PART IA 2004 PAPER 1

(WITH POST-EXAM COMMENTS)

Q1

a) Velocity will be larger at A to yield a larger pressure difference so more accurate measurement. Also streamlines will be parallel at A so easier (no calibration) to get mass flow. [3]

b) $\rho_{atm} = \dfrac{P}{RT} = \dfrac{100000}{287 \times 288} = \underline{\underline{1.210\ kg/m^3}}$ [2]

(Many candidates quoted data book value!)

Bernoulli: $P_{atm} = P_A + \frac{1}{2}\rho V_A^2$

$\frac{1}{2}\rho V_A^2 = P_{atm} - P_A = 0 - (-500) = 500\ N/m^2$

$V_A = \sqrt{2 \times 500 / 1.210} = \underline{\underline{28.75\ m/s}}$ [2]

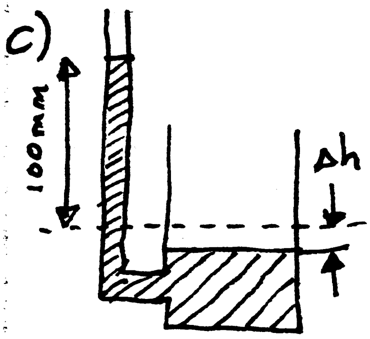
(Most candidates correctly applied Bernoulli)

$AREA = \pi D^2/4 = \pi \times 0.06^2/4 = 0.002827\ m^2$

$\dot{m} = \rho V_A \times AREA = 1.210 \times 28.75 \times 0.002827 =$

$\dot{m} = \underline{\underline{0.0983\ kg/s}}$ [2]

c)



$0.100 \times \dfrac{5^2}{4}\pi = \Delta h \times \dfrac{20^2}{4}\pi$ (VOL. CONSERVATION)

$\Delta h = 0.100 \times \left(\dfrac{1}{4}\right)^2 = \underline{0.00625\ m}$

$\underline{\underline{\text{TRUE HEIGHT} = 0.10625m}}$ [4]

(Many omitted sinking of reservoir level)

MANOMETER PRESSURE $= \rho_w g h = 1000 \times 9.81 \times 0.10625 = 1042.3\ N/m^2$

$V_A = \sqrt{2 \times \Delta P / \rho_{atm}} = \sqrt{2 \times 1042.3 / 1.210} = \underline{\underline{41.51\ m/s}}$ [2]

$\dot{m} = \rho_{atm} V_A \times AREA = 1.210 \times 41.51 \times 0.002827 = \underline{\underline{0.1420\ kg/s}}$ [2]

(Generally well done, except ~~of~~ for Δh sinking)

d) TRUE PRESSURE $= 100000 - 1042.3 = 98957.7\ N/m^2$

$\dfrac{\rho_A}{\rho_{atm}} = \left(\dfrac{P_A}{P_{atm}}\right)^{1/\gamma} = \left(\dfrac{98957.7}{100000}\right)^{1/\gamma} = 0.9925$

$\Rightarrow \dfrac{\Delta\rho}{\rho} < 1\%$ Hence Incompressible [3]

(Most answers were correct (ish!))

[Note slight change in mark distribution]

PART IA 2004 PAPER 1

(WITH POST EXAM COMMENTS)

$R = 287\ \mathrm{J/kgK}$

$C_p = 1005\ \mathrm{J/kgK}$.

Q2

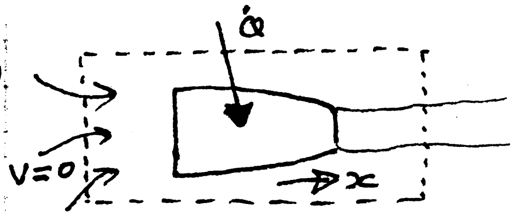
a) $P_{jet} = P_{atm}$ (PARALLEL STREAMLINES)

$$\rho_{jet} = \frac{P_{jet}}{R\,T_{jet}} = \frac{100000}{287 \times 500} = \underline{0.6969\ \mathrm{kg/m^3}}$$

$$\dot{m}_{jet} = \rho_{jet} A_{jet} V_{jet} = 0.6969 \times 0.5 \times 400 = \underline{\underline{139.4\ \mathrm{kg/s}}} \quad \boxed{2}$$

