1/ a) An ideal op-amp has infinite voltage gain A, infinite input resistance R_i, and zero output resistance, R_o.

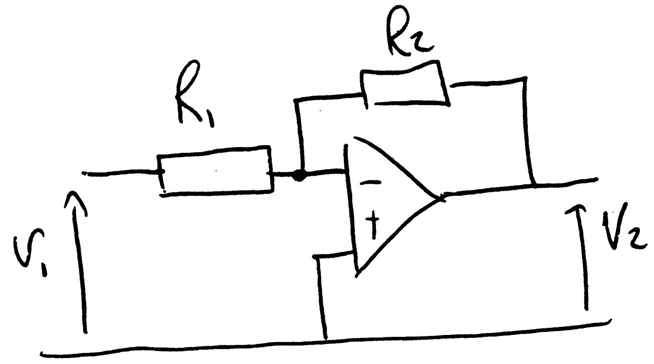
The simplifications that the ideal assumptions allow are:

$A \to \infty \Rightarrow V_+ - V_- \to 0$ so $V_+ = V_-$ - Virtual earth principle.

$R_i \to \infty \Rightarrow i_+ = i_- = 0$ so sum of currents at + and - inputs must be zero.

$R_o \to 0 \Rightarrow$ loading the output of the op-amp does not affect its output voltage.

b)

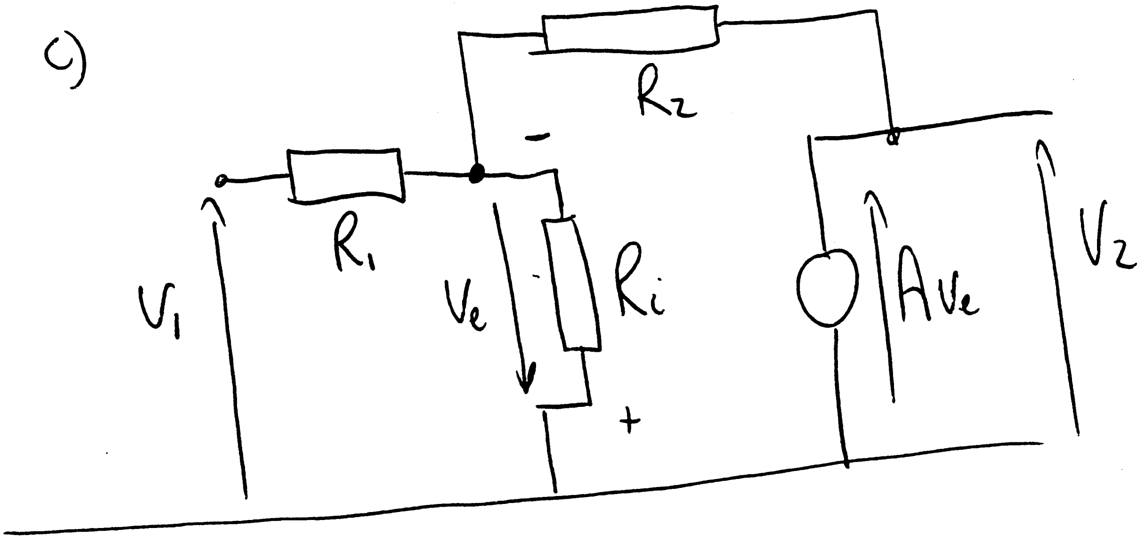


$V_- = 0V$ (Virtual earth principle)

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 0 \quad (R_i = \infty)$$

$$\underline{\underline{\frac{V_2}{V_1} = -\frac{R_2}{R_1}}}$$

c)



KCL at inverting input:

$$\frac{V_1+V_e}{R_1}+\frac{V_e}{R_i}+\frac{V_2+V_e}{R_2}=0 \qquad AV_e = V_2$$

$$\frac{V_1+V_2/A}{R_1}+\frac{V_2}{AR_i}+\frac{V_2+V_2/A}{R_2}=0$$

$\times AR_1R_2R_i$ $\quad AR_2R_iV_1 + V_2\left(R_2R_i+R_1R_2+AR_1R_i+R_1R_i\right)=0$

