ENGINEERING TRIPOS PART IA

Monday 9 June 2003 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer not more than **eight** questions, of which not more than **four** may be taken from section A and not more than **four** from section B.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

*Answer not more than **four** questions from this section.*

1 A tank is connected to a large reservoir of water through a pipe partly containing trapped air as shown in Fig. 1. The level of water in the reservoir remains constant. The end of the pipe is situated 0.1 m below the water level in the reservoir. All levels are measured with reference to the level in the reservoir. Initially, the level of water in the tank is H = 0.1 m and the levels in the pipe are respectively h_1 = 0.2 m and h_2 = 0.1 m. Pressure is atmospheric above the water in the tank and in the reservoir. The density of air may be neglected compared to that of water. Except in part (d), air compressibility may also be neglected.

(a) Show that this initial configuration is in equilibrium and find the pressure in the air trapped in the pipe. [20%]

(b) Water is slowly poured into the tank. For which value of H will air bubbles start appearing at the surface of the reservoir? [30%]

(c) For which value of H will water start to flow continuously from the tank to the reservoir? [25%]

(d) Determine the change to the answer to (b) when air compressibility is taken into account. [25%]

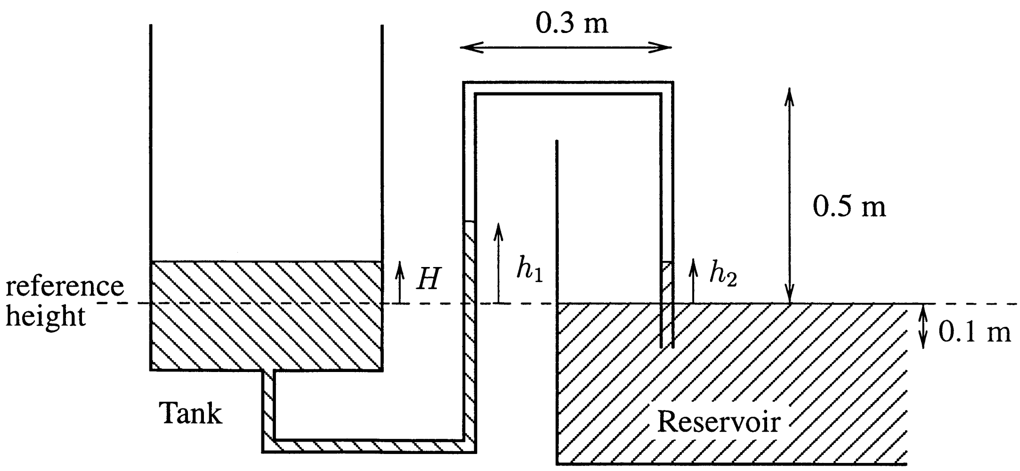

Fig. 1

2 Figure 2 shows a canal of rectangular cross-section with a width of 13 m and a horizontal bed. A bridge is supported by a single central pier of width 5 m. Upstream, the depth of the water is h_1 = 5 m and its velocity V_1 = 2.5 m s^{-1}. The flow is inviscid. Gravity can be assumed to be g = 10 m s^{-2} in this question.

(a) Find two independent equations involving the velocity of water V_2 and depth h_2 of the water between the pier and the bank. Hence, derive an equation for V_2 only and verify that V_2 = 5 m s^{-1} is the solution. Then determine the depth h_2. [50%]

(b) Determine the net force exerted by the flow on the pier. [50%]