(common error was to use ambient density)

b)



$$\dot{Q} - \underbrace{\dot{W}_x}_{=0} = \dot{m}\left[\left(h_2 + \tfrac{1}{2}V_2^2\right) - \left(h_1 + \underbrace{\tfrac{1}{2}V_1^2}_{=0}\right)\right]$$

(Most knew this)

$$\dot{Q} = \dot{m}\left[C_p(T_{jet} - T_{atm}) + \tfrac{1}{2}V_{jet}^2\right]$$

$$\dot{Q}/\dot{m} = 1005\,(500-288) + \tfrac{1}{2}400^2 = \underline{\underline{293.1\ \mathrm{kJ/kg}}} \quad \boxed{4}$$

c) SAME CONTROL VOLUME: F ON FLOW, −F ON ENGINE

PRESSURE UNIFORM, ON CV, SO MAY IGNORE $\boxed{2}$

TOTAL FORCE = Δ(MOM. FLUX)

$F = \dot{m}V_{jet} - \dot{m}V_1 \qquad V_1 = 0$ FAR UPSTREAM $\boxed{2}$

$F = 139.4 \times 400 = \underline{\underline{55.76\ \mathrm{kN}}}$ $\boxed{2}$

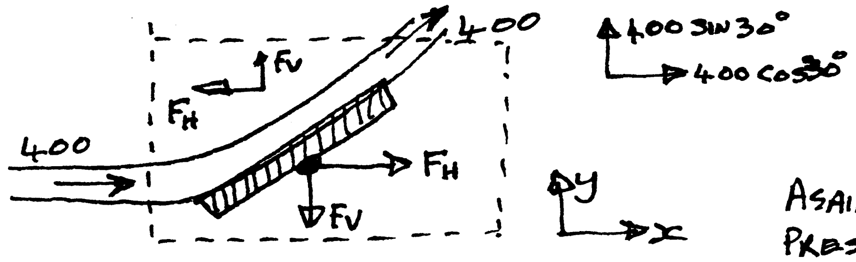
(Generally well done, most understood that $V_1 = 0$)

d) Parallel streamlines, so $P_{jet} = P_{atm}$ at exit of deflector

No loss due to deflector, so if P unchanged then T is unchanged, so from SFEE, V is unchanged $\boxed{4}$

(Some confusion, Bernoulli was also valid)

e)



F_H, F_V Force on Flow.

AGAIN MAY IGNORE PRESSURE AWAY FROM DEFLECTOR.

SFME $\Rightarrow$ TOTAL FORCE = Δ(MOM. FLUX)

HORIZ. (x) $\quad -F_H = \dot{m}\,400\cos 30° - \dot{m}\,400$

$F_H = \dot{m}\,400\,(1 - \cos 30°)$

$F_H = 139.4 \times 400 \times (1 - \cos 30°) = \underline{\underline{7.47\ \mathrm{kN}}}$ $\boxed{2}$

VERT. (y) $\quad F_V = \dot{m}\,400 \sin 30° - 0$

$F_V = \dot{m}\,400 \sin 30°$

$F_V = 139.4 \times 400 \times \sin 30° = \underline{\underline{27.88\ \mathrm{kN}}}$ $\boxed{2}$

(Only major problem was using C_v rather than C_p in SFEE)

PART IA 2004 PAPER 1

(WITH POST EXAM COMMENTS)

Q3 a)

Thermo-fluid system: Fixed mass, only heat and work can cross the system boundary. $Q - W = \Delta E$

Heat input $Q > 0$, Work output $W > 0$ [2]

b) $Q - W = \Delta E = \Delta U = m C_v \Delta T$

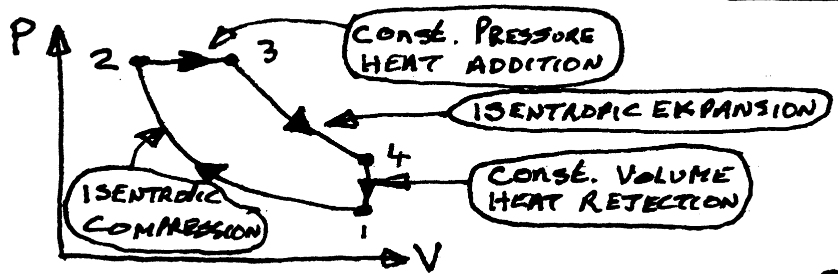
i) Const. Vol $\Rightarrow$ NO DISPLACEMENT WORK $\Rightarrow W = 0$

