PART IA 2004 PAPER 1 SECTION A

ANSWERS

Q1 a) Larger velocity and parallel streamlines

b) 1·210 kg/m³ , 28·75 m/s , 0·0983 kg/s

c) 0·10625 m , 41·51 m/s , 0·1420 kg/s

d) $\Delta\rho/\rho < 1\%$

Q2 a) 0·6969 kg/m³ , 139·4 kg/s

b) 293·1 kJ/kg

c) 55·76 kN

e) F_H = 7·47 kN , F_V = 27·88 kN

Q3 b) $mC_v\Delta T$, $mC_p\Delta T$

d ii) 953·3 K
iii) 2943·3 K , 3·087
iv) 1453·9 K , 828·5 kJ/kg
v) 58·6 %

Q4 b) [Q, L, θ] ⇒ 2 or more.
[M, L, T, θ] ⇒ 1 or more.

c) $\dot{q}/h\Delta T$ or $\dot{q}D/(\lambda\Delta T)$

d) $$\frac{\dot{q}}{h\,\Delta T} = \frac{1}{1+(hD/\lambda)}$$

Q5 c) 403129 N/m² , 0·6151 m³ , 5814 N

d) SLOW

e) 1·1905 m³ , 208289 N/m² , 10·72 m.

Engineering Tripos Part IA, 2004

Paper 1, Mechanical Engineering: Section B

Answers

6 (a) 4 m/s

(b) 0 m/s and 2 m/s; -16 m/s^2 (i.e. inwards) and 0 m/s^2

(c) $20r^4 - 12r^3 + 0.25 = 0$; 0.5 m

7 (a) (i) K.E. – no Momentum – yes (ii) $\dfrac{V_o M}{M + 2m_c}$

(iii) $2L$ (iv) $M + \dfrac{m_c x}{L}$

(b) $\dfrac{V_o ML}{ML + m_c x}$

(c) $-\dfrac{V_o{}^2 M^2 L^2 m_c}{(ML + m_c x)^3}$; $-\dfrac{V_o{}^2 m_c}{ML}$ at $x = 0$

8 (b) 395 mm/s at -81° to horizontal; 390 mm/s at +44° to horizontal

(c) 1.30 Nm; 1.39 Nm

9 (a) (i) F/m (ii) F/k (iii) $3F/2k$

(b) $\sqrt{\dfrac{3k}{m}}$, $\dfrac{\lambda}{2\sqrt{3km}}$ and $\dfrac{2z}{3}$

(c) 5.8 mm

(d) 6.06 N

10 (b) $\sqrt{\dfrac{k}{2J}}$ and $\sqrt{\dfrac{2k}{J}}$

(c) $\theta_2 = 0$ at $\omega = \sqrt{\dfrac{3k}{2J}}$; no movement in either rotor when $\omega = 0$ and $\omega = \infty$

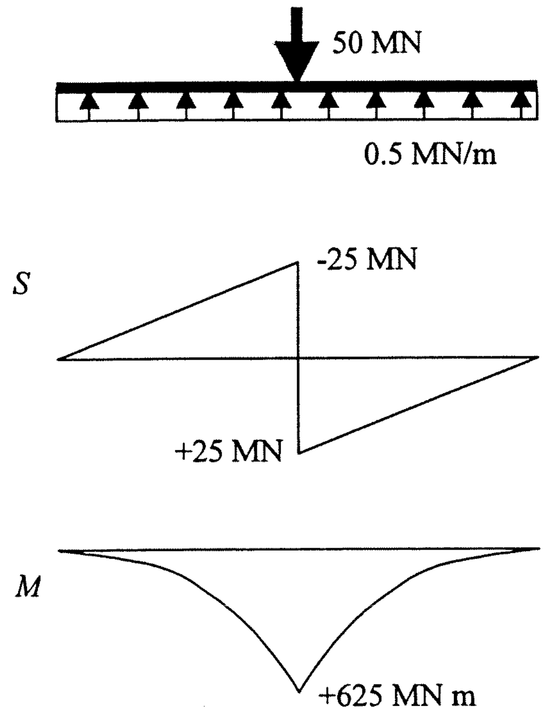
Engineering Tripos Part 1A, 2004
Section A of paper 2, structures and materials