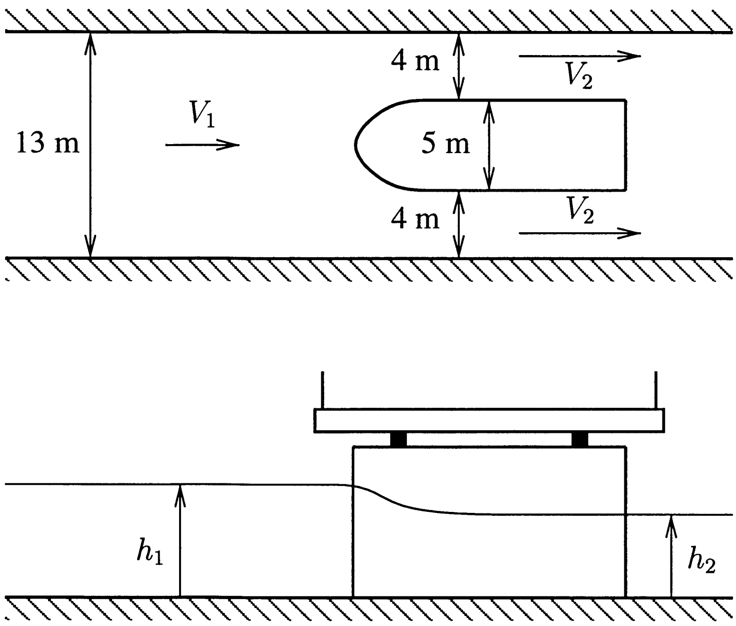

Fig. 2

3 A toy rocket is fixed to the ground for testing. As shown in Fig. 3, the rocket consists of a cylinder filled with air and water separated by a free piston. Initially, there is an equal volume of air and water, 10^{-3} m^3 each, the temperature is 300 K and the pressure is 1 bar. At some point in time, a heat pulse is transferred to the air, providing an amount of energy of 2 kJ. The air then expands, expelling the water through a small orifice at the back of the rocket. No heat transfer occurs between the air and the water nor between the rocket and the atmosphere. Moreover, despite the fact that the phenomenon is unsteady, the orifice is so small that, at each moment, the rocket can be considered to operate in a quasi-steady condition.

(a) Assuming that air can be treated as a perfect gas, show that the temperature and pressure of the air immediately after the heat transfer are 2698 K and 9 bar. [30%]

(b) Determine the velocity of the jet:

(i) immediately after the pulse; [10%]

(ii) just before all the water is expelled. [10%]

(c) The cross-section of the jet of water is 2 mm^2. Determine the thrust of the rocket at the two instants considered in (b). [20%]

(d) Neglecting any gravity effect, determine the total kinetic energy of the water once all of it has been expelled from the rocket. [30%]

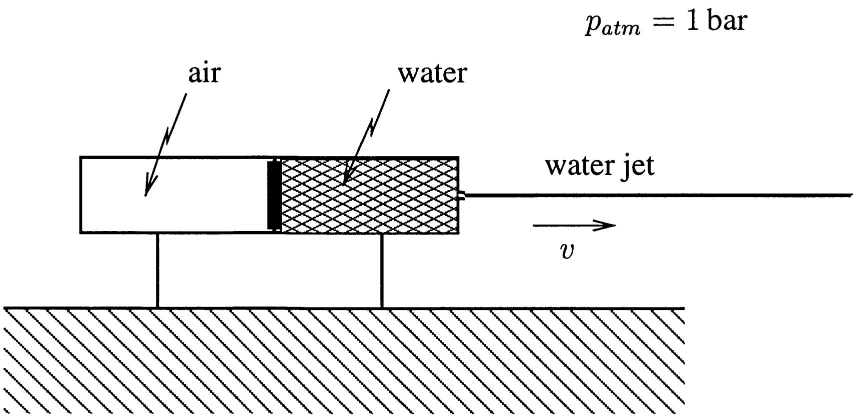

Fig. 3

4 In an adiabatic combustion chamber, combustion of hydrogen and oxygen takes place. At entry, the temperature of the flow is T_0 = 298.15 K and the total volumetric flow rate is 1 $m^3 s^{-1}$. The molar flow rates are 5×10^{-4} kmol s^{-1} for hydrogen and 2×10^{-3} kmol s^{-1} for oxygen. All species can be treated as semi-perfect gases (see page 12 of the Thermofluids Data Book) and enthalpies of reaction can be found on page 10 of the Thermofluids Data Book.

(a) Determine the partial pressures of hydrogen and oxygen for the inflow. What is the pressure in the combustion chamber? [25%]

(b) Combustion takes place so that the hydrogen is burned entirely and all products are gaseous. What is the composition of the gas in kmol s^{-1} after combustion? [25%]

(c) Verify that the temperature of the combustion products is about 1780 K. Discuss why the pressure being different from atmospheric does not affect that temperature. [50%]

5 An air-powered turbine operates in steady flow with an inlet pressure of 15 bar and an inlet temperature of 1500 K. The outlet pressure is 1 bar. The mass flow rate is 50 kg s^{-1}. Assume a fully resisted expansion and neglect the changes in kinetic and potential energies.

