ENGINEERING TRIPOS PART IA

Monday 9 June 2003 1.30 to 4.30

Paper 2

STRUCTURES AND MATERIALS

*Answer not more than **eight** questions, of which not more than **four** may be taken from Section A, and not more than **four** from Section B.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

(TURN OVER

SECTION A

Answer not more than **four** *questions from this section.*

1 A human arm is shown in an idealised way in Fig. 1. The maximum bending of the forearm occurs when the upper arm bone (*humerus*) is vertical and the main forearm bone (*ulna*) is horizontal. A reasonably fit adult can support a mass of 20 kg in that condition. The joint between the humerus and the ulna allows free rotation in the plane of the figure. The active muscle in this configuration is the biceps, which goes from the shoulder joint to a point about 10% of the way from the elbow to the wrist. [The other forearm bone (*radius*) and the many other muscles in the arm do not play a part in this loading configuration and can be ignored.]

(a) Calculate the forces that the humerus and muscle must apply to the ulna when loaded. Is the humerus in tension or compression? What forces must be applied at the shoulder? [40%]

(b) Draw a bending moment diagram for the ulna for the loading case shown. [20%]

(c) The ulna can be idealised as a beam with a constant bending stiffness EI. If the arm is in the position shown when unloaded, what shortening of the muscle must occur for the points A and C to remain at the same height? [40%]

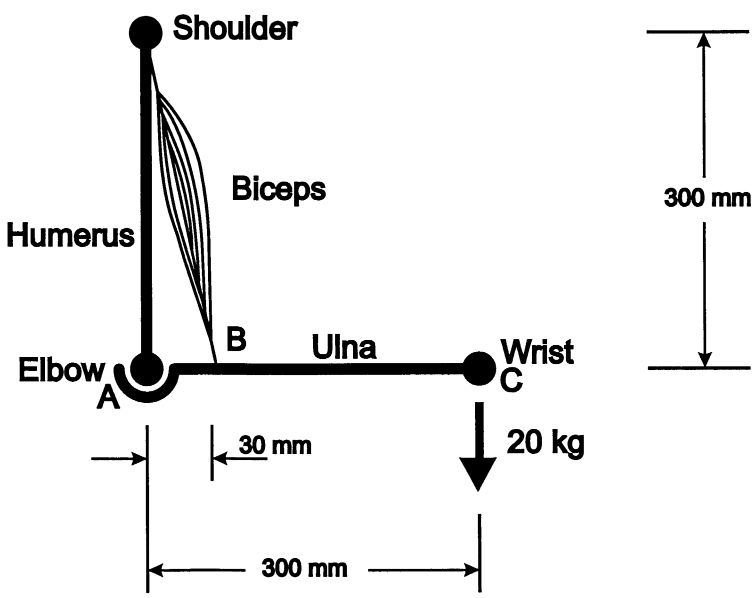


Fig. 1

2 A 2-storey tower takes the form of a pin-jointed truss, as shown in Fig. 2. It is loaded by a force P at B. All members have cross-sectional area A and Young's Modulus E.

(a) Determine all the member forces and corresponding extensions. [30%]

(b) Plot a displacement diagram for the structure on the extra sheet that has been provided. This sheet shows a suggested origin for your diagram and gives some grid lines at 45° intervals. A suggested scale is $PL/AE =$ 10 mm. [40%]

(c) Use your displacement diagram to find the change in distance between points A and D. [30%]