$$\frac{V_2}{V_1} = -\frac{AR_2R_i}{AR_1R_i+R_1R_2+R_i(R_1+R_2)}$$

d) i) $R_1 = 100\,\Omega$, $R_2 = 1\,k\Omega$, $R_i = 10\,k\Omega$, $A = 10^4$

$$\text{Gain} = -\frac{10^4\times10^3\times10^4}{10^4\times10^2\times10^4+10^2\times10^3+10^4(10^2+10^3)} = -\frac{10^{11}}{10^{10}+10^5+1.1\times10^7}$$

$$= \underline{\underline{-9.99}}$$

ii) $R_1 = 1M\Omega, R_2 = 10M\Omega$

$$\text{Gain} = -\frac{10^4 \times 10^7 \times 10^4}{10^4 \times 10^6 \times 10^4 + 10^6 \times 10^7 + 10^4(1.1 \times 10^7)}$$

$$= -\frac{10^{15}}{10^{14} + 10^{13} + 1.1 \times 10^{11}} = \underline{\underline{-9.08}}$$

e) Resistors in part i) give very small error in the desired gain, those in part ii) do not. This is because the resistors in part ii) are too large, such that the small input current which flows through R_i causes a significant voltage drop across such large resistors. Should choose circuit resistors to be $\ll R_i$, and $\gg R_o$.

4/ a) $V_{GS} = -2V$, $V_{DS} = 9V$, $I_D = 4mA$.

R_1 ties gate to 0V, and is also the input resistance, which is to be $1M\Omega$. $\therefore$ $\underline{\underline{R_1 = 1M\Omega}}$

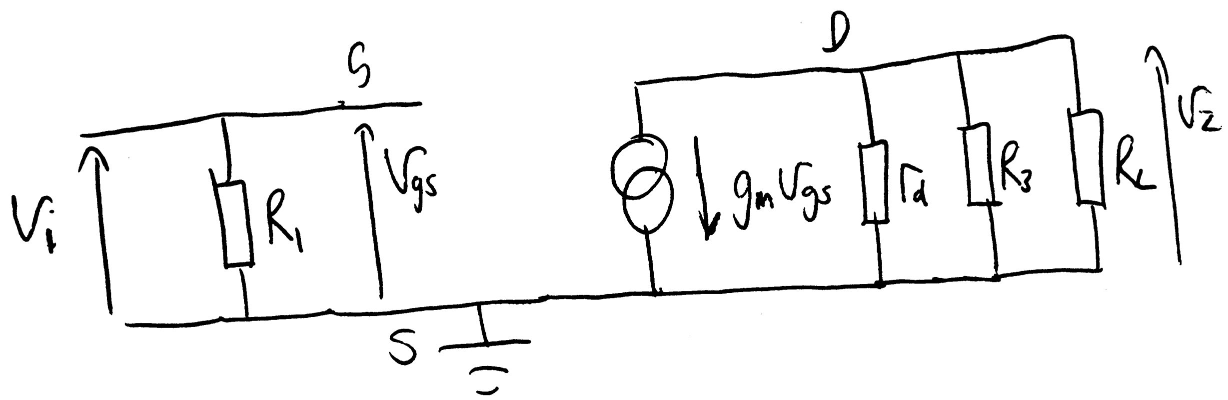
For $V_{GS} = -2V$, $V_S = +2V$

$$\therefore \frac{2}{R_2} = I_D = 4mA \quad \underline{\underline{R_2 = 500\Omega}}$$

$V_{DS} = 9V$, $V_S = 2V$ $\therefore$ $V_D = 11V$

$$\therefore \frac{20-11}{R_3} = 4mA \Rightarrow \underline{\underline{R_3 = 2.25\,k\Omega}}$$

b)



<u>Note</u> All capacitors are short-circuits at mid-band, hence source connected to ground.

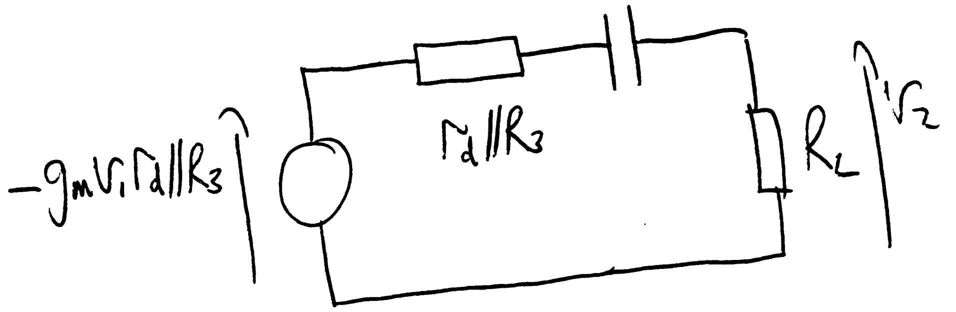
$V_{gs} = V_1 \qquad V_2 = -g_m V_{gs}\left(r_d // R_3 // R_L\right)$

