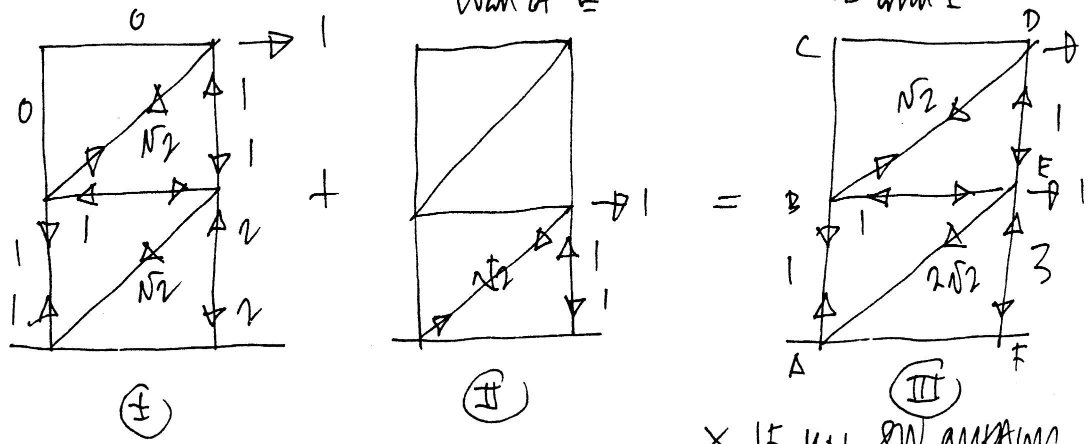
PART IA PAPER 2 SECTION A

1 (a) for unit load at D — from top panel, unit load at E — unit loads at D and E



× 15 kN for actual problem

(b) virtual work
compatible displacements are those produced by loading (III),
equilibrium bar forces and external loads correspond to loading I

bar	length	force (III)	extension (III)	tension (I)	$T_{ij} e_{ij}$
	1000 mm ×	15 kN ×	$\frac{15000}{EA}$ × mm		$\frac{15000}{EA}$ × mm
AB	1	~~0~~ + 1	1	1	1
BC	1	0	0	0	
CD	1	0	0	0	
DE	1	− 1	− 1	− 1	1
EF	1	− 3	− 3	− 2	6
DB	$\sqrt{2}$	+ $\sqrt{2}$	2	$\sqrt{2}$	$2\sqrt{2}$
BE	1	− 1	− 1	− 1	1
EA	$\sqrt{2}$	+ $2\sqrt{2}$	4	+ $\sqrt{2}$	$4\sqrt{2}$
					$9 + 6\sqrt{2}$

horizontal displacement at D $= \frac{15000}{6000}(9 + 6\sqrt{2}) = 43.7$ mm

1 (continued)

(c)

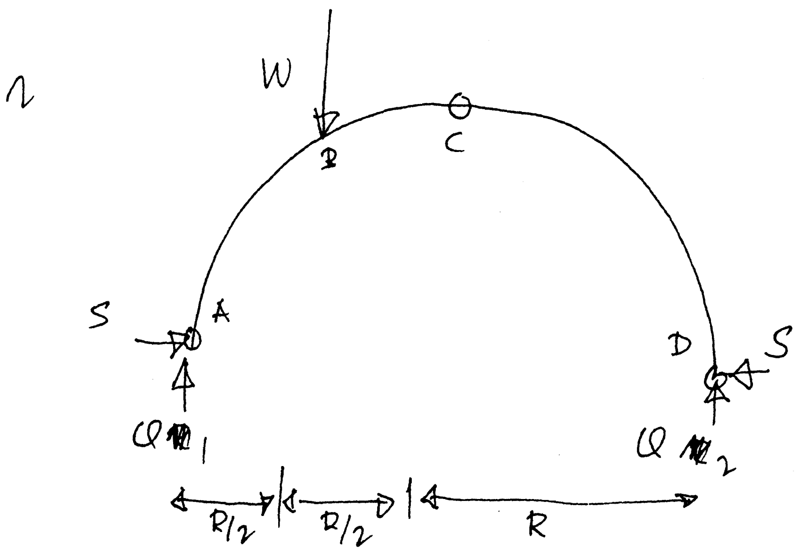
bar	extension	tension ($\mathcal{T}$)	$T_{ij} e_{ij}$ $\Delta \times$
AB	$\pm \Delta$	1	1
BC	"	0	0
CD	"	0	0
DE	"	-1	1
EF	"	-2	2
DB	"	$\sqrt{2}$	$\sqrt{2}$
BE	"	-1	1
EA	"	$+\sqrt{2}$	$\sqrt{2}$
			$(5+2\sqrt{2})\Delta$