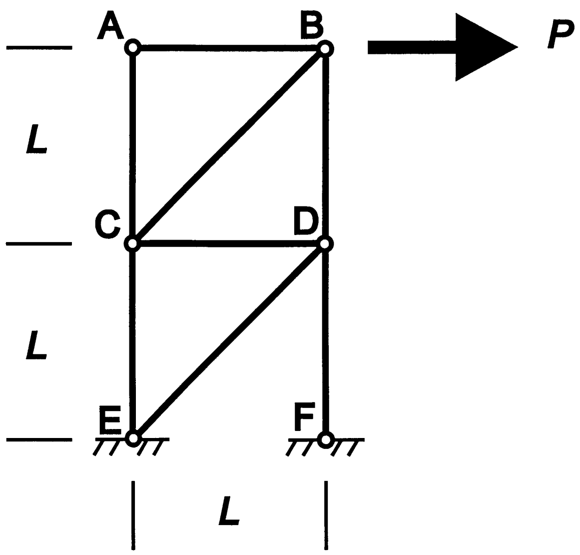


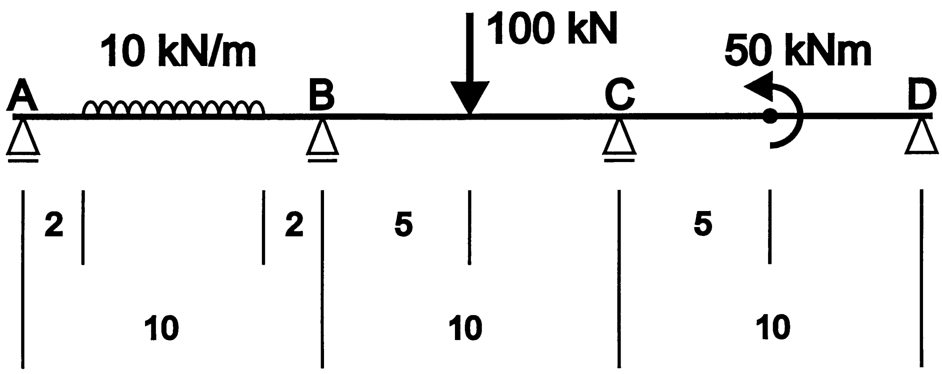
Fig. 2

3 A beam ABCD is continuous over three spans. Loads are applied to each span as shown in Fig. 3. The support reactions at A and B are measured and found to be 15 kN and 90 kN respectively.

(a) Determine the support reactions at C and D. [25%]

(b) Determine the bending moment in the beam at B and C. [25%]

(c) Plot, to scale and on graph paper, the shear force and bending moment diagrams, indicating the maximum and minimum values. Point out the relationship between the salient points on the two diagrams. [50%]



(Dimensions in m)

Fig. 3

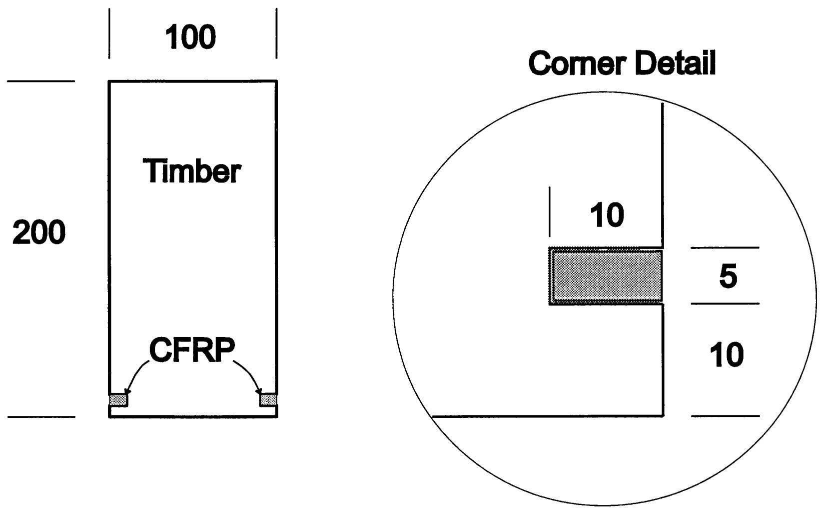
4 A system for reinforcing timber beams with Carbon Fibre Reinforced Plastic (CFRP) is known as 'Near Surface Mounting' (NSM). A slot is cut into the timber just large enough to contain a rectangular bar of CFRP with some adhesive to bond it to the timber. The surface can then be treated to make the reinforcement invisible.

A simply supported beam with the cross-section shown in Fig. 4 spans 4 m and after repair carries a load of 10 kN, uniformly distributed along its length. Assume that E_{CFRP} = 108 GPa, E_{WOOD} = 9 GPa. The beam was unstressed at the time of repair.

(a) Determine the second moment of area of the beam about its relevant neutral axis before and after adding the CFRP. [40%]

(b) Find the maximum bending stress in the timber and CFRP. [30%]

(c) Determine the average shear stress in the adhesive at the section of the beam where the shear force is a maximum. [30%]



(Dimensions in mm)

Fig. 4

5 An initially straight pin-ended strut is loaded by a force P applied at an eccentricity e, which represents a misalignment between the strut and the load it is designed to carry, as shown in Fig. 5.