ANSWERS

1(a) AB −15 kN, BC 0, CD 0, DE −15, EF −45, BD +15√2, BE −15, AE +30√2
(b) 43.7 mm
(c) 2.56 mm

2(a) at A, (3/4)W vertically, (1/4)W horizontally, acting to the right,; at D, (1/4)W vertically, (1/4)W horizontally, acting to the left,
(b) (1/2√2)W, at 45 degrees to horizontal
(c) -(3-√3)$WR/8$
(d) (√2-1)$WR/4$, at the midpoint of CD

3(a) 2.5 m
(b) (c) (d)



(e) vertical reaction from water slightly increased near the centre, slightly reduced at the ends; shear force diagram slightly curved, but maximum shear force unchanged,;bending moment diagram slightly more curved, and maximum bending moment slightly reduced.

4(a)(i) true, by a symmetry argument considering that a section of a beam in pure bending must deform in the same way when observed from either side of the beam (but not strictly true if the beam is not in pure bending but carries a shear force);
(ii) true, by considering the geometry of deformation, and the fact that there must be a neutral surface on which the longitudinal strain is zero;
(iii) true for either of the two materials individually, but the constant of proportionality is different;

(iv) false: this is not true in general, because the neutral axis is fixed by the condition that the longitudinal force is zero, though it might chance to be where the steel and the wood meet;
(v) false, because they have different moduli and different maximum distances from the neutral axis;
(vi) false: if the screws are removed the steel and the wood bend as separate beams and there is no composite action.

(b) $(1/3)E\kappa bd^3$

5(a) $(\pi^2/4)EI/L^2$
(b) 254×254×73kg/m UC
(c) 43 MPa
(d) discussion: see crib.

Engineering Tripos Part IA, 2004
Section B of Paper 2, Structures and Materials

ANSWERS

6. (d) $\Delta = -1.2\times10^{-4}$, $\delta V = -0.004\ \text{m}^3$.

7. (a) Plastic flow is caused by the movement of many dislocations. Dislocations will interact with each other. More dislocations mean more interaction, and hence it is more difficult to move them. This leads to an increase in the yield stress, resulting in the usual work hardening.
(c) Before necking, $d\sigma / d\varepsilon > \sigma$; at necking, $d\sigma / d\varepsilon = \sigma$; after necking, $d\sigma / d\varepsilon < \sigma$. A neck is stable if $d\sigma / \sigma > -dA / A$.
(d) Approximations: no work hardening assumed, elasticity effects ignored, only shear yielding along 45 degree planes considered, etc.

8. (a) A ceramic under 4-point bending is more critical than that in 3-point bending.
(c) $\sigma_{b1} / \sigma_t = 3\sqrt{2}$, $\sigma_{b2} / \sigma_t = 3$. The two ratios are larger than unity because less volume of material is stressed under maximum stress than that under uniaxial tension, so the chance of the 'weakest link' coinciding with a high stress region is reduced.

9. (b) (i) $\dfrac{W_{\min}}{\rho g l^3} = \dfrac{P}{\sigma_y l^2}\left[1+\dfrac{12}{\pi^2}\dfrac{\sigma_y}{E}\left(\dfrac{l}{b}\right)^2\right]$, $\dfrac{a}{l} = \dfrac{P}{\sigma_y l^2}\dfrac{l}{b}\left[1+\dfrac{12}{\pi^2}\dfrac{\sigma_y}{E}\left(\dfrac{l}{b}\right)^2\right]$;
(ii) $\dfrac{W_{\min}}{\rho g l^3} = 1.12\dfrac{P}{\sigma_y l^2}$, $\dfrac{a}{l} = 11.2\dfrac{P}{\sigma_y l^2}$. Material selection index = $\dfrac{\rho}{\sigma_y}$.