$\underline{\underline{Q = m C_v \Delta T}}$ [2]

ii) Const. PRESS. $\Rightarrow W = \int p\,dV = p \Delta V = m R \Delta T$

$Q = W + \Delta U = m R \Delta T + m C_v \Delta T = \underline{\underline{m C_p \Delta T}}$ [2]

c) i)



$T_1 = 300\text{K}$ [2]

$R = 287$ J/kgK

$C_p = 1005$ J/kgK

$C_v = 718$ J/kgK

ii) $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = T_1 \left(\dfrac{V_1}{V_2}\right)^{\gamma-1}$

$T_2 = 300(18)^{1.4-1} = \underline{\underline{953.3\ \text{K}}}$ [2]

iii) $\left.\begin{aligned} P_2 V_2 &= m R T_2 \\ P_3 V_3 &= m R T_3 \end{aligned}\right\} \quad \alpha = \dfrac{V_3}{V_2} = \dfrac{T_3}{T_2}$

CONSTANT PRESSURE HEATING $\Rightarrow Q/m = C_p \Delta T$

$T_3 - T_2 = 2000/1.005 = 1990\ \text{K}$

$T_3 = 953.3 + 1990 = \underline{\underline{2943.3\ \text{K}}}$ [2]

$\alpha = T_3/T_2 = 2943.3/953.3 = \underline{\underline{3.087}}$ [2]

iv) $\dfrac{T_4}{T_3} = \left(\dfrac{V_3}{V_4}\right)^{\gamma-1} \quad T_4 = T_3 \left(\dfrac{V_3}{V_2} \times \dfrac{V_2}{V_4}\right)^{\gamma-1} = T_3 \left(\dfrac{\alpha}{R_{VOL}}\right)^{\gamma-1}$

$T_4 = T_3 \left(\dfrac{3.087}{18}\right)^{\gamma-1} = 2943.3 \left(\dfrac{3.087}{18}\right)^{0.4} = \underline{\underline{1453.9\ \text{K}}}$ [2]

CONSTANT VOLUME $\Rightarrow Q/m = C_v \Delta T$

$Q/m = 0.718 \times (1453.9 - 300) = \underline{\underline{828.5\ \text{kJ/kg}}}$ [2]

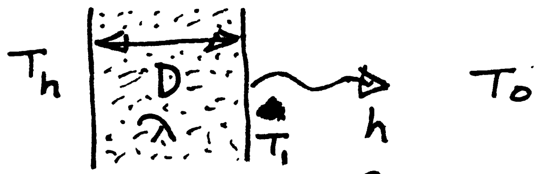
v) $\eta = \dfrac{W}{Q_{IN}} = \dfrac{Q_{IN} - Q_{OUT}}{Q_{IN}} = 1 - \dfrac{Q_{OUT}}{Q_{IN}}$

$\eta = 1 - \dfrac{828.5}{2000} = \underline{\underline{58.6\,\%}}$ [2]

(Straightforward, well done question)

PART IA 2004 PAPER 1
(WITH POST EXAM COMMENTS)

Q4



$$\left\{\begin{array}{l} 1W = 1J/s = 1 Nm/s \\ \quad = (MLT^{-2})L/T \\ Q = ML^2T^{-3} \end{array}\right\}$$

a) CONVECTIVE: $\dot{q} = \dot{Q}/A = h\Delta T$ } Heat transfer driven by bulk interchange of fluid. $\boxed{2}$

CONDUCTIVE: $\dot{q} = \dot{Q}/A = -\lambda\, dT/dx$ } Heat transfer by thermal conduction (mole. vib) down temperature gradient. $\boxed{2}$

(Most candidates knew both of these well.)