(a) Why is a small error in the load position more important when dealing with struts and columns, and less important when dealing with beams loaded primarily in flexure? [20%]

(b) By considering a deflection of the strut, v, measured from the undeformed position, derive the governing differential equation for the deflection. [25%]

(c) Show that the solution of this differential equation takes the form

$$v = A \sin \alpha x + B \cos \alpha x + C$$

What are the coefficients, A, B, C and α? [35%]

(d) A strut has an eccentricity e that is 1% of its length. If it is loaded by an axial force that is 50% of the Euler buckling load of the strut, determine the lateral displacement of the strut at its centre and hence determine its maximum bending moment. [20%]

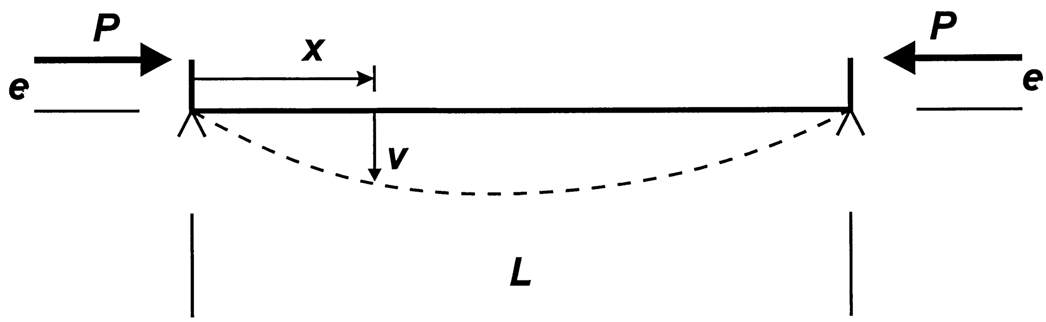


Fig. 5

SECTION B

Answer not more than **four** *questions from this section.*

6 (a) A stone column of uniform cross-section is of height H, with the material properties given below. By considering the stress on a general section at a height h above the base, derive expressions for the variation of stress and strain as functions of h. Hence find the length of column which will shorten under its own weight by 0.001%, when raised to the vertical from an initial horizontal position. What height of column would be required for the stone at the base to fail in compression? Comment on the result. [40%]

Material properties for stone: Young's modulus = 125 GPa, density = 2750 kg/m^3, compressive strength = 1000 MPa.

(b) The Young's modulus of a material may be measured by vibrating a cantilever of length l and flexural rigidity EI using a mass M_o (much larger than the mass of the cantilever) and measuring the natural frequency f. Use dimensional analysis to find the dependence of Young's modulus on f, assuming that no other variables are involved. [20%]

(c) Sketch the unit cell of the face centred cubic (FCC) atomic structure, and calculate the atomic packing factor. Calculate the theoretical density of aluminium, given that the atomic radius is 1.432 nm, and compare this with the measured value. [40%]

7 (a) Define *yield stress*, *hardness* and *ductility*. Which of these properties depend on the dimensions of the test piece? [20%]

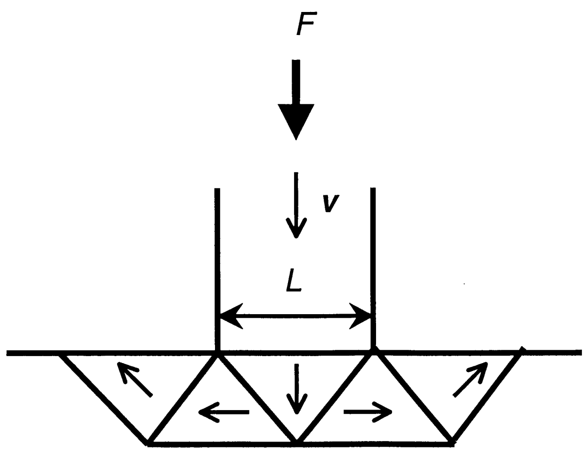
(b) A relationship between hardness and yield stress may be derived by considering the force (per unit depth into the page) F required to push a long flat punch of width L into a surface (Fig. 6). The material is assumed to deform as five rigid blocks, sliding at their interfaces with a constant shear stress $k = \sigma_y / 2$. The punch descends at a velocity v.

(i) Find the lengths L_i of each interface across which there is a relative sliding velocity.

