2003 Paper 3 Question 1 A/B

(a) Output voltage = $A(V_+ - V_-)$, independent of frequency (infinite bandwidth). Op-amp is otherwise ideal, so infinite input impedance and zero output impedance.

(b) By potential divider, $V_- = \left(\frac{R_2}{R_1+R_2}\right) V_2$.

So $V_2 = A(V_+ - V_-) = A\left(V_1 - \frac{V_2 R_2}{R_1+R_2}\right)$

$$\Leftrightarrow \quad AV_1 = V_2\left(1 + \frac{AR_2}{R_1+R_2}\right) = V_2\left(\frac{R_1+R_2+AR_2}{R_1+R_2}\right)$$

Hence $\frac{V_2}{V_1} = \frac{A(R_1+R_2)}{R_1+R_2+AR_2}$. Values $\Rightarrow \frac{V_2}{V_1} = 99.0$

(c) Standard inverting amplifier, $\frac{V_4}{V_3} = -\frac{R_4}{R_3} = -10.$

(d) Replace R_4 with $R_4 \| C_4 = \frac{R_4}{1+j\omega C_4 R_4}$

Hence $\frac{V_4}{V_3} = -\frac{R_4}{R_3(1+j\omega C_4 R_4)}$

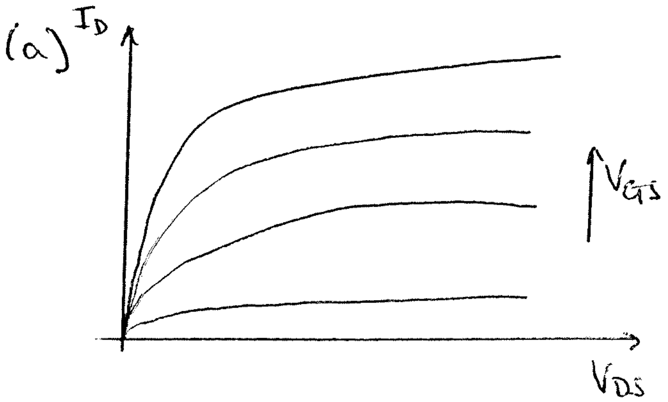
Values @ 10Hz $\Rightarrow \frac{V_4}{V_3} = 10\angle 180°$

@ 1.6MHz $\Rightarrow \frac{V_4}{V_3} = 7.05\angle 135°$ (ie 3dB point)

(e) Fig 1(a) has a very high input impedance, Fig 1(b) has input impedance $\approx R_3$ (ie easily controllable).
Fig 1(a) has a phase change of 0°.
Fig 1(b) has a phase change of 180° at midband, 135° at 3dB and 90° as $\omega \to \infty$.

2003 Paper 3 Question 2

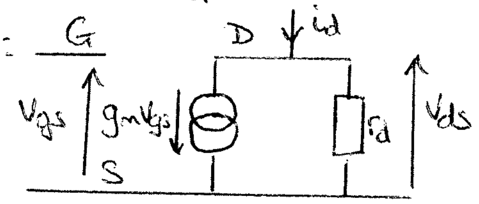
(a)



From the FET's output characteristic, we can estimate $\frac{\partial I_D}{\partial V_{DS}}$ (slope of the lines in the linear region) and $\frac{\partial I_D}{\partial V_{GS}}$ (spacing of the lines in the linear region). For variations, simple mathematics tells us that

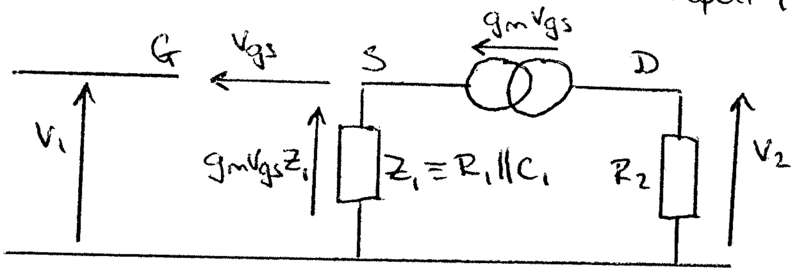
$$\Delta I_D = \frac{\partial I_D}{\partial V_{DS}} \Delta V_{DS} + \frac{\partial I_D}{\partial V_{GS}} \Delta V_{GS}.$$

Defining $\frac{1}{r_d} \equiv \frac{\partial I_D}{\partial V_{DS}}$, $g_m \equiv \frac{\partial I_D}{\partial V_{GS}}$, $i_d = \Delta I_D$, $V_{ds} = \Delta V_{DS}$, $V_{gs} = \Delta V_{GS}$, we get $i_d = \frac{V_{ds}}{r_d} + g_m V_{gs}$. This describes the following equivalent circuit:



The use of the equivalent circuit greatly facilitates the small signal analysis of FET circuits (gain, input/output impedance etc...)