b)

$$\dot{q} = fn(\ h\ ,\ \lambda\ ,\ D\ ,\ T_h - T_o\ )$$

$\dot{q}$	h	λ	D	T_h-T_o	
W/m^2	W/m^2K	W/mK	m	K	θ = [TEMP]
Q/L^2	$Q/L^2\theta$	$Q/L\theta$	L	θ	Q = [POWER]
					L = [LENGTH]

$\boxed{5}$

5 VARIABLES, 3 DIMENSIONS $\quad 5-3 = 2$ OR MORE N-D GROUPS $\boxed{1}$

{Note: USING M, L, T, θ HAVE:
MT^{-3} $\quad MT^{-3}\theta^{-1}$, $MLT^{-3}\theta^{-1}$, L, θ
5 VAR, 4 DIMS $\Rightarrow 5-4 = 1$ OR MORE N-D GROUPS
HENCE, USING $MLT\theta$ NEED "OR MORE"}

c) $\dfrac{hD}{\lambda} = \dfrac{Q}{L^2\theta}\cdot\dfrac{L}{Q}\cdot L\theta = 1 \quad \Rightarrow \quad \dfrac{hD}{\lambda}$ = NON-DIM. $\boxed{2}$

OTHER GROUP: $\dfrac{\dot{q}}{h\Delta T}$ or $\dfrac{\dot{q}D}{\lambda\Delta T}$ $\quad (\Delta T = T_h - T_o)$ $\boxed{2}$

d) CONVECTION: $T_1 - T_o = \dot{q}/h \quad \Rightarrow R_T = 1/h$ $\boxed{1}$

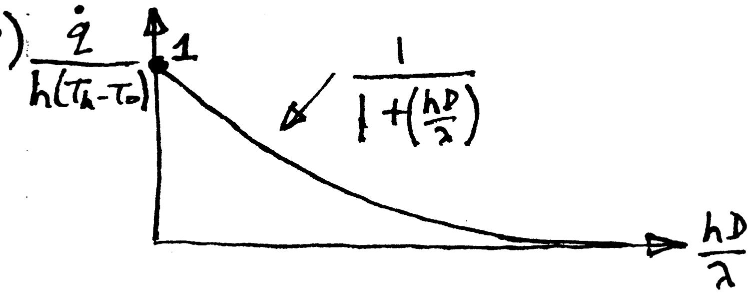
CONDUCTION: $T_h - T_1 = \dot{q}D/\lambda \quad \Rightarrow R_T = D/\lambda$ $\boxed{1}$

$$\Rightarrow T_h - T_o = \dot{q}\left(\frac{1}{h} + \frac{D}{\lambda}\right) = \frac{\dot{q}}{h}\left(1 + \frac{hD}{\lambda}\right)$$

$$\Rightarrow \frac{\dot{q}}{h(T_h - T_o)} = \left(1 + \frac{hD}{\lambda}\right)^{-1} = \frac{1}{1 + (hD/\lambda)} \qquad \boxed{2}$$

(Some candidates confused over "Potential" = ΔT and "current" = $\dot{q}$ in electrical analogue)

e)



$\boxed{2}$

(Very few reached correct graph)

PART IA 2004 PAPER 1

(WITH POST EXAM COMMENTS)

Q5 a)

Upthrust = Weight of water displaced. $\boxed{2}$

b) $\rho_a << \rho_w \Rightarrow$ Hydrostatic pressure gradient within air is much less than in sea water. Hence reasonable to assume the air is at a uniform pressure given by that at the air-water interface $\boxed{2}$

c) $P = \rho_{sw} g D + P_{atm} = 1030 \times 9.81 \times 30 + 100000 = \underline{\underline{403129 \text{ N/m}^2}}$

(Many candidates forgot about ambient pressure) $\boxed{4}$

$PV = mRT \Rightarrow V = \frac{mRT}{P} = \frac{3 \times 287 \times 288}{403129} = \underline{\underline{0.6151 \text{ m}^3}}$ $\boxed{1}$

Weight of sea water Displaced $= V\rho_{sw} g$

$= 0.6151 \times 1030 \times 9.81 = \underline{\underline{6215 \text{ N}}}$ $\boxed{1}$

Weight of air added $= m_a g$

(Most forgot this!) $= 3 \times 9.81 = \underline{\underline{29.4 \text{ N}}}$ $\boxed{1}$