10. (a) (i) In one mechanism of hydrogen embrittlement, hydrogen atoms pin the movement of dislocations and hence restrict plasticity. This causes the crack to break in a brittle manner, leading to a lower fracture toughness. In stress corrosion cracking (for some materials and under certain environments), cracks grow steadily under a constant load well below the fast fracture load.
(ii) Al has a larger driving force, and Fe has a higher rate of reaction in water and hence needs galvanizing. With galvanizing, the protective layer (e.g., Zn) shields Fe, and Zn preferentially corrodes due to more negative electrochemical potential to protect Fe.
(b) $a = 0.25$ mm.
(c) (i) The plot shows the influence of the applied stress σ on growth of an arrested indentation crack. The additional term in K leads to a minimum in the total K, corresponding to a critical crack length a_c. The crack undergoes stable growth as σ is increased. Once $a = a_c$, the condition for final (unstable) failure is met.
(ii) At unstable crack growth, $dK / da = 0$ and $K = K_{IC}$. The critical stress is $\sigma_c = 1.96(H / E)^{1/6} K_{IC}^{4/3} / P^{1/3}$.

2004 Paper 3 Electrical and Information Engineering

Numerical and Algebraic Answers

Section A

1. b) Gain = $-R_2/R_1$ d) i) Gain = -9.99 ii) Gain = -9.08

2. a) $R_1 = 1\ M\Omega$ $R_2 = 500\ \Omega$ $R_3 = 2.25\ k\Omega$ b) Mid-band gain = -3.24

c) $C_1 = 5.61\ \mu F$ d) $R_L = 1.84\ k\Omega$

3. a) $V_{TH} = R_2V/(R_1 + R_2)$ $R_{TH} = R_1R_2 / (R_1+R_2)$ b) $I_{rms} = 9.80$ A Phase = 78.7^0

$I_{peak} = 13.9$ A c) C = 318 μF $V_{cap} = 1000$ V (rms) $\angle -90^0$

4. b) i) $I_2' = 16.0$ A ii) $P_{load} = 2.56$ kW $Q_{load} = 1.28$ kVAr iii) $P_{in} = 3.65$ kW

$Q_{in} = 3.45$ kVAr iv) $I_{in} = 20.9$ A Power factor = 0.726 lagging

v) Efficiency = 70.2 % c) C = 191 μF

Section B

5. b) Z = A.B + C.D + A.C + A.D + B.C + B.D (sum of products)

Z = (A+B+C).(A+B+D).(A+C+D).(B+C+D) (product of sums)

c) i) $Z = \overline{\overline{A.B}\,.\,\overline{C.D}\,.\,\overline{A.C}\,.\,\overline{A.D}\,.\,\overline{B.C}\,.\,\overline{B.D}}$ i.e. Six 2-input NAND gates and one 6-input NAND gate so 7 gates in total.

ii) $Z = \overline{\overline{(A+B+C)} + \overline{(A+B+D)} + \overline{(A+C+D)} + \overline{(B+C+D)}}$ i.e Four 3-input NOR gates and one 4-input NOR gate so 5 gates in total.

d) 15 2-input NAND gates required so 4 chips for NAND implementation, 5 for NOR implementation.

6. a) 000 100 110 111 011 001 and 010 101 b) ii) $J_A = K_A = Q_B.Q_C$

$J_B = K_B = Q_C$ $J_C = K_C = 1$

7. a) ii) $2^{16} = 65536$ bistables Capacity = 8 kbytes iii) 8 memory devices

$\overline{CS} = \overline{A_{13} . \overline{A_{14}} . A_{15}}$

b) i) $FF_{16} = -1_{10}$ $8C_{16} = -116_{10}$ $3B_{16} = +59_{10}$

Most positive = $7F_{16} = +127_{10} = 01111111_2$
Most negative = $80_{16} = -128_{10} = 10000000_2$

ii) V flag is set ($V = N \oplus C$ and $C = 1$, $N = 0$) $-53 + (-115) = -168$ is outside the 2's complement range, hence 2's complement overflow.