$$\therefore \frac{V_2}{V_1} = -g_m\left(r_d // R_3 // R_L\right) = -5\times10^{-3}\left(\frac{1}{\frac{1}{10\times10^3}+\frac{1}{2.25\times10^3}+\frac{1}{10^3}}\right)$$

$$= \underline{\underline{-3.24}}$$

c) Capacitor C_1 tends towards an open circuit for low frequencies since $X_c = 1/\omega C \to \infty$ as $\omega \to 0$. $\therefore$ Current through R_L will tend to zero, so $V_2 = R_L i_L$ also tends to zero.

Convert to Thevenin form:



$$V_2 = \frac{-g_m V_1\, r_d//R_3 \times R_L}{r_d//R_3 + 1/j\omega C_1 + R_L}$$

For -3dB point

$$r_d//R_3 + R_L = 1/\omega C_1 \qquad 2837 = \frac{1}{2\pi\times10\times C_1}$$

$$\underline{\underline{C_1 = 5.61\,\mu F.}}$$

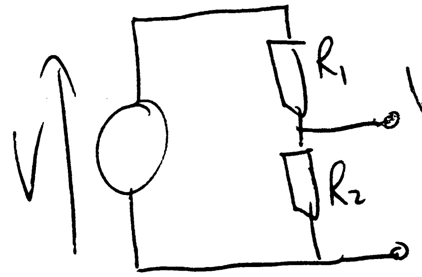
d) 1 kHz input voltage means the effect of C_1 may be ignored. For maximum load power, $R_L = R_{out}$ by maximum power transfer theorem.

$$R_{out} = r_d // R_3 = 1837\ \Omega$$

$$\therefore \underline{\underline{R_L = 1.84\ k\Omega.}}$$

3/ a) Thevenin equivalent circuit represents a linear circuit, insofar as the load is concerned, as an ideal voltage source in series with an impedance, such that:

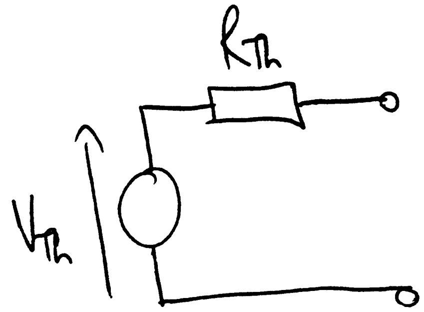
$$V_{Th} = V_{oc} \qquad R_{Th} = \frac{V_{oc}}{I_{sc}}.$$



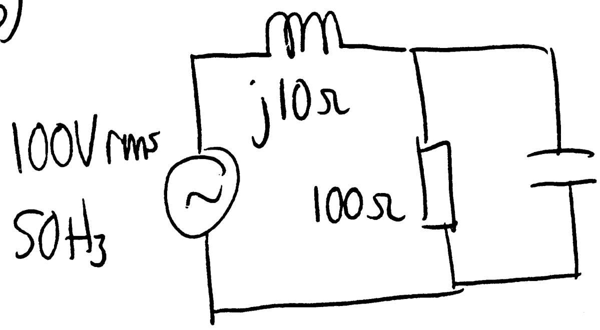
$$V_{Th} = V_{oc} = \frac{R_2}{R_1+R_2} V \qquad I_{sc} = V/R_1$$

$$\text{so } R_{Th} = \frac{R_1 R_2}{R_1+R_2}$$

$\Rightarrow$



b)



$$\bar{Z}_L = j\omega L = j \times 2\pi \times 50 \times 31.8 \times 10^{-}$$

$$= j10\Omega$$

$$\bar{Z}_C = \frac{1}{j\omega C} = \frac{-j}{2\pi \times 50 \times 159 \times 10^{-6}}$$