(b)



(c) (i) From diagram, $V_2 = -g_m V_{gs} R_2$ and $V_1 = g_m V_{gs} Z_1 + V_{gs}$
$= V_{gs}(1 + g_m Z_1)$

Eliminating V_{gs} between the two equations gives

$$V_1 = -\frac{V_2}{g_m R_2}(1 + g_m Z_1), \text{ so } \frac{V_2}{V_1} = \frac{-g_m R_2}{1 + g_m Z_1}$$

$$\text{So } \frac{V_2}{V_1} = \frac{-g_m R_2}{1 + \frac{g_m R_1}{1 + j\omega C_1 R_1}} = \frac{-g_m R_2 (1 + j\omega C_1 R_1)}{1 + g_m R_1 + j\omega C_1 R_1}$$

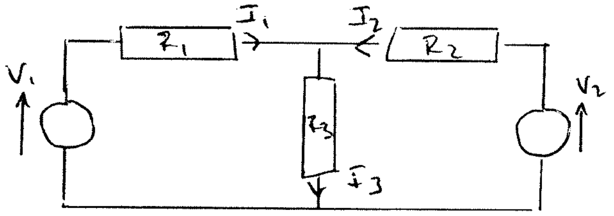
(ii) As $\omega \to \infty$, $\frac{V_2}{V_1} \to \frac{-g_m R_2 j\omega C_1 R_1}{j\omega C_1 R_1} = -g_m R_2$.

Values $\Rightarrow$ $\frac{V_2}{V_1} \to -5 \times 10^{-3} \times 15 \times 10^{3} = -75$.

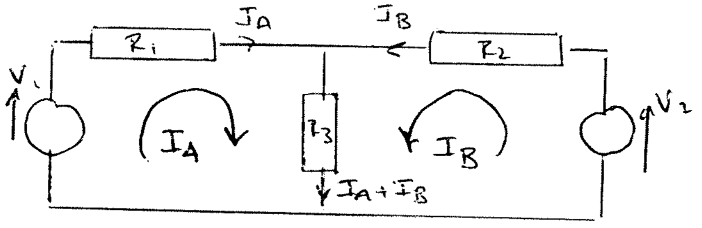
So $V_2 \approx -75 V_1$ for high ω.

2003 Paper 3 Question 3

(a) Mesh/loop current analysis refers to the application of Kirchoff's voltage law to solve for unknown currents in a circuit. Some writers like to distinguish between mesh and loop analysis. In mesh analysis, the unknown currents flow through the individual circuit elements, and Kirchoff's current law needs to be applied to enforce current conservation. In loop analysis, the unknown currents flow around each circuit loop, and the currents through the individual elements are appropriate sums of these loop currents. The number of equations and unknowns is equal to the number of independent circuit loops.

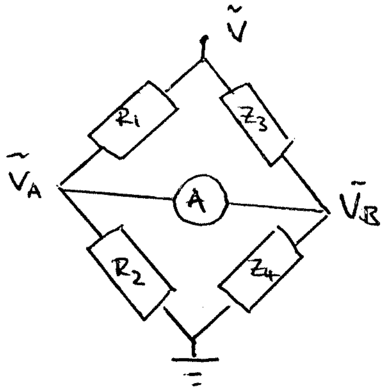


Mesh analysis.
KCL $\Rightarrow I_3 = I_1 + I_2$
Then solve KVL ×2



Loop analysis
Solve KVL ×2.

(b)



$$\bar{V}_A = \frac{R_2 \bar{V}}{R_1 + R_2} \qquad \bar{V}_B = \frac{Z_4 \bar{V}}{Z_3 + Z_4}$$

For balance, $\bar{V}_A = \bar{V}_B \Rightarrow \dfrac{R_2}{R_1 + R_2} = \dfrac{Z_4}{Z_3 + Z_4}$

$$\Leftrightarrow R_2 Z_3 + R_2 Z_4 = R_1 Z_4 + R_2 Z_4$$

$$\Leftrightarrow \underline{R_2 Z_3 = R_1 Z_4}$$

(c) Now $Z_4 = \dfrac{R_4}{1 + j\omega C_4 R_4}$ and $Z_3 = R_3 - j/\omega C_3$

Substitute $\Rightarrow R_2 R_3 - \dfrac{jR_2}{\omega C_3} = \dfrac{R_1 R_4}{1 + j\omega C_4 R_4}$