(a) Assuming that the flow is adiabatic, determine the temperature at the outlet and find the shaft power. [25%]

(b) In fact the flow is not adiabatic. Heat is lost at a rate equal to 1 % of the rate of decrease of the internal energy of the air during the expansion: $\delta q = 0.01\, du$. Using the first law of thermodynamics in a differential form and the equation of state for a perfect gas (air), show that the relationship between temperature and pressure has the following equation:

$$\frac{T}{p^{\frac{\gamma-1}{\gamma-0.01}}} = constant$$ [50%]

(c) Under the conditions of (b), determine the temperature at the outlet and the total shaft power. Comment on the results. [25%]

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SECTION B

*Answer not more than **four** questions from this section.*

6 Figure 4 shows the mechanism of Hackworth's valve gear for a reciprocating steam engine. The crank CM is 20 mm long and is rotating anticlockwise with an angular velocity of 20 rad s^{-1}. The jack-link MN (100 mm long) is hinged to the crank-pin at M, and is hinged at N to a trunnion-block which is constrained to move along the axis of the slide XY. The valve-rod PV is hinged to the jack-link at point P (40 mm from M), and is hinged to the valve-spindle at V. The valve-spindle is constrained to move along the horizontal axis of the valve-chest.

(a) Draw the velocity diagram for the mechanism in the position shown, using a scale of 1 mm to represent a velocity of 5 mm s^{-1}. Hence find the velocity of V relative to the valve-chest. [70%]

(b) The movement of the valve-spindle is resisted by a frictional force of 200 N caused by steam pressure acting on the face of the valve. Assuming that there is no other friction in the mechanism, find:

(i) the torque required to turn the crank; [10%]

(ii) the normal reaction force between the trunnion-block and the slide. [20%]

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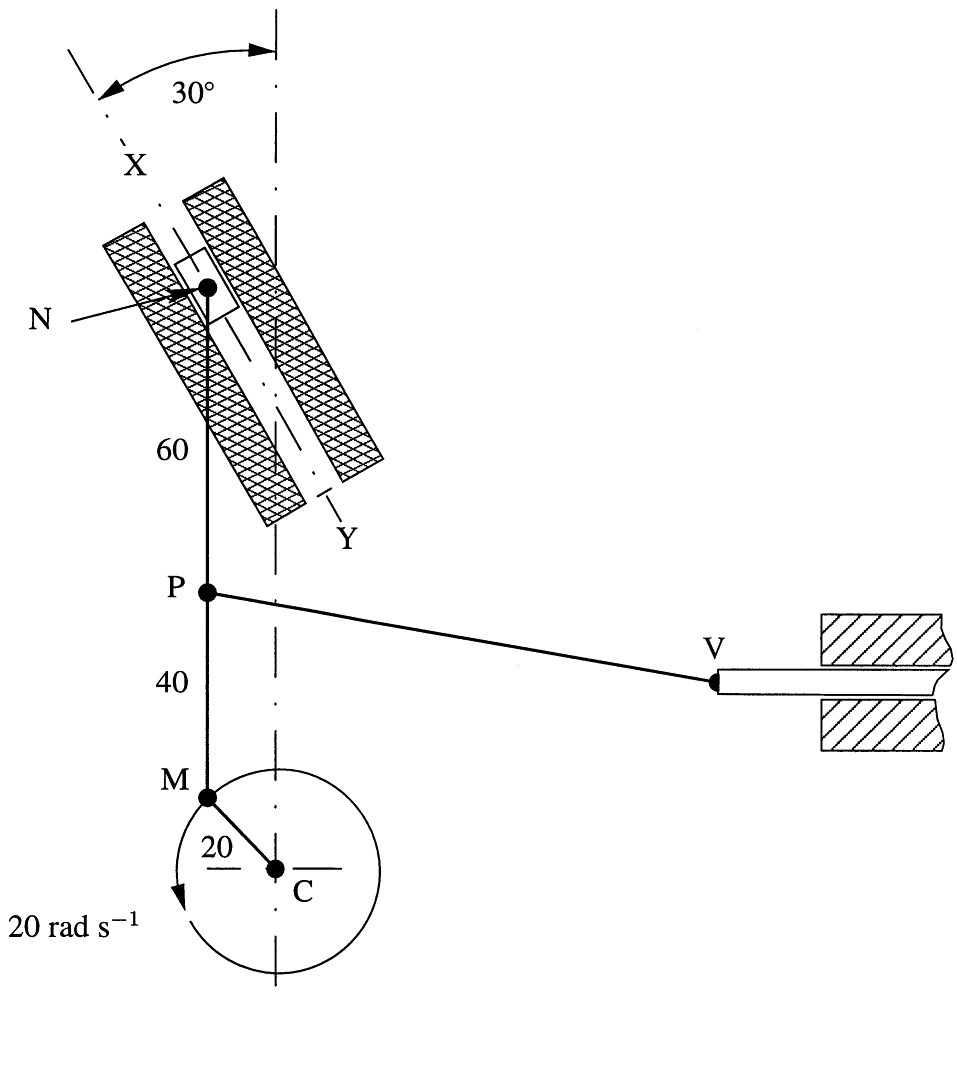