NEW TENSION FORCE $= 12000 - 6215 + 29.4 = \underline{\underline{5814 \text{ N}}}$ $\boxed{1}$

D) Slow expansion to allow time for heat transfer $\boxed{2}$

e) Air Volume $V \Rightarrow$ Weight of sea water displaced $= V\rho_{sw} g$

$$\text{TENSION} = 12000 - V\rho_{sw} g + 29.4 = 0$$

$$\Rightarrow V\rho_{sw} g = 12029.4 \text{ N}$$

$$\Rightarrow V = \frac{12029.4}{1030 \times 9.81} = \underline{\underline{1.1905 \text{ m}^3}}$$ $\boxed{2}$

$$P = \frac{mRT}{V} = \frac{3 \times 287 \times 288}{1.1905} = \underline{\underline{208289 \text{ N/m}^2}}$$ $\boxed{2}$

$$P = \rho_{sw} g D + P_{atm} = 1030 \times 9.81 \times D + 100000 = 208289$$

$$D = \frac{208289 - 100000}{1030 \times 9.81} = \underline{\underline{10.72 \text{ m}}}$$ $\boxed{2}$

Common mistakes were

(i) Forget ambient pressure in (c)

(ii) Omit mass of air added

(iii) Very confused over conditions for isothermal process.

ENGINEERING TRIPOS PART IA, 2004
PAPER 1, SECTION B SOLUTIONS

6 a) If $r = 0.5$ m, extension $= 0.2$ m
so force in spring $= \underline{8\ N}$

$$\therefore \frac{mV^2}{r} = \frac{0.25\,V^2}{0.5} = 8$$

So $V^2 = 16 \rightarrow V = \underline{\underline{4\ m/s}}$

b) Momentum is conserved in the collision, but energy is not. Since the mass doubles, the velocity must halve.
Hence $V_{RADIAL} = \underline{\underline{0}}$, $V_{TANGENTIAL} = \underline{\underline{2\ m/s}}$

The combined mass still has 8 N acting on it, towards O

Hence $A_{RADIAL} = \frac{8}{0.5} = \underline{\underline{16\ m/s^2 \text{ inwards}}}$
and $A_{TANGENTIAL} = \underline{\underline{0}}$

c) The particle continues to travel round O, with varying r - it does <u>not</u> travel in an ellipse (this only happens if the central force is proportional to $1/r^2$, as in satellite orbits). Energy, and Moment of Momentum about O, will be conserved.

After the collision, energy $= \frac{1}{2}mV^2 + \frac{1}{2}ke^2$ (e = Spring extension)

$$= \tfrac{1}{2} \times 0.5 \times 2^2 + \tfrac{1}{2} \times 40 \times 0.2^2$$
$$= 1 + 0.8 = \underline{1.8\ J}$$

$M\ of\ M = 0.5 \times 2 \times 0.5 = \underline{0.5\ kg\,m^2/s}$

6 c) (CONT)

Whenever velocity is perpendicular to OP, $mVr = 0.5 \rightarrow \underline{V = 1/r}$

So by energy, at max. and min. r,

$$\frac{1}{2} \times 0.5 \times \left(\frac{1}{r}\right)^2 + \frac{1}{2} \times 40 \times (r-0.3)^2 = 1.8$$

$$0.25 + 20r^4 - 12r^3 + 1.8r^2 = 1.8r^2$$

$$\underline{\underline{20r^4 - 12r^3 + 0.25 = 0}}$$

This has solutions at $r \approx 0.391$ and $r = 0.5$, which are the minimum and maximum values for r after the collision.

7. a) i) Kinetic energy is <u>not</u> conserved, because each link of the chain is 'jerked' into motion. Momentum <u>is</u> conserved, because no external forces act on the system (car + chains).

ii) By momentum,

$$(M + 2m_c) V_F = M V_0$$

$$\rightarrow \underline{\underline{V_F = V_0 \frac{M}{M + 2m_c}}}$$

iii) Car must travel $\underline{\underline{2L}}$ for chains to straighten

iv) $m = M$ @ $x = 0$, and $m = M + 2m_c$ @ $x = 2L$.