always taking + sign so as to maximise displacement

$$(5+2\sqrt{2})\Delta \ngtr 20$$

$$\Delta \ngtr 2.55 \text{ mm}$$

2



(a) moments about D for whole frame

$$Q_1 \cdot 2R = W \cdot \tfrac{3}{2}R$$

$$Q_1 = \tfrac{3}{4}W$$

from vertical equilibrium

$$Q_2 = \tfrac{1}{4}W$$

moments about C for CD

$$SR = Q_2 R$$

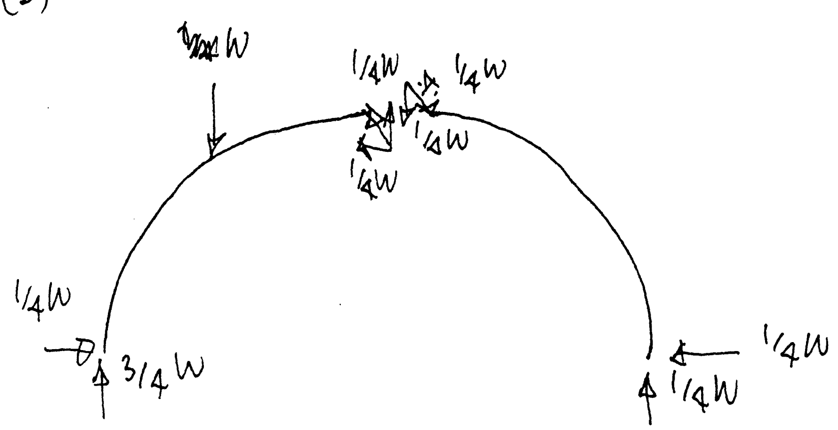
$$S = Q_2 = \tfrac{1}{4}W$$

[check
moments about C for AC

$$S\cancel{R} + W\tfrac{\cancel{R}}{2} = \tfrac{3}{4}W\cancel{R}$$

$$S = \tfrac{1}{4}W \quad]$$

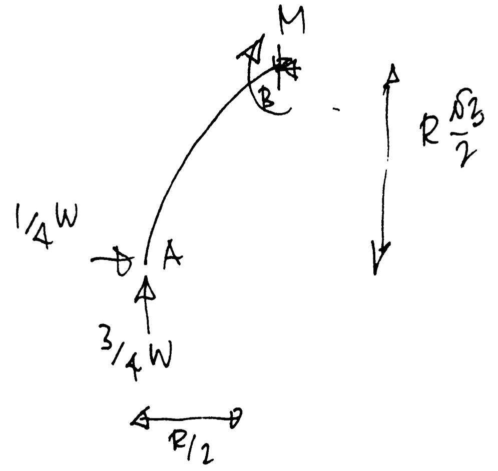
(b)



at C, reaction acts upward and to the left on CA, downward and to the right on CD

2 (continued)

(c) fbd for BA

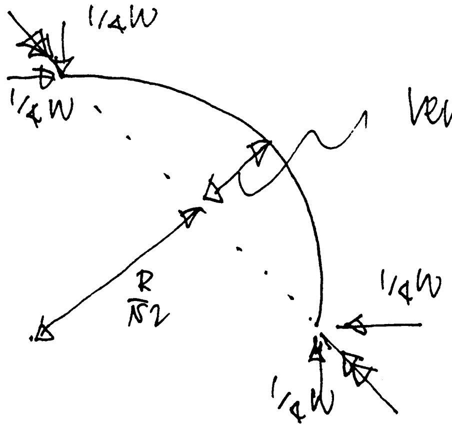


moments about B

$$M + \frac{3}{4}W\frac{R}{2} = \frac{1}{4}W\frac{\sqrt{3}}{2}R$$

$$M = -WR\,\frac{3-\sqrt{3}}{8}$$

(d) forces on CD



lever arm greatest at midpoint

$$\text{moment} = \frac{1}{4}W\sqrt{2}\left(R - \frac{R}{\sqrt{2}}\right)$$

$$= \frac{1}{4}WR\left(\sqrt{2}-1\right)$$

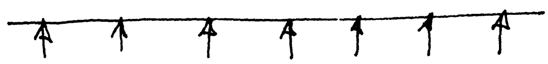
3(a) Plan area of barge is $100 \times 20 = 2000\ m^2$

For each m barge sinks, additional buoyancy force $= 10\ kN/m^3 \times 2000\ m^2$

$= 20\ MN$

$\therefore$ barge sinks by 2.5 m

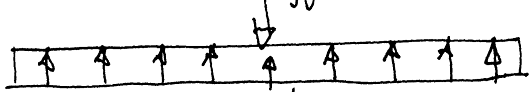
(b) additional loading



0.5 MN/m

(c)

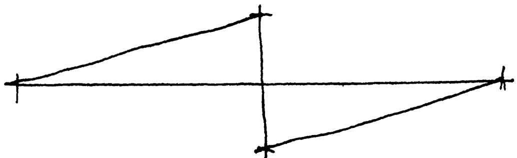
50



0.5 /m

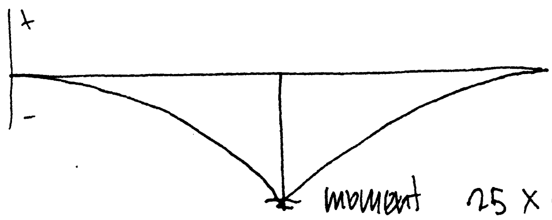
shear force

−25 MN



+25

(d) bending moment

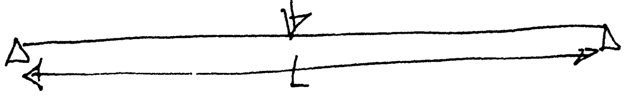


moment $25 \times 25 = 625$ MN m

(e) from data book, superpose two standard cases

central deflection

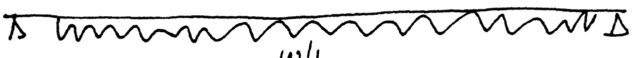
W



L

$\frac{1}{48}\frac{WL^3}{EI}$ ↓

+

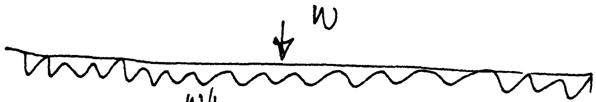


W/L

$\frac{5}{384}\frac{\frac{W}{L}L^4}{EI}$ ↑

=

W



W/L

$\frac{3}{384}\frac{WL^3}{EI}$ ↓

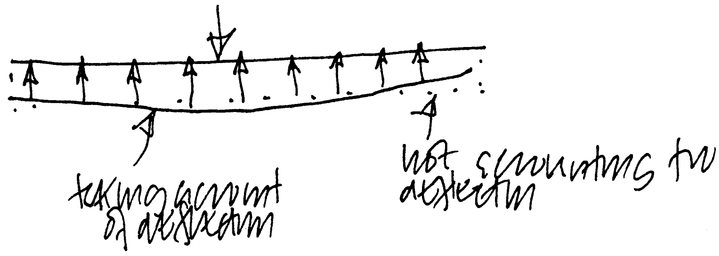
(e) $\frac{3}{384}\frac{(5\times10^{7})(10^{2})^{3}}{2\times10^{12}} = 0.195 \text{ m}$