Fig. 4

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7 Figure 5 shows the force-extension characteristics of a bungee rope. The length of the rope under zero load is 15 m. One end of the rope is attached to a bridge deck, and the other end is attached to a person standing on the deck. The person then jumps off the deck, and descends vertically until arrested by the rope.

(a) Find the maximum mass of the person for a successful arrest, m_{max}. [20%]

(b) For a person of mass 60 kg, find:

(i) the extension of the rope at the point of arrest;

(ii) the force on the rope at the point of arrest. [60%]

(c) Figure 5 shows that the unloading line is 0.5 kN below the loading line. Comment on the practical significance of this to bungee jumping. [10%]

(d) Indicate and explain the dangers when a person of mass $m \simeq m_{max}$ jumps using the rope. [10%]

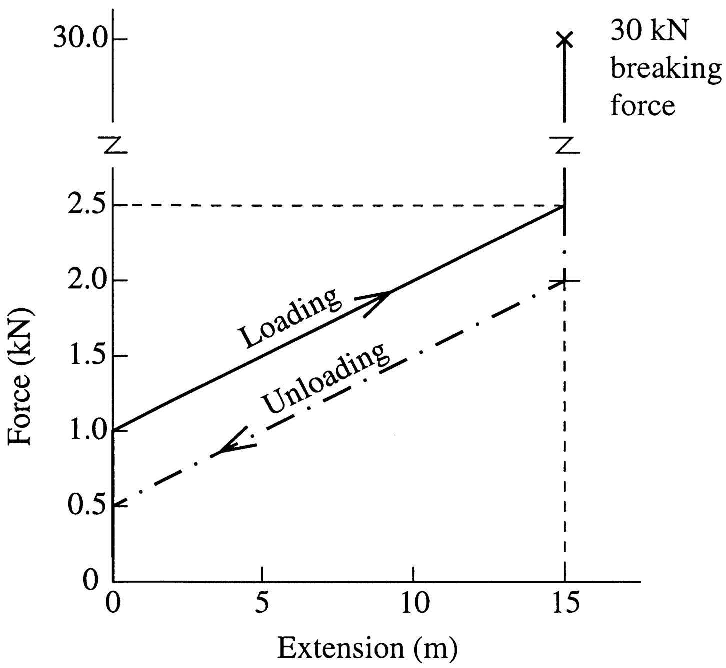

Fig. 5

8 (a) Figure 6(a) shows a uniform solid disc of mass m and radius a. The disc is released from rest and falls a height h. Derive an expression for the time t that it takes for the disc to fall in terms of h and g, the acceleration due to gravity. [20%]

(b) The disc is now made into a simple 'yo-yo' by attaching one end of a length of string to the circumference, and wrapping the string around the circumference a number of times. The other end of the string is attached to a fixed support P, as shown in Fig. 6(b). The yo-yo is released from rest and falls a height h. Derive an expression for the time t that it takes for the yo-yo to fall in terms of h and g. [70%]

(c) Give a physical reason why t should be greater for the yo-yo than for the simple disc. [10%]