So $\underline{\underline{m = M + \frac{m_c x}{L}}}$

b) By momentum, $mV = M V_0$

i.e. $V = \frac{M}{m} V_0$

Substituting for m using (a)(iv) gives

$$\underline{\underline{V = \frac{ML}{ML + m_c x} V_0}}$$

Check: when $x = 2L$, $V = V_F = \frac{M}{M + 2m_c} V_0$ ✓

c) $a = \frac{dV}{dt} = \frac{V_0 ML}{ML + m_c x} \frac{d}{dx}\left(\frac{V_0 ML}{ML + m_c x}\right)$

$$\therefore \underline{\underline{a = -\frac{V_0^2 M^2 L^2 m_c}{(ML + m_c x)^3}}}$$

This is a maximum when $x = 0$

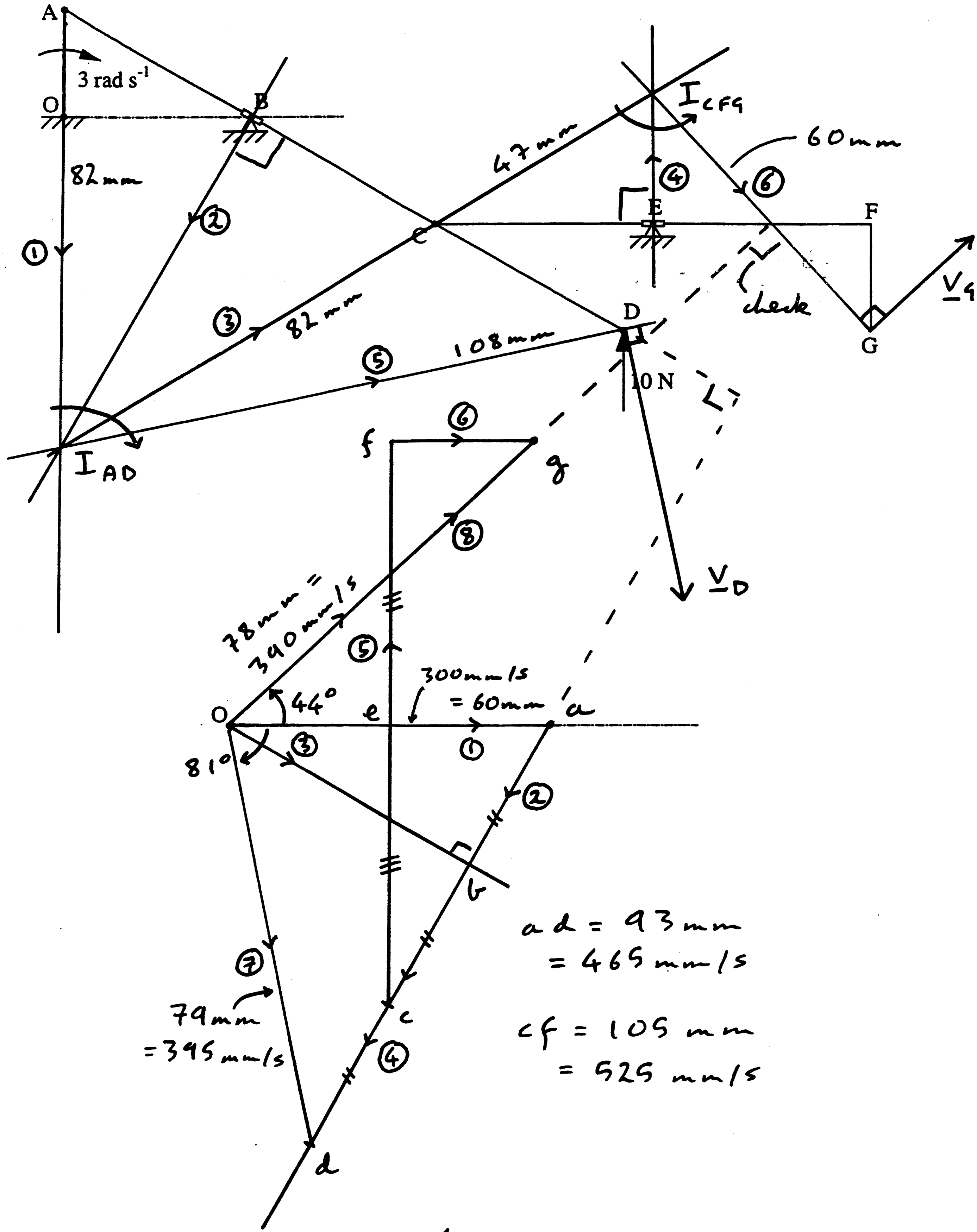
giving $\underline{\underline{a = \frac{-V_0^2 m_c}{ML}}}$