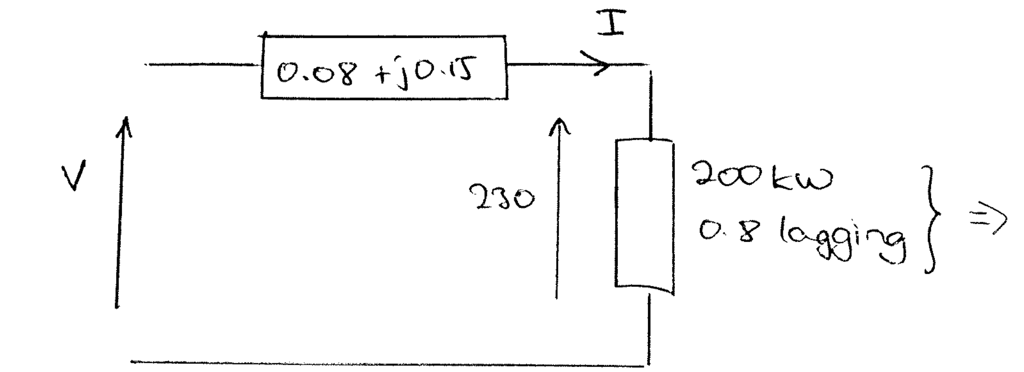
$\times\, \omega C_3 (1 + j\omega C_4 R_4) \Rightarrow R_2 R_3 \omega C_3 (1 + j\omega C_4 R_4) - jR_2(1 + j\omega C_4 R_4) = R_1 R_4 \omega C_3$

$$\Leftrightarrow R_2 R_3 \omega C_3 + R_2 \omega C_4 R_4 + jR_2(\omega^2 C_4 R_4 R_3 C_3 - 1) = R_1 R_4 \omega C_3$$

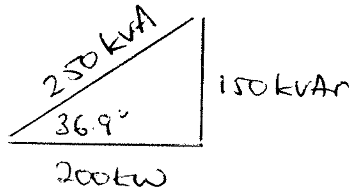
Real part tells us nothing about ω (it cancels).

Imaginary part $\Rightarrow \underline{\omega^2 = \dfrac{1}{C_3 C_4 R_3 R_4}}$

2003 Paper 3 Question 4



Power triangle for factory



(a) (i) $230 \times I = 250 \times 10^3$ VA $\Rightarrow$ $\underline{I = 1.09 \text{ kA}}$

(ii) $P_{line} = I^2 \times 0.08 = \underline{94.5 \text{ kW}}$

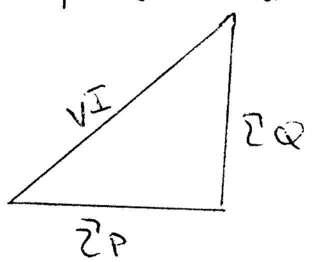
(iii) $Q_{line} = I^2 \times 0.15 = 177$ kVAr.

$\sum P = 200 + 94.5 = 294.5$ kW

$\sum Q = 150 + 177 = 327$ kVAr

So $V = \dfrac{\sqrt{(\sum P)^2 + (\sum Q)^2}}{I} = \underline{405 \text{ Volts}}$

Power triangle for power station



(b) (i) Capacitor must absorb 150 kVAr at 230 Volts, so

$V^2 \omega C = 150 \times 10^3$ $\Rightarrow$ $230^2 \times 2\pi \times 50 \times C = 150 \times 10^3$

$\Leftrightarrow$ $\underline{C = 9.03 \text{ mF}}$

(ii) Assuming factory voltage is to remain at 230 Volts, we now have no VArs, so $230 \times I = 200 \times 10^3$ Watts

$\Leftrightarrow$ $\underline{I = 870 \text{ A}}$

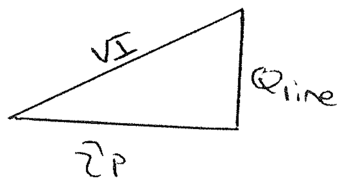
(iii) $P_{line} = I^2 \times 0.08 = \underline{60.5 \text{ kW}}$