N m²

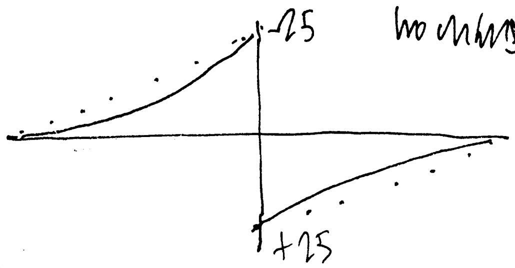
3 (continued)

(f) the barge deflects and sinks slightly more at the centre and slightly less at the ends

loading

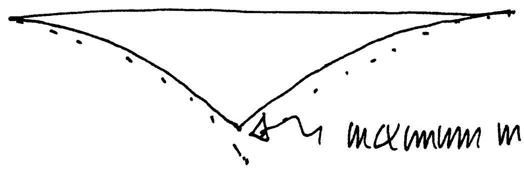


shear force



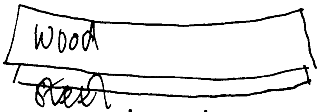
no change in maximum

bending moment



maximum moment slightly reduced

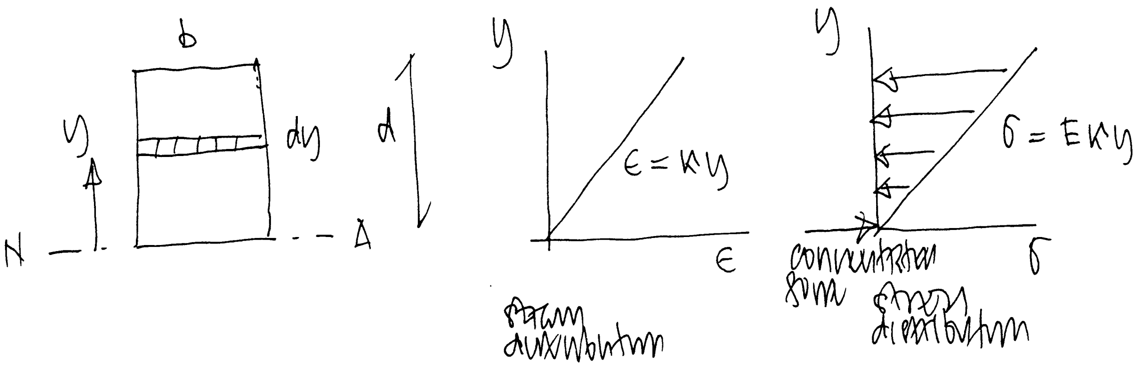
4(a)(i) true, by a symmetry argument considering that a section of a beam in pure bending must deform in the same way when observed from either side of the beam (but not strictly true if the beam is not in pure bending but carries a shear force);
(ii) true, by considering the geometry of deformation, and the fact that there must be a neutral surface on which the longitudinal strain is zero;
(iii) true for either of the two materials individually, but the constant of proportionality is different;
(iv) false: this is not true in general, because the neutral axis is fixed by the condition that the longitudinal force is zero, though it might chance to be where the steel and the wood meet;
(v) false, because they have different moduli and different maximum distances from the neutral axis;
(vi) false: if the screws are removed the steel and the wood bend as separate beams



and there is no composite action.

(this part of the question does not require any calculation: cross-section (a) was included to fix ideas about what kind of beam is being considered)

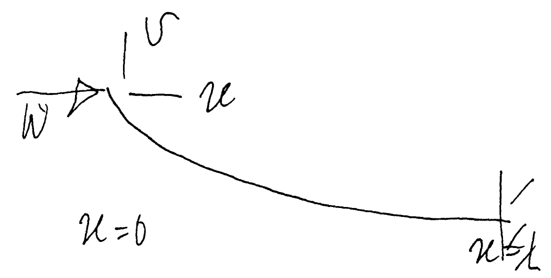
(b) the material cannot deform in compression: therefore the neutral axis must be at the bottom (or top) of the cross-section. The same result can be secured by applying the transformed section method.



take moments about $y = 0$

$$\text{moment} = \int_0^d \sigma b y \, dy = \int_0^d E\kappa b y^2 \, dy = \frac{1}{3} E\kappa b d^3$$

5(a)



FBD

$$EI\frac{d^2v}{dx^2} = M = W(-v)$$

$$EI\frac{d^2v}{dx^2} + Wv = 0$$

$$\frac{d^2v}{dx^2} + \alpha^2 v = 0 \quad \text{where } \alpha^2 = \frac{W}{EI}$$

GS $\quad v = c_1 \sin \alpha x + c_2 \cos \alpha x$

$$\frac{dv}{dx} = \alpha c_1 \cos \alpha x - \alpha c_2 \sin \alpha x$$

At $x = 0$, $v = 0$: $\quad 0 = c_2$

$x = L$, $\frac{dv}{dx} = 0$: $\quad 0 = \alpha c_1 \cos \alpha L - \alpha c_2 \sin \alpha L$

If $c_1 \neq 0$, $\alpha \neq 0$: $\quad 0 = \cos \alpha L$

$$\alpha L = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$$

taking lowest root

$$L\sqrt{\frac{W}{EI}} = \frac{\pi}{2}$$

$$W = \frac{\pi^2}{4}\frac{EI}{L^2}$$

(b) $\quad 4 \times 10^5 \text{ N} = \frac{\pi^2}{4} \frac{2.1 \times 10^5 \text{ N/mm}^2}{6000^2 \text{ mm}^2} I$

$$I = 4 \times 10^5 \times \frac{4}{\pi^2} \times \frac{6000^2}{2.1 \times 10^5} = 2.779 \times 10^7 \text{ mm}^4$$