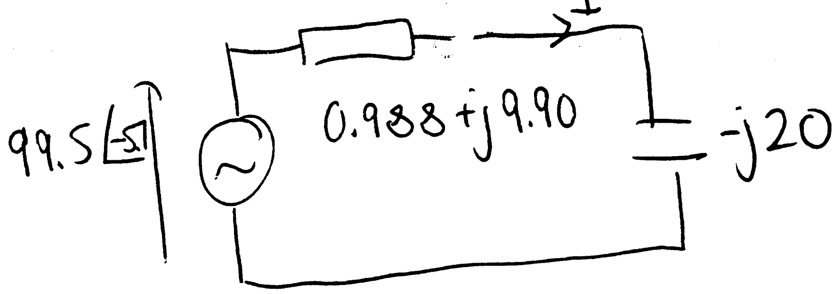
$$= -j20\Omega$$

Regarding $\bar{Z}_c$ as load:-

$$\bar{V}_{Th} = \frac{100}{100+j10} \times 100 = 99.5V \angle -5.7^\circ$$

$$\bar{Z}_{TH} = \frac{j10 \times 100}{j10+100} = \frac{1000\angle 90^\circ}{100.5 \angle 5.7^\circ} = 9.95\Omega \angle 84.3^\circ$$

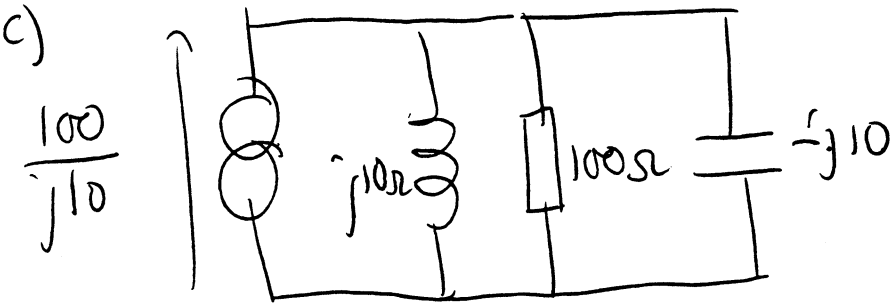
$$= (0.988 + j\,9.90)\,\Omega$$



$$\bar{I} = \frac{99.5\angle -5.7}{0.988-j10.1} = \frac{99.5\angle -5.7^\circ}{10.15\angle -84.4^\circ} = \underline{\underline{9.80A\angle 78.7^\circ}}$$

$$\hat{I}_c = \sqrt{2}\,I_{c_{rms}} = \sqrt{2}\times 9.8 = \underline{\underline{13.86\ A}}$$

c)



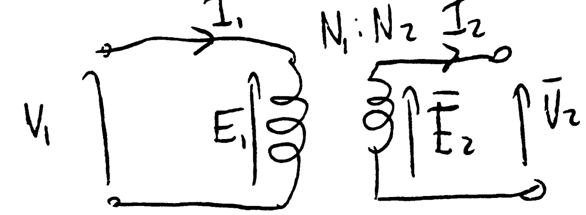
At resename the load is purely real, so $\overline{Z}_c = -j10\Omega$

$$\frac{1}{\omega C} = 10 \quad \frac{1}{2\pi \times 50 \times C} = 10 \quad \underline{\underline{C = 318\mu F}}$$

$$\overline{V}_c = \frac{100}{j10} \times 100 = -j1000\,V = \underline{\underline{1000V\angle -90°}}$$

d) At resonance a large amount of energy is transfered to and fro between the inductor and capacitor in comparison to the energy dissipated in the resistor. This results in large values of current and voltage.

4 a) An ideal transformer transforms input voltage/current to new values depending on its turns ratio with no real or reactive power being consumed:



$$\frac{\overline{E}_2}{\overline{E}_1} = \frac{\overline{V}_2}{\overline{V}_1} = \frac{N_2}{N_1} \qquad N_1\overline{I}_1 = N_2\overline{I}_2$$