8 Using sheet from question paper:

ENGINEERING TRIPOS PART IA

7 June 2004 9 to 12 Paper 1: MECHANICAL ENGINEERING

Sheet to be handed in with your answer to Question 8.



8 b) (CONT)

The velocities of D and G can be found from the Instantaneous Centres alone, as follows:

$$V_D = V_A \times \frac{I_{AD}D}{I_{AD}A} = 300 \times \frac{108}{82} = \underline{\underline{395\ \text{mm/s}}}$$

$$V_C = V_A \times \frac{I_{AD}C}{I_{AD}A} = V_A$$

$V_G = V_C \times \dfrac{I_{CFG}G}{I_{CFG}C} = 300 \times$ [illegible]/46 $= \underline{\underline{390\ \text{mm/s}}}$

Directions are 81° and 44°, as shown.

c) i) Downwards velocity of V

$$= 395 \sin 81° = \underline{390\ \text{mm/s}}$$

Using work, $T\omega = 390 \times 10$

$$\therefore\ T = \frac{390 \times 10}{3} = 1300\ \text{N mm}$$

$$= \underline{\underline{1.30\ \text{N m}}}$$

ii) $\omega_{AD} = \dfrac{465}{600} = 0.775$ rad/s

$\omega_{CF} = \dfrac{525}{400} = 1.313$ rad/s

A.V.'s are in opposite directions, so work equation gives:

$$3T = 2\,(1.313 + 0.775)$$

$$T = \underline{\underline{1.392\ \text{Nm}}}$$

9. a) i) The spring has no mass, so the force F will immediately act on the mass m. Since there are initially no other forces acting on the mass, it's acceleration will be $\underline{\underline{F/m}}$

ii) Eventually the mass will come to rest, and there will be no force in the dashpot. The lower spring will have a tension of F, so the displacement of the mass will be $\underline{\underline{F/k}}$

iii) The upper spring will have a compressive force of F, so the total displacement of A will be

$$F/2k + F/k = \underline{\underline{3F/2k}}$$

(Many candidates found this quite difficult! Some gave the answers $\ddot{y}$, y and z to the questions above - these were not valid answers.)

b) If the compressive force in the upper spring is f, we can write:

$$m\ddot{y} + \lambda\dot{y} + ky = f = 2k(z - y)$$

9 b) (CONT)

So $m\ddot{y} + \lambda\dot{y} + 3ky = 2kz$

$$\frac{\ddot{y}}{3k/m} + \frac{\lambda}{3k}\dot{y} + y = \tfrac{2}{3} z$$

This is case (a) in the D.B., with

$$\omega_n = \sqrt{\frac{3k}{m}}$$

$$\frac{2\zeta}{\omega_n} = \frac{\lambda}{3k} \rightarrow \zeta = \frac{\lambda}{2\sqrt{3km}}$$

and $x = \tfrac{2}{3} z$

c) For this system, $\omega_n = \sqrt{\frac{400}{1}} = 30$ rad/s

$6\ \text{hz} = 12\pi$ rad/s $\rightarrow \omega/\omega_n \approx \underline{1.26}$

$$\zeta = \frac{\lambda}{2\sqrt{3km}} = \frac{12}{2\times 30} = \underline{0.2}$$

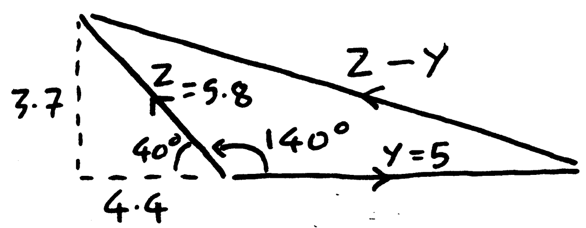
From the graph on page 9 of the databook,