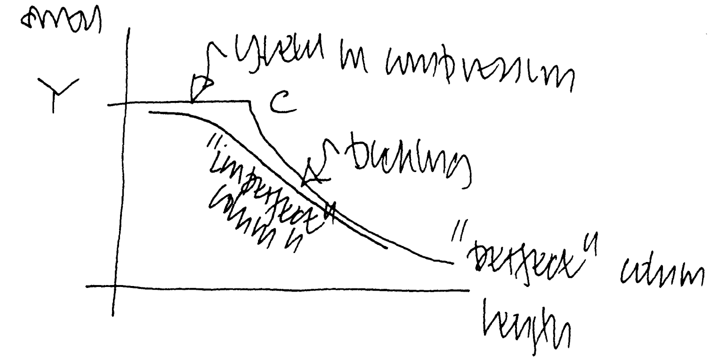
$$= 2779 \text{ cm}^4$$

254 × 254 × 73 UC section has $I = 3908 \text{ cm}^4$ in weaker direction

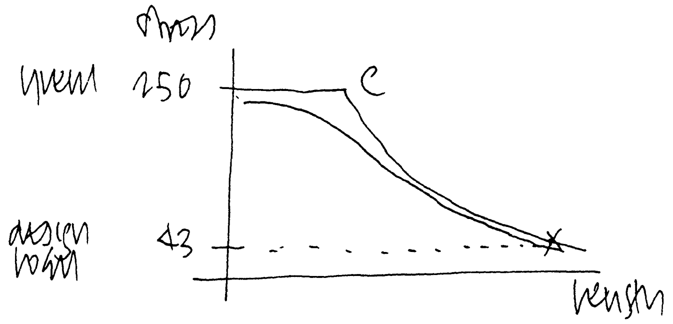
area = 93.1 cm^2 = 9310 mm^2

(c) $\quad$ mean stress $= \frac{4 \times 10^5 \text{ N}}{9310 \text{ mm}^2} = 43.0 \text{ N/mm}^2$

(d) The relationship between mean stress and length is

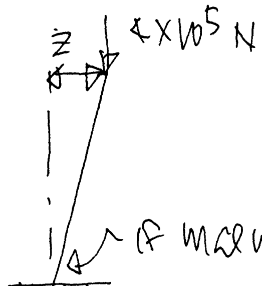


Imperfection sensitivity is highest near C. In this case



We are well away from C, and the column is not highly sensitive to imperfection.

However, if the column is a long way out of vertical the bending stress may approach yield



if maximum stress here is yield stress

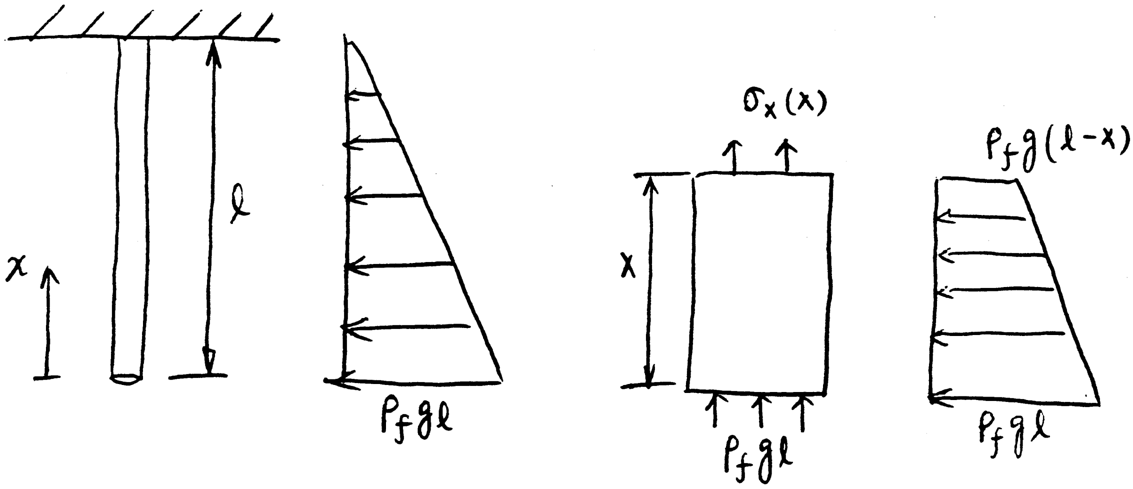
$$250 = 43.0 + \frac{(4 \times 10^5 \text{ N}) \, Z \text{ mm} \, (127.3 \text{ mm})}{\cancel{2.779}\ 3.908 \times 10^7 \text{ mm}^4}$$

$$207.0 = 1.303 Z$$

$$Z = 159 \text{ mm}$$

which is more than "slightly" out of vertical.