(iv) $Q_{line} = I^2 \times 0.15 = 113$ kVAr

$\sum P = 200 + 60.5 = 260.5$ kW

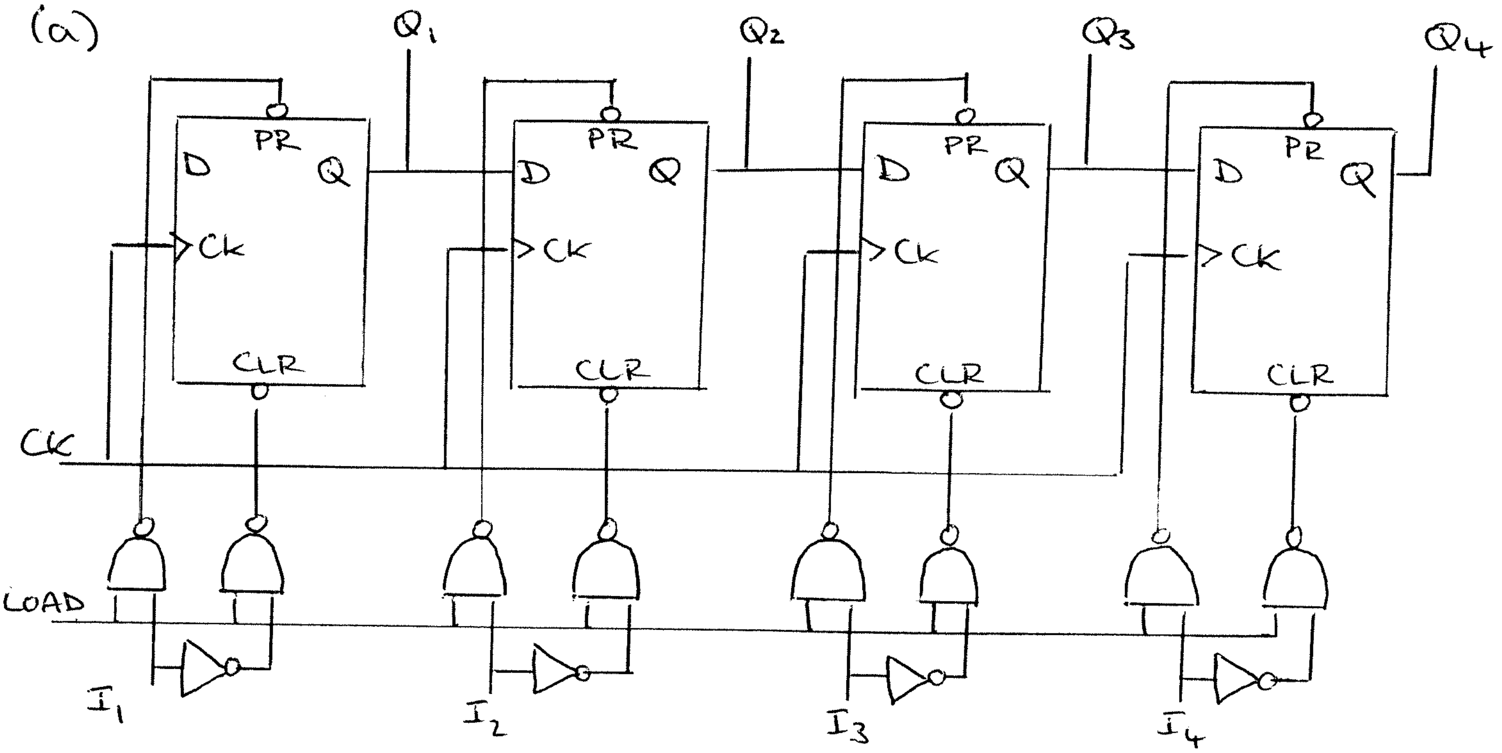
So $V = \dfrac{\sqrt{(\sum P)^2 + Q_{line}^2}}{I} = \underline{327 \text{ Volts}}$