$Y/X \approx 1.30 \rightarrow Z = 3/2 \times \frac{5}{1.30} = \underline{\underline{5.8\ \text{mm}}}$

(Exact calculated value is 1.304)

d) From the databook, the phase angle between input and output $\approx 140°$

Hence:



$$|Z - Y| = \sqrt{3.7^2 + (5 + 4.4)^2} = \pm 10.1\ \text{mm}$$

$$\therefore |F| = 2k \times 10.1 \times 10^{-3} = \underline{\underline{\pm 6.06\ N}}$$

10. a) Equilibrium for each rotor:

$$2J\ddot{\theta}_1 = k(\theta_2 - \theta_1) - 2k\theta_1$$

$$J\ddot{\theta}_2 = T + k(\theta_1 - \theta_2)$$

So in matrix form,

$$\underset{\text{MASS MATRIX}}{\begin{bmatrix} 2J & 0 \\ 0 & J \end{bmatrix}}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \underset{\text{STIFFNESS MATRIX}}{\begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix}}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ T \end{bmatrix}$$

b) For natural modes, $T = 0$, $\ddot{\theta} = -\omega^2\theta$

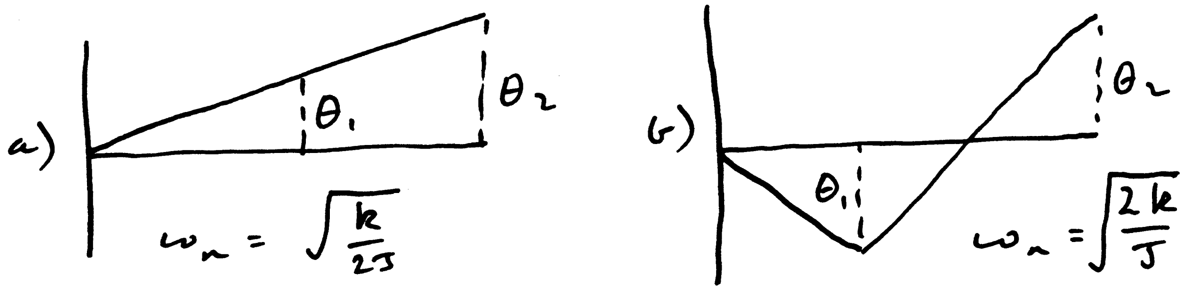
$$\therefore \begin{bmatrix} 2J\omega^2 - 3k & k \\ k & J\omega^2 - k \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = 0$$

$$\text{DET} = 2J^2\omega^4 - 5kJ\omega^2 + 2k^2 = 0$$

$$\frac{J\omega^2}{k} = \frac{+5 \pm \sqrt{25-16}}{4} = 2, \tfrac{1}{2}$$

$$\omega_n = \sqrt{\frac{k}{2J}}, \sqrt{\frac{2k}{J}}$$

Corresponding modal shapes are:



(In (a), $\theta_2 = 2\theta_1$ and in (b) $\theta_2 = -\theta_1$, but this was not asked for.)

Many candidates (wrongly!) suggested that $\theta_2 = \theta_1$ in mode (a)

10. c) As in (b), $\ddot{\theta} = -\omega^2 \theta$, so

$$\begin{bmatrix} 2J\omega^2 - 3k & k \\ k & J\omega^2 - k \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ T \end{bmatrix}$$

So $$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \frac{1}{DET} \begin{bmatrix} J\omega^2 - k & -k \\ -k & 2J\omega^2 - 3k \end{bmatrix} \begin{bmatrix} 0 \\ T \end{bmatrix}$$

For $\theta_1 = 0$, $-kT = 0$: No solution

for $\theta_2 = 0$, $(2J\omega^2 - 3k)T = 0$

i.e. $\underline{\underline{\omega = \sqrt{\frac{3k}{2J}}}}$

Also – $\dot{\theta} = \omega\theta$, so there is no <u>movement</u> in either rotor when $\omega = 0$

And – at $\omega = \infty$, $DET = \infty^2$, so θ_1, θ_2 (and $\dot{\theta}_1$, $\dot{\theta}_2$) are all zero.

———— " ————

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July 2004