6. (a)



Force equilibrium in x-direction:

$$\sigma_x(x)\,\pi a^2 + \rho_f g l\,\pi a^2 = \rho g\,\pi a^2 x$$

$$\Rightarrow \quad \sigma_x(x) = \rho g x - \rho_f g l$$

(b) Maximum stress at $x = l$:

$$\sigma_x^{max} = (\rho - \rho_f)\, g\, l_{max} = (7900 - 1020) \times 9.8 \times l_{max} \quad (N/m^2)$$

But $\sigma_x^{max} = \sigma_y / 10$

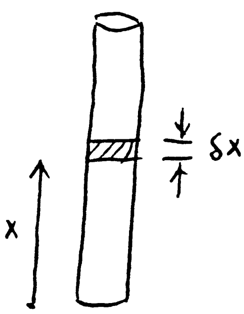
$$\Rightarrow \quad l_{max} = \frac{700 \times 10^6}{10 \times (7900-1020) \times 9.8} = 1038.2\ m$$

(c) At x position:

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z), \quad \sigma_y = \sigma_z = -\rho_f g(l-x)$$

$$\Rightarrow \quad \varepsilon_x = \frac{\rho g x - \rho_f g l}{E} + \frac{2\nu}{E}\rho_f g (l-x)$$

$$= \frac{\Delta(\delta x)}{\delta x}$$



$$\Rightarrow \quad \Delta l = \int_0^{\Delta l} d(\delta x) = \int_0^l \varepsilon_x\, dx$$

$$= \frac{g}{E}\int_0^l (\rho x - \rho_f l + 2\nu\rho_f l - 2\nu\rho_f x)\, dx$$

$$= \frac{g}{E}\left[(\rho - 2\nu\rho_f)\frac{x^2}{2} - (\rho_f - 2\nu\rho_f)\, l x\right]_0^l$$

6. (c) continued

$$\Rightarrow \quad \Delta l = \frac{g}{E}\left[(\rho - 2\nu\rho_f)\frac{l^2}{2} - \rho_f l^2(1-2\nu)\right]$$

Let $l = l_{max}$

$$\Delta l = \frac{9.8}{200\times10^9}\left[(7900 - 2\times0.3\times1020)\frac{1038^2}{2} - 1020\times1038^2\times(1-2\times0.3)\right]$$

$$= 0.171 \text{ m}$$

(d)

$$\underbrace{\Delta}_{\text{dilatation}} = -\frac{\overbrace{P}^{\text{hydrostatic pressure}}}{\underbrace{K}_{= \frac{E}{3(1-2\nu)} \text{ bulk modulus}}}$$

At 2000 m deep, $P = \rho_f g \times 2000$

$$\Rightarrow \quad \Delta = -\frac{2000\,\rho_f g \times 3(1-2\nu)}{E}$$

$$= -\frac{2000\times1020\times9.8\times3(1-2\times0.4)}{200\times10^9}$$

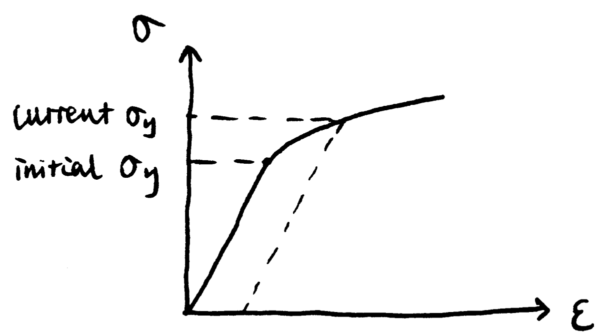
$$= \underbrace{-1.2\times10^{-4}}_{\text{negative means shrinkage}}$$

Change in volume

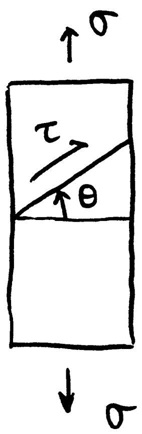
$$\delta V = \Delta \times V = \Delta \cdot \pi a^2 l_{max}$$

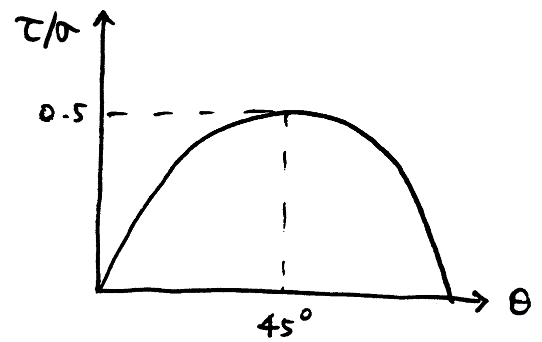
$$= -1.2\times10^{-4}\times3.14\times0.1^2\times1038 = -0.004 \text{ m}^3$$

7. (a) Plastic flow is caused by the movement of many dislocations. Dislocations will interact with each other. More dislocations mean more interaction, and hence it is more difficult to move the dislocations. This leads to an increase in σ_y and τ_y, resulting in the usual work hardening. During work hardening, dislocation density typically increases from $10^9\ mm^2$ to $10^{13}\ mm^2$.



(b)





$\tau = \sigma \sin\theta \cos\theta$

$\cos 45° \sin 45°$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 0.5$

τ at maximum when $\theta = 45° \Rightarrow \tau_{max} = \frac{\sigma}{2}$

For single crystal, $\tau_{max} = \tau_y = \frac{\sigma_y}{2}$ at yield $\Rightarrow \boxed{\sigma_y = 2\tau_y}$

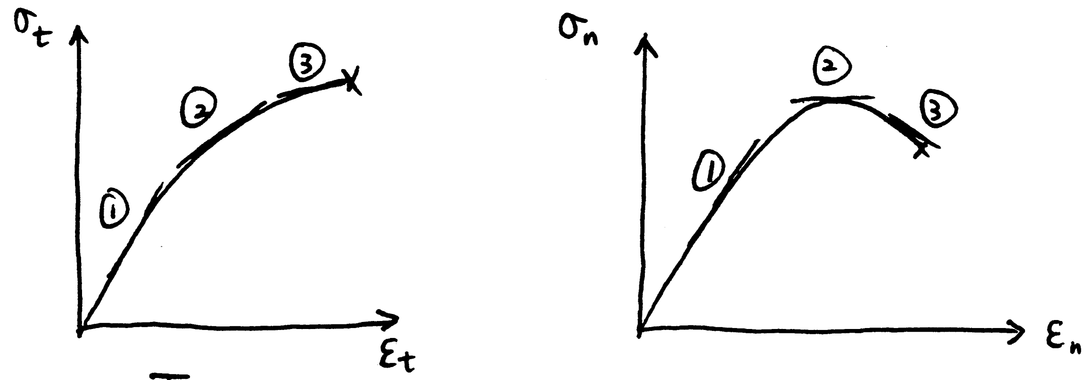
For polycrystalline material, yielding is more difficult due to grains lying in different orientations.