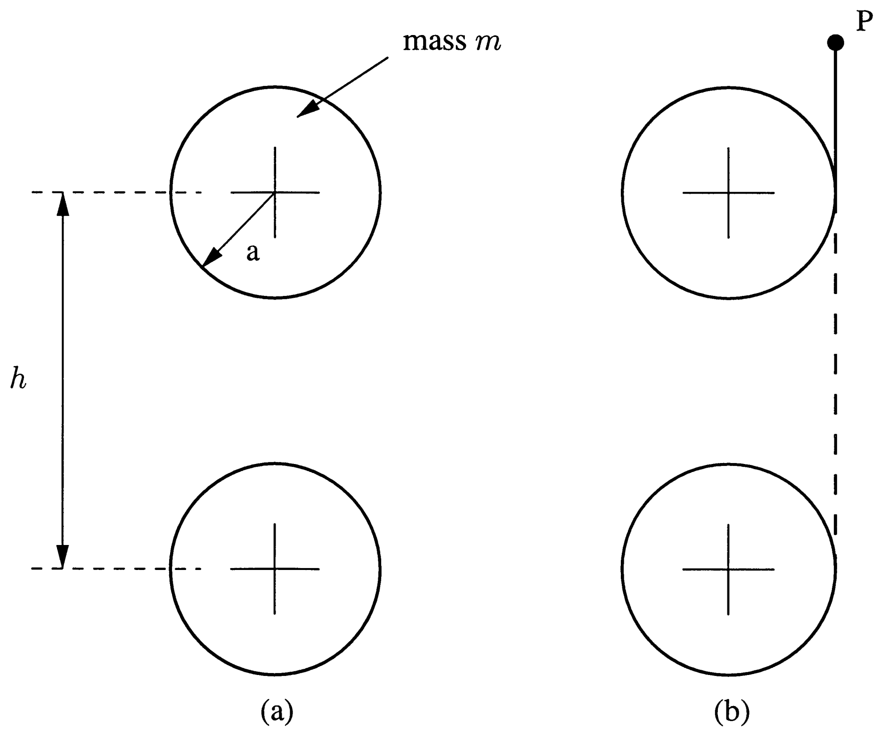

Fig. 6

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9 The brakes fail on a train as it approaches the end of the line at a railway station. As a result, the train collides with the buffer-stops and is brought to rest by them. The buffer-stops consist of a hydraulic ram and cylinder with a damping rate λ. The velocity of the train at the moment of collision is u, and the train remains in contact with the buffer-stops throughout the arrest process. The mass of the train is m.

(a) Derive a first-order differential equation for the arrest process in terms of the instantaneous velocity v and acceleration $\mathrm{d}v/\mathrm{d}t$ of the train. [30%]

(b) Hence derive an expression for v in terms of u, λ, m and t, and find the time constant T of the arrest process. [20%]

(c) Derive an expression for the distance L the ram moves during the arrest process in terms of u, m and λ. [20%]

(d) How long does it take for the train to come to rest after the collision? Explain why this time is not realistic in practice. [10%]

(e) How would you modify the buffer-stops to improve their performance? [20%]

10 A testing machine is installed on the top of a laboratory bench. In order to isolate the machine from the bench, an anti-vibration mounting is placed between the base of the machine and the bench-top. The mounting consists of a spring of stiffness k in parallel with a damper of rate λ. The mass m of the machine is 20 kg. General vibration in the laboratory results in a vertical sinusoidal movement x (absolute) of the bench-top, with amplitude $X = 50$ μm and frequency 10 Hz. This causes a vertical sinusoidal movement y (absolute) of the machine with amplitude Y. In order to make accurate measurements, Y must not exceed 10 μm.

(a) Derive a second-order differential equation for the system, and show that your equation is identical in form to Case (c) on p12 of the Mechanics Data Book. [25%]

(b) The damper is initially set to give zero damping. Find the value of k required to achieve the specified 10 μm isolation of the machine. [25%]

(c) It is not unknown for careless students to throw their bags on the bench when the machine is being used. The bench-top then moves with the same amplitude of 50 μm, but with a frequency well below 10 Hz. The amplitude of the machine must not exceed 100 μm if damage is to be avoided. Find the damping factor which must be used to achieve the specified 100 μm isolation. [25%]

(d) Without further calculation, indicate how adding this damping changes the response of the system to the usual 10 Hz input. Explain how you reach your conclusion. [25%]

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