Power triangle for power station



2003 Paper 3 Question 5

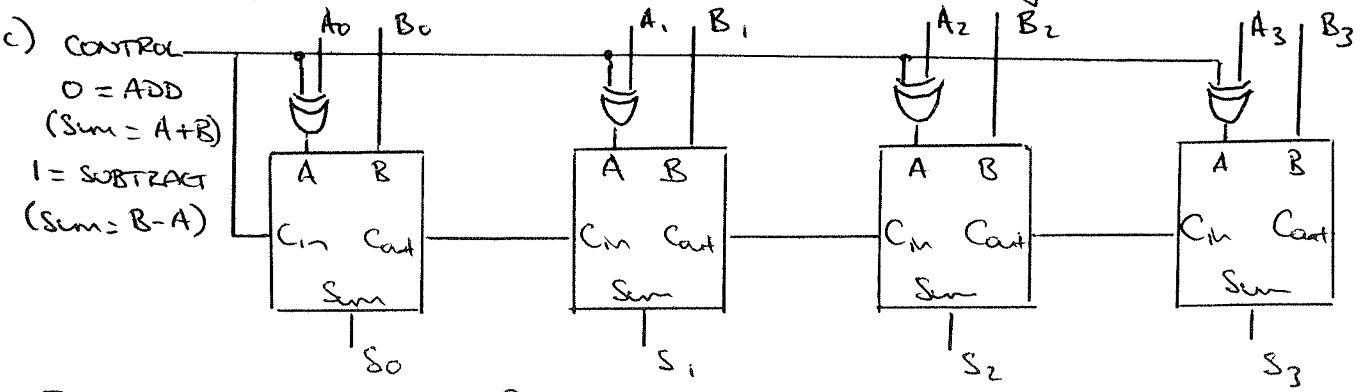
(a)



When the LOAD line goes high, the data on the parallel inputs $I_1 \ldots I_4$ is loaded into the four D-type flip flops. On the rising edge of each clock (Ck) pulse, the data is shifted one place to the right, so it all eventually appears, serially, at output Q_4. The circuit therefore functions as a parallel to serial converter.

(b) (i) 2's complement is a technique for using about half of the n-bit bit patterns to represent positive numbers, and half to represent negative numbers. Numbers with a 1 in the most significant bit are negative, the rest are zero or positive. To convert between 2's complement positive and negative forms, change 1's to 0's and 0's to 1's, then add 1. It is an attractive scheme because the same logic can be used for adding/subtracting 2's complement numbers as for adding unsigned numbers, the distinction being made entirely through how the bit patterns are interpreted. (ii) 8-bit 2's complement range is −128 to +127.

(c)



Each box contains a full-adder, whose logic can be derived using truth tables and karnaugh maps in the usual way.

2003 Paper 3 Question 6

(a) E = emergency stop, $S_1 \ldots S_3$ = sensors, M = "keep line moving"

E	S_1	S_2	S_3	M
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

(b) Karnaugh map

ES_1 \ S_2S_3	00	01	11	10
00	1	1	0	1
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

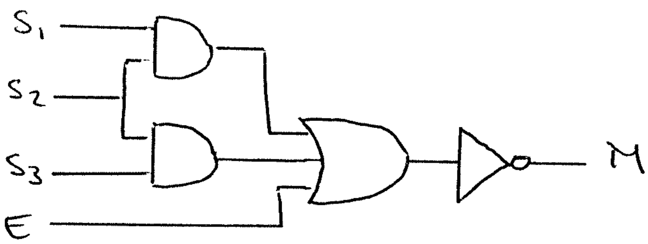
From 1's on karnaugh map (solid lines)

$M = \bar{E}\bar{S_2} + \bar{E}\bar{S_1}\bar{S_3} = \bar{E}\cdot(\bar{S_2} + \bar{S_1}\bar{S_3})$

Requires 4 inverters, one AND, one OR = 6 gates.

Try 0's on karnaugh map (dotted lines) for possibly fewer gates:

$\bar{M} = E + S_1S_2 + S_2S_3$, so $M = \overline{E + S_1S_2 + S_2S_3}$. Requires 2 AND, one OR and one inverter = 4 gates.



(c)(i) $M = \bar{E}\bar{S_2} + \bar{E}\bar{S_1}\bar{S_3} = \overline{\overline{\bar{E}\bar{S_2}} \cdot \overline{\bar{E}\bar{S_1}\bar{S_3}}}$ – ie. 3 NAND gates

(ii) $\bar{M} = E + S_1S_2 + S_2S_3 \iff M = \overline{E + \overline{\bar{S_1} + \bar{S_2}} + \overline{\bar{S_2} + \bar{S_3}}}$

– ie. 3 NOR gates

(d) NAND solution incurs two propagation delays ⇒ 16ns

NOR solution incurs two propagation delays ⇒ 20ns

So NAND solution is faster by 4ns.

2003 Paper 3 Question 7

(a) The input and output devices are presented to the microprocessor as if they were memory locations. To input data, the microprocessor performs a read operation. To output data, the microprocessor performs a write operation.

(b) Program counter – a 16-bit register that stores the memory address of the next program instruction to be executed. Incremented automatically after fetching each instruction, and also updated by jump and branch operations.