Taylor factor

Shear yield stress of polycrystal $= k = 1.5\ \tau_y$

But $k = \frac{\sigma_y}{2} \Rightarrow \underline{\sigma_y = 2 \times 1.5\ \tau_y = 3\tau_y}$

7. (c)



True　　　　　　　　Nominal

① Before necking $\quad \frac{d\sigma}{d\varepsilon} > \sigma$

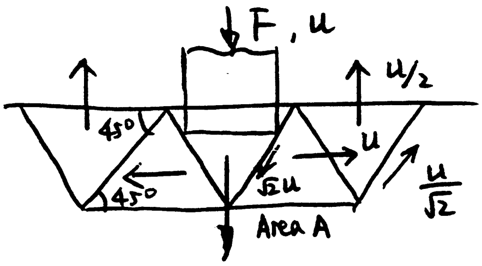
② At necking $\quad \frac{d\sigma}{d\varepsilon} = \sigma$

③ After necking $\quad \frac{d\sigma}{d\varepsilon} < \sigma$

At equilibrium
$$\underbrace{\underset{\text{Force}}{\delta F} = \delta(\underset{\text{Area}}{A}\sigma) = 0 \Rightarrow A\,d\sigma + \sigma\,dA = 0}_{\text{critical conditions of stability}}$$

When $\delta F > 0$, namely, σA increases under constant external load, specimen remains stable, with $\frac{d\sigma}{\sigma} > -\frac{dA}{A}$

(d)



Net work done $= Fu$

$$= \frac{2kA}{\sqrt{2}} u\sqrt{2} + 2Aku + 4k\frac{A}{\sqrt{2}} \cdot \frac{u}{\sqrt{2}}$$

$$= 6Aku \Rightarrow F = 6Ak$$

But $k = \frac{\sigma_y}{2} \Rightarrow \underline{H = \frac{F}{A} = 6k = 3\sigma_y}$

Approximations

① No work hardening*

② Elasticity ignored

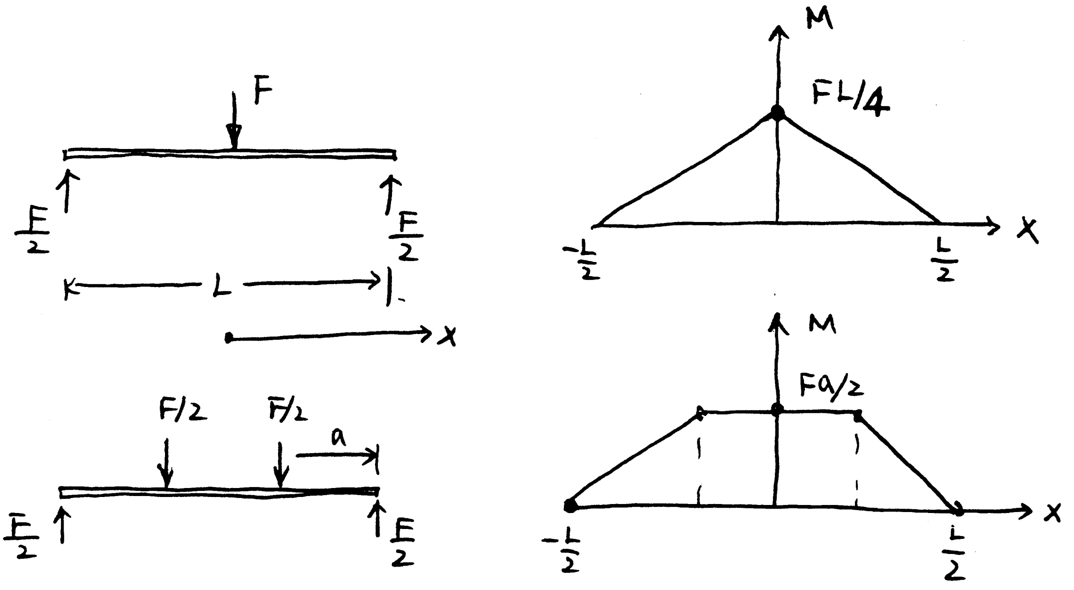
③ Only shear yield at 45° planes (i.e., polycrystals)

④ Deformation within each block ignored

⑤ 3D effects ignored

⑥ Changes in dimensions ignored

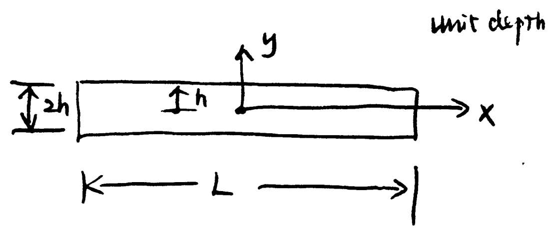
8. (a)



$$\sigma = \frac{My}{I}$$

In 3-point bending, σ_{max} occurs at one point ($x=0$) only, whereas in 4-point bending, σ_{max} occurs in a volume bounded by $|x| \le \frac{L}{2} - a$. In general, it is expected that the ceramic under 4-point bending is more critical than that in 3-point bending.