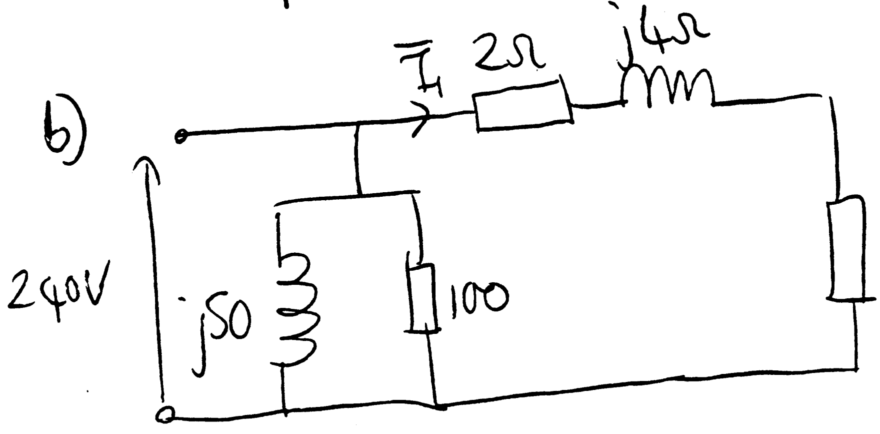
R_1, R_2 - primary and secondary winding power loss due to resistance

X_1, X_2 - " " " " reactive power loss due to leakage flux (flux which is not mutual flux)

X_m - magnetising reactance, representing the fact that some input current is required to drive the mutual flux of the transformer

R_0 - represents transformer iron losses.

b)



$$\left(\frac{N_1}{N_2}\right)^2 Z_L = 100(0.1 + j0.05)$$
$$= (10 + j5)\,\Omega$$

i) $$\overline{I}_1 = \frac{240}{2 + j4 + 10 + j5} = \frac{240}{12 + j9} = \underline{\underline{16\text{A}}}$$

ii) $P_L = I_1^2 R_L = 16^2 \times 10 = \underline{\underline{2.56 \text{ kW}}}$

$Q_L = I_1^2 X_L = 16^2 \times 5 = \underline{\underline{1.28 \text{ kVAr}}}$

iii) $$P_{in} = \frac{V^2}{R_0} + I_1^2(R_1 + R_2') + P_L = \frac{240^2}{100} + 16^2 \times 2 + 2560$$

$$= \underline{\underline{3.648 \text{ kW}}}$$

$$Q_{in} = \frac{V^2}{X_0} + I_1^2(X_1 + X_2') + Q_L = \frac{240^2}{50} + 16^2 \times 4 + 1280$$

$$= \underline{\underline{3.456 \text{ kVAr}}}$$

iv) $S_{in} = (P_{in}^2 + Q_{in}^2)^{1/2} = 5025 \text{ VA} = 240 \times I_{in}$

$I_{in} = \underline{\underline{20.9 \text{ A}}}$

$\cos\phi = \frac{P_{in}}{S_{in}} = \underline{0.726 \text{ lagging}}.$

v) $\eta = \frac{P_L}{P_{in}} = \frac{2560}{3648} \times 100\% = \underline{\underline{70.2\%}}$

c) Capacitor must generate VArs equal to Q_{in}

$$\therefore \frac{V^2}{1/\omega C} = 3456$$

$$2\pi \times 50 \times C \times 240^2 = 3456$$

$$\underline{\underline{C = 191\mu F.}}$$

5/ a)

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

b) $Z = A.B + C.D + A.C + A.D + B.C + B.D$

For product of sums, map $\overline{Z}$

$$\overline{Z} = \overline{A}.\overline{B}.\overline{C} + \overline{A}.\overline{B}.\overline{D} + \overline{C}.\overline{D}.\overline{A} + \overline{C}.\overline{D}.\overline{B}$$

De Morgan:

$$Z = (A+B+C).(A+B+D).(A+C+D).(B+C+D)$$

c) i) Using sum of products solution

$$\overline{Z} = \overline{A.B + C.D + A.C + A.D + B.C + B.D}$$

$$= \overline{A.B}.\overline{C.D}.\overline{A.C}.\overline{A.D}.\overline{B.C}.\overline{B.D}$$

$$Z = \overline{\overline{A.B}.\overline{C.D}.\overline{A.C}.\overline{A.D}.\overline{B.C}.\overline{B.D}}$$

i.e. 6 2 input NAND gates and 1 6 input NAND gate

$= \underline{7 \text{ gates}}.$

ii) Using product of sums expression

$$\overline{Z} = \overline{A+B+C} + \overline{A+B+D} + \overline{A+C+D} + \overline{B+C+D}$$

$$Z = \overline{\overline{A+B+C} + \overline{A+B+D} + \overline{A+C+D} + \overline{B+C+D}}$$