Index register – a general purpose 16-bit register, which can be used to store addresses in the same way that the accumulators store data. Particularly useful for manipulating arrays of data via "indexed" addressing.

(c)

INSTRUCTION	ADDRESSING MODE	CLOCK CYCLES (μs)
LDAA #00	Immediate	2
LDAB $50	Direct	3
CMPB $51	Direct	3
BEQ 01	Relative	4
COMA	Implied	2
STAA $E001	Extended	5

(d) (i) Equal ⇒ COMA not executed ⇒ 17 μs

(ii) Not equal ⇒ COMA executed ⇒ 19 μs

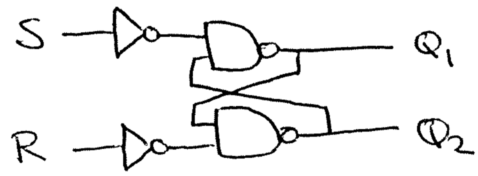
(e)

	LDAA #10	2 clock cycles	
	NOP	2	
loop	NOP	2	} $8 \times 10 = 80$
	DECA	2	
	BNE loop	4	

Total = 84 clock cycles

2003 Paper 3 Question 8

(a)

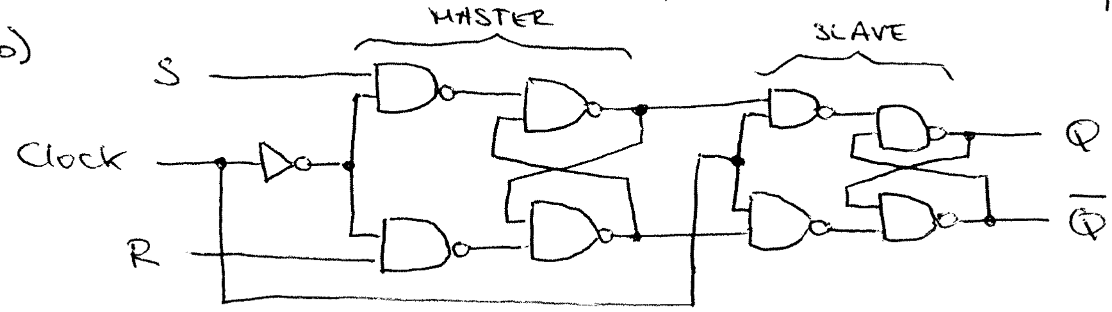


$$Q_1 = \overline{\bar{S}\,Q_2} = S + \overline{Q_2}$$

$$Q_2 = \overline{\bar{R}\,Q_1} = R + \overline{Q_1}$$

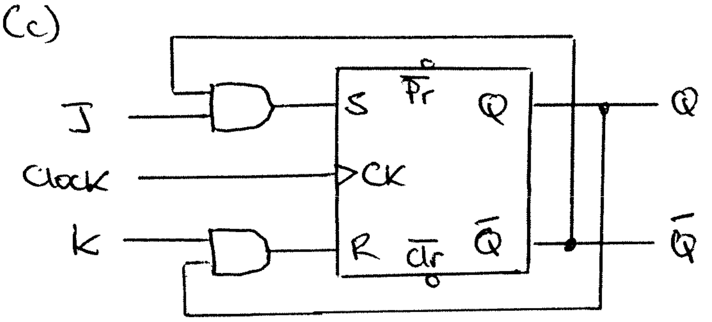
For useful operation, avoid S=R=1. S=1, R=0 turns on Q_1, off Q_2. S=0, R=1 turns on Q_2, off Q_1. S=R=0 leaves outputs as they are.

(b)



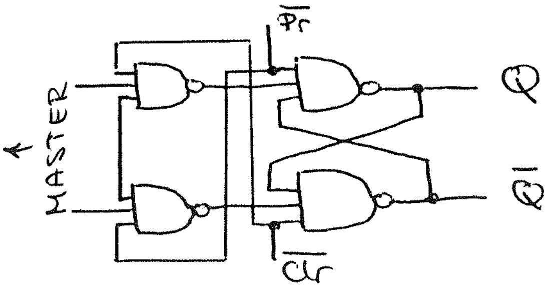
When the clock is low, the master bistable is sensitive to the S/R inputs, while the slave outputs remain as they are. As the clock rises, the state of the master bistable is transferred to the slave. It is still necessary to avoid S=R=1.
Using master-slave bistables, it is possible to build synchronous systems in which all the outputs change on the edge of the clock. Without this synchronicity, there is a high risk of race hazards.