(b) $M = \frac{F}{2}\left(\frac{L}{2} - x\right)$



$$\Rightarrow \sigma = \frac{My}{I} = \sigma_b \frac{y}{h}\left(1 - \frac{x}{L/2}\right)$$

Probability of survival in 3-point bending

$$P_{sb} = \exp\left\{ \int_V -\left(\frac{\sigma}{\sigma_0}\right)^m \frac{dV}{V_0} \right\}$$

ignore compression

$$= \exp\left\{ \int_V - \frac{1}{V_0}\left(\frac{\sigma_{b1}}{\sigma_0}\right)^m \int_{-L/2}^{L/2} \left(1 - \frac{x}{L/2}\right)^m dx \int_0^h \left(\frac{y}{h}\right)^m dy \right\}$$

$$= \exp\left\{ -\frac{1}{V_0}\left(\frac{\sigma_{b1}}{\sigma_0}\right)^m \cdot \frac{L}{m+1} \cdot \frac{h}{m+1} \right\} = \exp\left\{ -\frac{V}{2V_0}\left(\frac{\sigma_{b1}}{\sigma_0}\right)^m \frac{1}{(m+1)^2} \right\}$$

$V = 2hL$

Probability of survival in uniform uniaxial tension.

$$P_{st} = \exp\left\{ -\frac{V}{V_0}\left(\frac{\sigma_t}{\sigma_0}\right)^m \right\}$$

Let $P_{sb} = P_{st}$ $\Rightarrow$ $\exp\left\{ -\frac{V}{2V_0}\left(\frac{\sigma_{b1}}{\sigma_0}\right)^m \frac{1}{(m+1)^2} \right\} = \exp\left\{ -\frac{V}{V_0}\left(\frac{\sigma_t}{\sigma_0}\right)^m \right\}$

$$\Rightarrow \underline{\frac{\sigma_{b1}}{\sigma_t} = \left[2(m+1)^2\right]^{1/m}}$$

8. (c) $$\frac{\sigma_{b1}}{\sigma_t} = \left[2(m+1)^2\right]^{1/m}$$

$$\frac{\sigma_{b2}}{\sigma_t} = \left[\frac{4(m+1)^2}{2+m}\right]^{1/m}$$

When $m=2$, $\sigma_{b1}/\sigma_t = 3\sqrt{2}$, $\sigma_{b2}/\sigma_t = 3$. Note $\sigma_{b1} > \sigma_{b2}$, which is consistent with (a). Also, $\sigma_{b1} > \sigma_t$ and $\sigma_{b2} > \sigma_t$ because less volume of material is stressed under maximum stress than that under uniaxial tension, so the chance of the 'weakest link' coinciding with a high stress region is reduced.

9. (b)

l, b fixed

(i) Column weight $W = \rho g l a b$ (1)

Minimise W, subject to

$$P \le P_B$$

$$\frac{1}{P_B} = \frac{l^2}{\pi^2 E} \cdot \frac{12}{a b^3} + \frac{1}{\sigma_y a b} \qquad (2)$$

Note that: choose $I = ab^3/12$ not $I = a^3 b/12$ because $a \ge b$.

Let $P = P_B$

From (2) $\Rightarrow$

$$ab = P\left[\frac{1}{\sigma_y} + \frac{12}{\pi^2 E}\left(\frac{l}{b}\right)^2\right] \qquad (3)$$

Substitute (3) into (1) to get

$$W_{min} = \frac{\rho g l P}{\sigma_y}\left[1 + \frac{12}{\pi^2}\frac{\sigma_y}{E}\left(\frac{l}{b}\right)^2\right] \qquad (4)$$

Nondimensionlise

$$\frac{W_{min}}{\rho g l^3} = \frac{P}{\sigma_y l^2}\left[1 + \frac{12}{\pi^2}\frac{\sigma_y}{E}\left(\frac{l}{b}\right)^2\right] \qquad (5)$$

From (3):

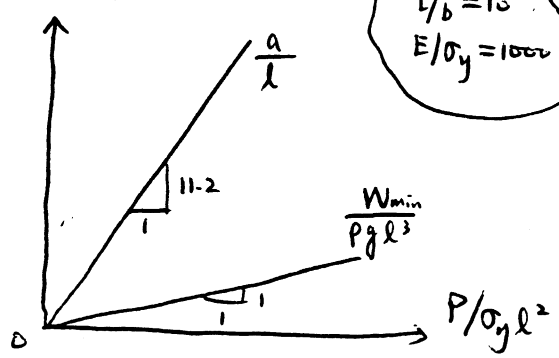
$$\frac{a}{l} = \frac{P}{\sigma_y l^2} \cdot \frac{l}{b}\left[1 + \frac{12}{\pi^2}\frac{\sigma_y}{E}\left(\frac{l}{b}\right)^2\right] \qquad (6)$$

(ii) $l/b = 10$, $\sigma_y / E = 1/1000$

$$\Rightarrow \frac{W_{min}}{\rho g l^3} = \frac{P}{\sigma_y l^2}\left(1 + \frac{12}{\pi^2}\cdot\frac{1}{1000}\cdot 100\right) = 1.12\,\frac{P}{\sigma_y l^2}$$

$$\frac{a}{l} = \frac{P}{\sigma_y l^2} \times 10 \times 1.12 = 11.2\,\frac{P}{\sigma_y l^2}$$

$l/b = 10$
$E/\sigma_y = 1000$



9. (a) (ii) continued

$$W_{min} = \frac{\rho}{\sigma_y} P l g \left[1 + \frac{12}{\pi^2} \frac{\sigma_y}{E} \left(\frac{l}{b}\right)^2 \right]$$

The material selection index is $I = \rho / \sigma_y$. Minimise I to minimise W_{min}.