This requires 4 3 input NOR gates and 1 4 input NOR gate

i.e. 5 gates in total

$\therefore$ NOR gate implementation appears best since it uses less gates.

d) Rewrite Z as :-

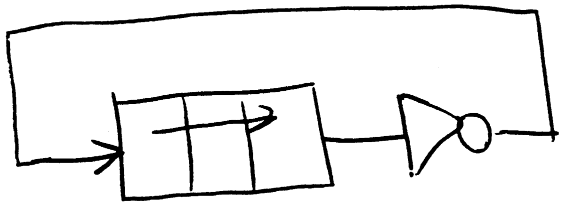
$$Z = \overline{\overline{\overline{\overline{\overline{\overline{A.B}.\overline{C.D}}}.\overline{\overline{\overline{A.C}.\overline{A.D}}}}}.\overline{\overline{\overline{B.C}.\overline{B.D}}}}$$

This requires $6 + 3\times2 + 2 + 1 = 15$ 2 input NAND gates

(a NOT gate may be formed by )

$\therefore$ 4 ICs required for NAND implementation, 5 ICs for NOR since 17 gates needed. $\therefore$ NAND gate implementation is better.

6/ a)



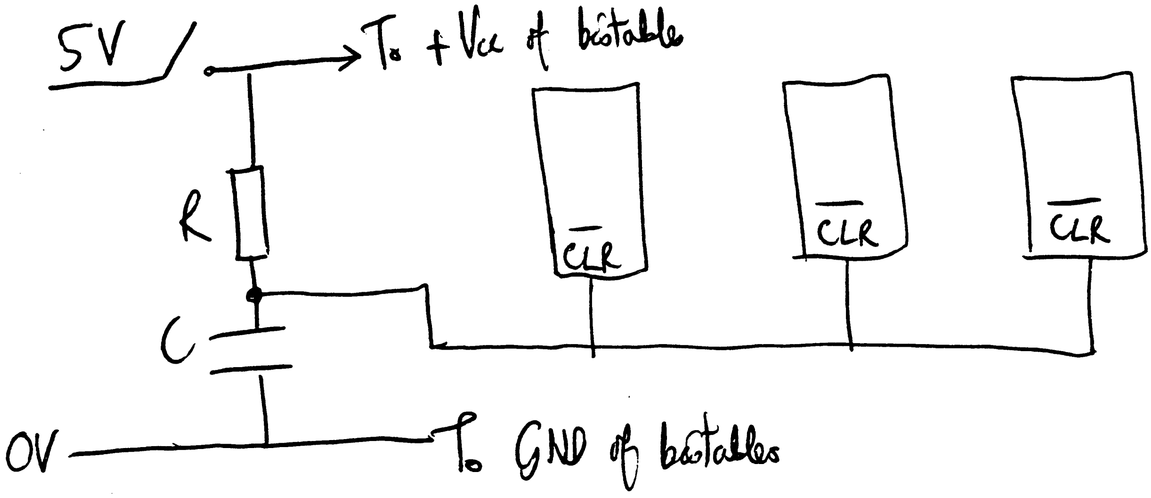
i) 000
100
110
111
011
001

Count of six

010
101

Count of two

ii) Need $\overline{CLR}$ inputs to be held low when power is applied.



An RC network as shown will cause $\overline{CLR}$ inputs to be held low whilst power is applied, so ensuring that $Q_A Q_B Q_C = 000$.

b) i)

	Present			Next			Bistable inputs					
	Q_A	Q_B	Q_C	Q_A	Q_B	Q_C	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	0	0	1	0	X	0	X	1	X
1	0	0	1	0	1	0	0	X	1	X	X	1
2	0	1	0	0	1	1	0	X	X	0	1	X
3	0	1	1	1	0	0	1	X	X	1	X	1
4	1	0	0	1	0	1	X	0	0	X	1	X
5	1	0	1	1	1	0	X	0	1	X	X	1
6	1	1	0	1	1	1	X	0	X	0	1	X
7	1	1	1	0	0	0	X	1	X	1	X	1

ii) By inspection $J_C = K_C = 1$ (treat all don't cares as '1')