(c)



Inside the box is a master-slave bistable, as in (b) above.
Unlike the S-R master-slave bistable, the JK flip flop has a role for the J=K=1 input: the output changes state each time the clock rises.

The preset ($\overline{Pr}$) and clear ($\overline{Clr}$) inputs are active low. They are wired into the slave bistable as follows.

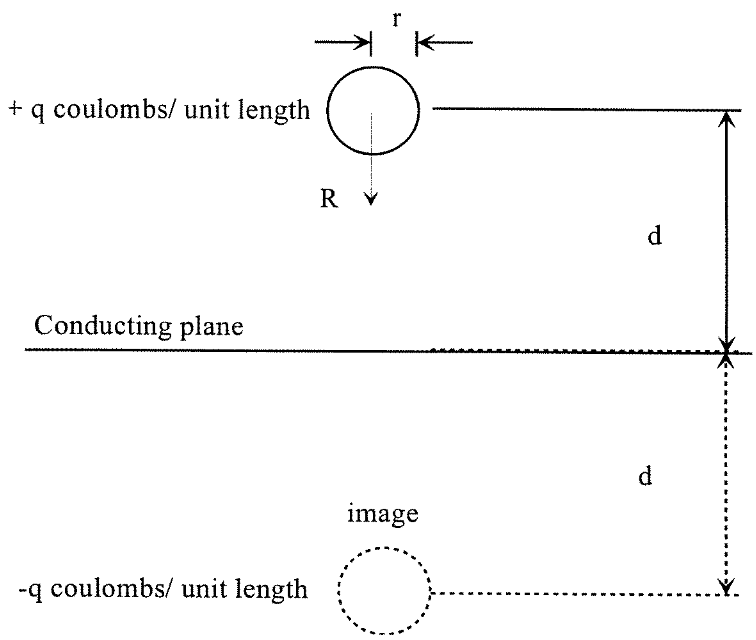


The preset and clear inputs act immediately - the clock is not relevant to their operation.

Part IA --- Paper 3 – Electrical and Information Engineering

Section C – Exam crib

9 a)



Use Method of Images

Gauss's Law gives us a general expression for the electric field at distance R from a wire as follows:

$$\int_c D.ds = Q$$

$$\Rightarrow 2\pi RDl = ql$$

$$D = \varepsilon_0 E$$

$$\Rightarrow E = \frac{q}{2\pi R\varepsilon_0}$$

Where l is the length into the page. We are calculating in free space hence permittivity is ε_0

The coordinate system assumes that R is zero at the centre of the wire

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Therefore for the conductor and its image we get:

$$\Rightarrow E = \frac{q}{2\pi R\varepsilon_0} + \frac{q}{2\pi(2d-R)\varepsilon_0} = -\frac{dV}{dR}$$

Integrate between the edge of the wire and the conducting plane to get the potential difference between the wire and the conducting plane.

$$V = -\frac{q}{2\pi\varepsilon_0}\int_d^r \left(\frac{1}{R} + \frac{1}{2d-R}\right) dR$$

$$= \frac{q}{2\pi\varepsilon_0} \ln\frac{(2d-r)}{r}$$

Since Q=CV we have

$$C = {}^{2\pi\varepsilon_0}\Big/_{\ln\frac{(2d-r)}{r}} \quad \text{Farads/per unit length}$$

b) Amperes's law states:

$$\int \underline{H}.d\underline{L} = I$$

Evaluate this for a loop around the wire of radius R:

$$\Rightarrow 2\pi RH = I$$

Substitute in for H using:

$$B = \mu_0 H$$

Then

$$B = \frac{\mu_0 I}{2\pi R}$$

c) The second wire should be placed in exactly the same position as the image wire used to solve part a). To achieve field cancellation both wires should have the same current in the same direction. The total field is given by:

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$$B(R) = \frac{\mu_0 I}{2\pi R} - \frac{\mu_0 I}{2\pi(2d - R)}$$

Its a lso w orth remembering that almost total field cancellation can be achieved by having a second wire carrying a current in the opposite direction and placed as close as possible to the first. This is the principle behind twisted pair.

d) force is given by F = BIL and is **attractive**. Note the B is the field at one wire due to the other NOT the total field: i.e.

$$F = \frac{\mu_0 I}{2\pi R} * I * L \text{ where R = 2d}$$

$$\Rightarrow F = \frac{\mu_0 I^2}{4\pi d} \text{ N/unit length}$$

10.