(b) For stress applied in the longitudinal direction:

Strain in layer 1 = strain in layer 2 = strain in composite

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} = \frac{\sigma}{E_{//}} = \varepsilon$$

But $\sigma = V_1 \sigma_1 + (1-V_1)\sigma_2$, $\quad V_1 = \frac{t_1}{t_1 + t_2}$

$$\Rightarrow \quad E_{//} \cancel{\varepsilon} = V_1 E_1 \cancel{\varepsilon} + (1-V_1) E_2 \cancel{\varepsilon}$$

$$\Rightarrow \quad E_{//} = V_1 E_1 + (1-V_1) E_2 = (t_1 E_1 + t_2 E_2)/(t_1 + t_2)$$ "rule of mixtures"

For stress applied in the transverse direction:

Stress in layer 1 = stress in layer 2 = stress in composite

$$\sigma_1 = E_1 \varepsilon_1 = \sigma_2 = E_2 \varepsilon_2 = \sigma = E_\perp \varepsilon$$

But, displacement Δ over length $t_1 + t_2$

$$\Delta = \varepsilon (t_1 + t_2) = \varepsilon_1 t_1 + \varepsilon_2 t_2$$

$$\Rightarrow \quad \frac{\cancel{\sigma}}{E_\perp}(t_1 + t_2) = \frac{\cancel{\sigma}}{E_1} t_1 + \frac{\cancel{\sigma}}{E_2} t_2$$

$$\Rightarrow \quad \frac{1}{E_\perp} = \frac{t_1/(t_1+t_2)}{E_1} + \frac{t_2/(t_1+t_2)}{E_2}$$

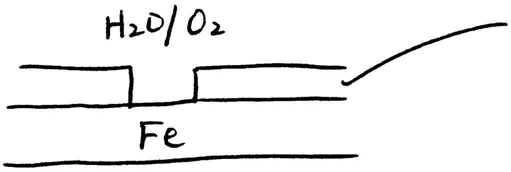
$$\Rightarrow \quad E_\perp = \left\{ \frac{V_1}{E_1} + \frac{1-V_1}{E_2} \right\}^{-1}$$

10. (a) (i) In one mechanism of hydrogen embrittlement, hydrogen atoms pin the movement of dislocations and hence restrict plasticity. This causes the crack to break in a brittle manner, leading to a lower fracture toughness.

In stress corrosion cracking (for some materials and under certain environments), cracks grow steadily under a constant load well below the fast fracture load.

(ii) Al has a larger driving force — see Data Book

Fe has a higher rate of reaction in water, and hence needs galvanizing.



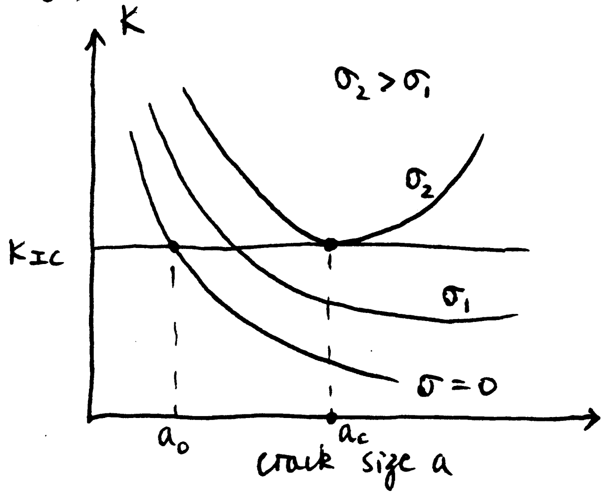
protective layer (e.g., Zn, Ni) shields Fe

Broken film: Zn preferentially corrodes due to more negative electrochemical potential to protect Fe.

(b) $K = K_{IC}$

$$0.016\sqrt{\frac{300}{3}}\ \frac{100}{a^{3/2}} = 4\times10^6 \Rightarrow \underline{a = 0.25\ \text{mm}}$$

(c) (i)



This plot shows the influence of applied stress σ on growth of an arrested indentation crack, length a_0. The additional term in K leads to a minimum in the total K, corresponding to a crack length of a_c. The crack undergoes stable growth as σ is increased. Once $a = a_c$, the condition for final (unstable) failure is obtained.

$$K = 0.016\,(E/H)^{1/2}\,P/a^{3/2} + 1.12\,\sigma\sqrt{\pi a}$$

10. (c) (ii)

Let $\sigma = \sigma_c$ at unstable crack growth, at which

$$\frac{dK}{da} = 0 \quad \& \quad K = K_{Ic}$$

$$\Rightarrow \quad 0.016\,(E/H)^{1/2} P \left(-\tfrac{3}{2}\right) a^{-5/2} + 1.12\,\sigma_c \sqrt{\pi} \cdot \tfrac{1}{2} a^{-1/2} = 0$$

$$\Rightarrow \quad a_c^2 = 0.024 \frac{P}{\sigma_c} \left(\frac{E}{H}\right)^{1/2}$$

But

$$K_{Ic} = 0.016 \left(\frac{E}{H}\right)^{1/2} \frac{P}{a_c^{3/2}} + 1.12 \times 0.024 \sqrt{\pi} \frac{P}{a_c^{3/2}} \left(\frac{E}{H}\right)^{1/2}$$

$$= 0.21 \left(\frac{E}{H}\right)^{1/2} \frac{P}{a_c^{3/2}}$$

$$\Rightarrow \quad a_c^{3/2} = 0.21 \left(\frac{E}{H}\right)^{1/2} \frac{P}{K_{Ic}} \quad \Rightarrow \quad \underline{a_c = 0.35 \left(\frac{E}{H}\right)^{1/3} \left(\frac{P}{K_{Ic}}\right)^{2/3}}$$

$$\Rightarrow \quad \sigma_c = \frac{0.024\,P\,(E/H)^{1/2}}{0.35^2\,(P/K_{Ic})^{4/3}\,(E/H)^{2/3}} = \underline{1.96\,(H/E)^{1/6}\,K_{Ic}^{4/3}/P^{1/3}}$$