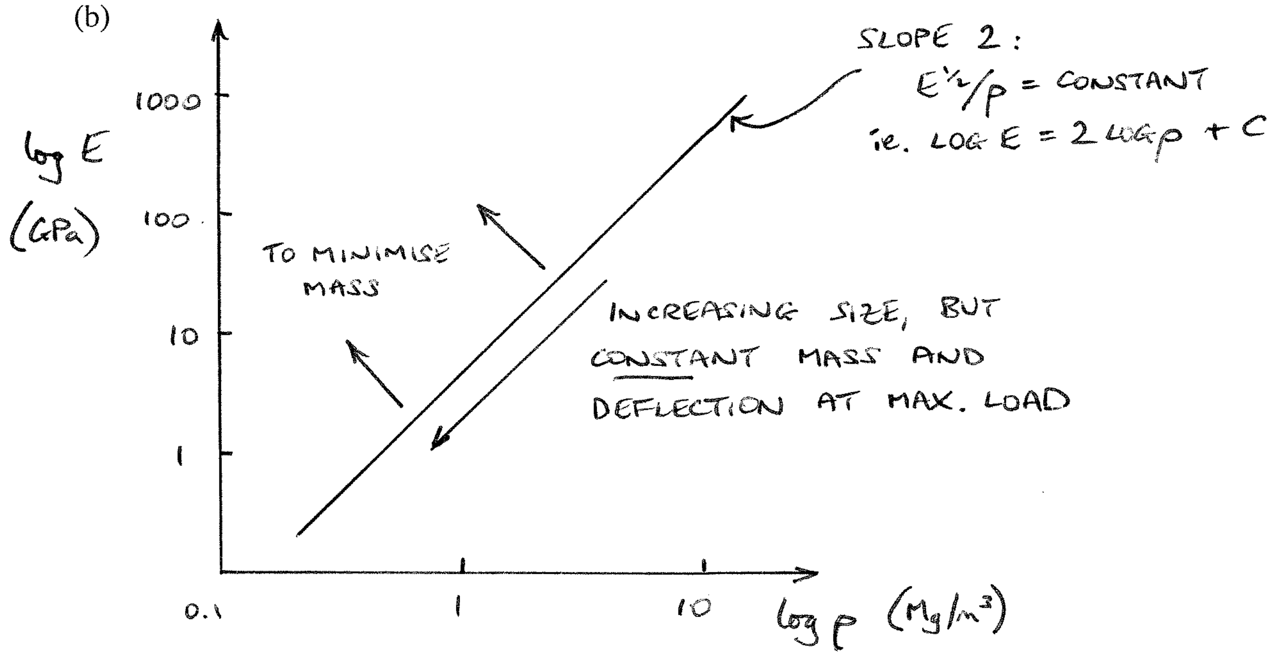
Hence: $b = d$ (from 3); $c = 1$ (from 1); $d = -1/2$ (from 2); and $b = -1/2$

Hence: $m \propto L^{5/2}\,(\delta/F)^{-1/2}\,\rho\,E^{-1/2}$

For fixed L, (δ/F): minimum mass $\Rightarrow$ maximise $E^{1/2}/\rho$

Section size is not an independent variable – given (δ/F) and L, the modulus E will fix the section size (via EI). Conversely, size is the free variable which enable the stiffness constraint to be met as the material varies.

(b)




(c) To avoid failure while minimising mass, derive a second performance index – either by dimensional analysis (for which the information given in insufficient) or from objective and constraint:

Objective: $m = \rho A L$

Constraint: $\sigma_{max} = \sigma_f$ (failure stress of material), with $\sigma_{max}$ depending on section size (through $I/y_{max}$).

Eliminate free variable (size) to find merit index for minimum mass which doesn't fail.

To find whether stiffness or strength is limiting for each material:

- find *actual* mass required to meet each constraint
- take the heavier of the two for each material, i.e. to just meet the more stringent of the two
- optimum material has the lowest of these heavier masses.

This requires that the design parameters are defined numerically (not just specified as constant), i.e. length, load and allowable deflection. Also need the bending mode to give the numerical constants in the bending formulae for deflection and stress (e.g. cantilever, 3-point bending etc).

(Notes: it is meaningless to directly compare the values of the 2 merit indices for a given material, since though each is proportional to mass, it is a different mass in each case – one which provides adequate stiffness, the other which is strong enough.
Another misconception is that the materials can be ranked on the first constraint, and then the maximum stress calculated from the top down until a material is found which is below its failure stress. This procedure does *not* find the lightest – for example, if *all* the materials are strength-limited, the first to come through on the basis of low mass-for-stiffness need not be the lightest in relation to strength.)

Examiner's comments

Moderately popular question (attempted by 70% of candidates), below average marks.
Part (a) threw up similar trouble with dimensional analysis as Q.6, though this problem was similar to an Examples question. Simple errors in algebra led many astray, e.g. when gathering terms on dimensions, $(1/T^{-2})^b$ came through as –2b.
In part (b), many candidates had clearly not appreciated that a given merit index line of constant value corresponded to equal mass and equal stiffness, with size varying – which is the whole point of the analysis. Some interesting merit indices such as $E^{90}/\rho^{19}$ emerged, and the student ploughed on doggedly to plot this on a selection chart. Descriptions of how to extend the analysis to include a second constraint were poor – clearly people prefer to just do the sums, rather than to outline the steps involved. Many thought the second constraint set a limit on a new property, rather than the property for each material setting a new requirement on section size (to avoid failure). One character suggested using the selection software, so you could "hand all the decisions over to a computer". It is to be hoped they find another career rather than Engineering.

H.R. Shercliff
June 2003