a) Ampere's law

$$\int \underline{H}.d\underline{L} = NI$$

NI = 2 x 0.01 H + ~~iron~~

The iron can be ignored since it has a relative permeability of 10^4 and therefore adds little to the overall reluctance of the magnetic circuit. This is the key assumption and is frequently used in the analysis of magnetic circuits. It is not necessary (or correct) to assume that the cross-sectional area of the car roof is the same as the magnet. It is also assumed that the iron does not saturate i.e. that the permeability remains at 10^4 for the whole of the problem.

$$B = \mu_0 H = \frac{\mu_0 NI}{0.02}$$

$$B = \frac{2000 * I * 4\pi * 10^{-7}}{0.02}$$

$$= 0.126 \text{ Tesla/Amp}$$

b) Magnetic Energy / unit volume =

$$\int B.dH = \frac{BH}{2} = \frac{B^2}{2\mu_0}$$

Hence by considering the change in energy when the car moves by a small distance dx and equating that to the work done, we get

$$Fdx = \frac{B^2}{2\mu_0} * A * dx$$

and

$$\Rightarrow F = \frac{B^2 A}{2\mu_0} = mg$$ where m is the mass of the car. Note A is the total air gap area i.e. both of the air gaps added together.

$$\Rightarrow I = \frac{1}{0.126}\sqrt{\frac{2\mu_0 mg}{A}} = \frac{1}{0.126}\sqrt{\frac{2*4\pi*10^{-7}*1200*9.81}{2*\pi*0.05^2}}$$

I= 10.98 Amps

(note B = 1.37 Tesla which is sensible)

c) The field applied is 1% greater than that required to lift it. Hence we have an energy balance as follows:

change in magnetostatic energy = change in potential energy + change in kinetic energy.

$$\frac{(B')^2}{2\mu_0} * A * \Delta h = mg\Delta h + \frac{1}{2}mv^2$$

But in part b) we have

$$\frac{B^2}{2\mu_0} * A = mg$$

$$B' = 1.01 * B$$

$$\Rightarrow \frac{(B')^2}{2\mu_0} * A * \Delta h = 1.01^2 * mg\Delta h$$

and therefore

$$(1.01^2 - 1)mg\Delta h = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\left(2*0.01*9.81*(1.01^2 - 1)\right)} = 0.063ms^{-1}$$

d) Saturation leads to less flux per amp. In other words more current will be needed to lift the car in part b)

11.

a) Ampere's law

$$\int \underline{H}.d\underline{L} = NI$$

$$\Rightarrow 2\pi RH = NI$$

Where R is the mean radius of the toroid

maximum H before losses = 50 A/m

Hence $I = \dfrac{2\pi * {(6+3.5)}/{2} * 10^{-2} * 50}{100}$

$$= 0.149A$$

Note some students assume that N is the total number of windings i.e. primary plus secondary. This is incorrect.

$$e = -N\frac{d\phi}{dt} = -N * 2\pi f * A * (2\hat{B})$$

Where A is the cross-sectional area of the toroid we are assuming a sinousoidal variation of current and we use 2 B peak because we are assuming that the driving current is a.c and therefore we are going from –H to +H .

$$e = -100 * 2\pi 100 * \left(1.25 * 10^{-2}\right)^2 * \pi * (2 * 1.6)$$

$$= 98.7V$$

Since this represents the whole cycle (i.e. –peak to plus peak) , the peak voltage is actually half

i.e 49.3 V peak

b) Loss per unit volume

$= \int BdH$ = area under BH curve = 100*3.2=320 J/m^3/cycle

Volume of iron=

$$2\pi * 4.75 * 10^{-2} * \pi * \left(1.25 * 10^{-2}\right)^2 = 1.46 * 10^{-4} m^3$$

therefore loss per cycle = 0.0468 J/cycle

(At 100 Hz this is equal to 4.68 Watts, note some students forgot to multiply by the volume of iron and ended up with iron losses in the order of kilowatts)

c) The voltage on the secondary is also given by:

$e = -N\dfrac{d\phi}{dt}$ but the secondary has only 50 turns (the primary has 100) hence the peak voltage is

= 49.3 / 2 V

Power to the speakers is therefore

$$= \frac{24.7^2}{8*2}$$ (the factor of 2 on the bottom gives us an rms value)

= 38 Watt

Hence we are providing 38 Watt of useful power and losing < 5 Watt in magnetic losses. Therefore the efficency is:

38/ (38 + 5) = 88%

T. A. Coombs