8. c) Time = 66 μsecs

Section C

9. a) 110 V b) $C_{new}/C_{old} = 1.375$ c) Thickness = 1.99 μm

10. a) $C = \varepsilon_0 A/d$ b) $C = 4\pi\varepsilon_0 r_0$ c) $c = 2\pi\varepsilon_0 / \ln((2s - a)/a) \approx 2\pi\varepsilon_0 / \ln(2s/a)$

11. a) Flux density = 8.38 T b) Flux density = 0.303 T c) Weight = 5.59 kN

ENGINEERING TRIPOS Part IA 2004

Paper 4 – Mathematical Methods

Answers:

1. a) (ii) $\dfrac{\underline{a}.(\underline{b}\times\underline{c})}{|\,\underline{a}\times\underline{b}+\underline{b}\times\underline{c}+\underline{c}\times\underline{a}\,|}$ (iii) $\dfrac{1}{2}|\,\underline{a}\times\underline{b}+\underline{b}\times\underline{c}+\underline{c}\times\underline{a}\,|$ (iv) $\dfrac{1}{6}\underline{a}.(\underline{b}\times\underline{c})$

   b) $\underline{r} = (0, 4, -6) + \lambda(1, 1, -3)$ (N.B. not unique)

2 a) (i) 2 (ii) $\dfrac{\pi}{2\sqrt{3}}$

   c) (i) $4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$ (ii) $e^{-\pi/4}$

3 a) $x(t) = -\dfrac{t}{4}e^{-2t} + \dfrac{\sin t}{8}$

   b) (i) $\dfrac{1\pm\sqrt{5}}{2}$ (ii) $L_n\,L_{n+1}$ (iii) $\left(\dfrac{1+\sqrt{5}}{2}\right)^2$

4 a) (ii) $A_2:\ \alpha$ (twice), $[1\ 0]^T, [0\ 1]^T$. $A_3:\ \alpha, \alpha^{-1}$, $[1\ 0]^T, [0\ 1]^T$

   (iii) $A_2: \left[\alpha^{100}x_1 \quad \alpha^{100}x_2\right]^T$, $A_3: \left[\alpha^{100}x_1 \quad 0\right]^T$

5 b) (i) $\dfrac{1-e^{-\alpha t}}{\alpha}$ $t \ge 0$ 0 $t<0$ (ii) $e^{-\alpha t}$ $t\ge 0$ 0 $t<0$

   c) (ii) $\dfrac{t}{\alpha} - \dfrac{1}{\alpha^2} + \dfrac{e^{-\alpha t}}{\alpha^2}$ $t\ge 0$ 0 $t<0$

6 a) $a_0 = \dfrac{1}{4}$ $a_n = \dfrac{2}{n^2\pi^2}\left(1-\cos\dfrac{n\pi}{2}\right)$ $n>0$ $b_n = \dfrac{1}{n\pi} - \dfrac{2}{n^2\pi^2}\sin\dfrac{n\pi}{2}$

   b) (ii) $\sum_{n=1}^{\infty} 2b_n \sin nt$

8 a) $\dfrac{3}{8}e^t - \dfrac{11}{40}e^{-3t} - \dfrac{1}{10}\cos t - \dfrac{1}{5}\sin t$

9 a) (ii) $\alpha = 12, \beta = 2$, $f = 6x^2y + \dfrac{y^3}{x} + \cos x + \text{const}$

   b) (ii) saddles at (0,0), (0,2), minimum at $\left(1, \dfrac{1}{\sqrt{3}}\right)$, maximum at $\left(1, -\dfrac{1}{\sqrt{3}}\right)$

11 a) 13 c) 3.5616 ; accuracy .000049 d) 20/13 e) 25 iterations