(ii) Draw a hodograph (a velocity, or displacement, diagram) for the five sliding blocks and the punch, relative to the solid material substrate. Hence find the relative velocities v_i across each interface.

(iii) Calculate the internal work rate (per unit depth) W in the material, given by $W = \sum_i k L_i v_i$ and equate this to the work rate (per unit depth) of the punch, to find an expression for F. Hence derive the relationship between hardness and yield stress. [60%]

(c) Explain why the spacing between obstacles is important in determining the resistance to dislocation motion in metals. For two hardening mechanisms, state what microstructural parameters control this spacing. [20%]



Unit depth into page.

All sliding blocks are 45° right-angled triangles.

Fig. 6

8 (a) Distinguish briefly between the mechanisms of tensile and compressive failure in ceramics. Which leads to the higher strength? [20%]

(b) A batch of cylindrical ceramic specimens of length 30 mm and radius 3 mm were tested in tension. Half of the specimens failed at or below a stress $\sigma = 500$ MPa. A second batch of specimens of the same ceramic had a square section 8×8 mm and length 50 mm. The Weibull modulus for the ceramic is $m = 8$.

(i) Find the tensile stress σ_t which must be applied to the square section specimens to give the same probability of failure as for the cylindrical specimens.

(ii) The square section specimens are tested in pure bending (Fig. 7). Find an expression for $\sigma(y)$ in terms of the maximum tensile stress σ_{max} at the surface. Hence find the value of σ_{max} which gives the same probability of failure in bending as did the tensile stress σ_t in (i).

(iii) Explain why there is a difference between σ_{max} and σ_t, and comment on how this difference would change if the value of m was higher. [80%]

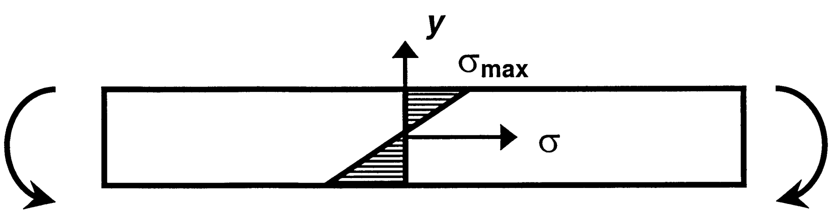


Fig. 7

9 (a) Stress and strain values were recorded in a tensile test on a sample of polyethylene, as follows:

ε	0.08	0.2 - 2.6	3.0 (failure)
σ (MPa)	25	22	80

Sketch the stress-strain curve and account for its shape. [20%]

(b) Briefly explain two methods by which the toughness of thermosets may be enhanced. [20%]

(c) (i) In the sliding and rolling contact between two metal components, define what is meant by the nominal and true contact areas. Explain how the true contact area depends on applied load, hardness and nominal contact area. How does the nature of the contact differ for elastomers?

(ii) The contact between two dry metal components carries a normal force W and a friction force F. Make a numerical estimate of the coefficient of friction, μ, if the shear yield stress is equal to $\sigma_y/2$, stating any assumptions made. [40%]

(d) Give two examples of problems caused by aqueous corrosion, and in each case outline a method by which it could be prevented. [20%]

10 A simple component may be modelled as a beam of length L loaded in bending, and is designed for minimum mass with a specified compliance (= deflection per unit load, δ/F). The beam has a square section, which can vary in size. It may be assumed that an expression for the mass m will take the form

$$m \propto L^a \, (\delta / F)^b \, \rho^c \, E^d$$

where ρ is the density and E is Young's modulus.

(a) Use dimensional analysis to find a performance index for minimum mass, if it is known that $a = 5/2$. Explain why it is not necessary to include the section size in the expression for mass. [35%]

(b) Sketch a material selection chart showing log E against log ρ, and show a line corresponding to a constant value of the performance index. Indicate the direction in which to move the line to optimise the design. Explain whether the following parameters vary along a given line: mass, section size, deflection at maximum design load. [30%]

(c) Discuss (without further analysis) how to proceed to select the lightest material if, *in addition*, the beam must not fail at the specified design load. Explain carefully what additional information is required, and how to determine for a given material whether the design is limited by stiffness or strength. [35%]

END OF PAPER

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Sheet to be handed in with solution for Question 2

A suggested origin for your diagram is shown.
A suggested scale is $PL/AE =$ 10 mm.
Grid lines at 45° intervals are shown for convenience