J_A

Q_C \ Q_AQ_B	00	01	11	10
0	0^0	0^2	X^6	X^4
1	0^1	1^3	X^7	X^5

K_A

Q_C \ Q_AQ_B	00	01	11	10
0	X	X	0	0
1	X	X	1	0

J_B

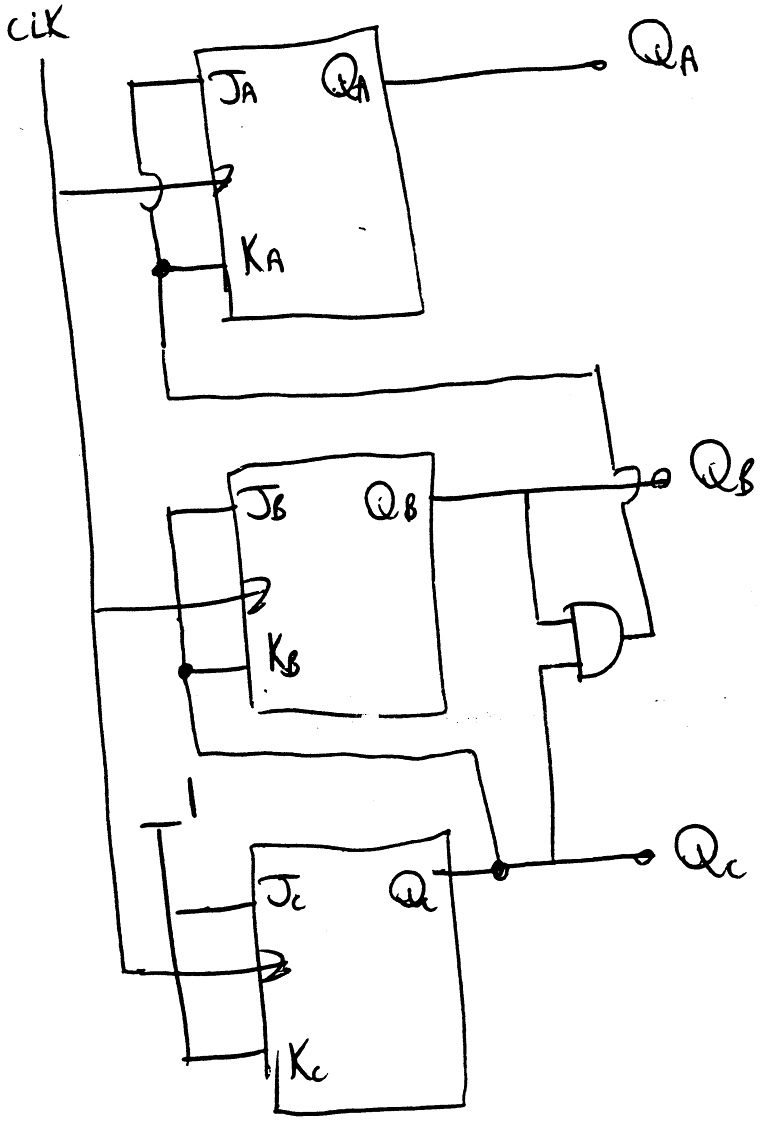
Q_C \ Q_AQ_B	00	01	11	10
0	0	X	X	0
1	1	X	X	1

K_B

Q_C \ Q_AQ_B	00	01	11	10
0	X	0	0	X
1	X	1	1	X

So $J_A = K_A = Q_B . Q_C$ $\quad J_B = K_B = Q_C$

iii)



b)

	Mnemonic	Op-code	#	~	Address	N° of times
	LDX #$1000	CE1000	3	3	0000	1
next	LDAA 0,X	A600	2	5	0003	16
	LDAB 0,X	E600	2	5	0005	16
	RORB	56	1	2	0007	16
	BCS label	25 02	2	4	0008	16
	STAA 10,X	A710	2	6	000A	8
label	INX	08	1	4	000C	16
	CPX #$1010	8C1010	3	3	000D	16
	BEQ end	27 03	2	4	0010	16
	JMP next	7E 0003	3	3	0012	15
end					0015	

For BCS label need to jump from 000A → 000C ie 2 bytes

For BEQ end need to jump from 0012 → 0015 ie 3 bytes

For JMP next need to go to address of next label ie 0003.

c) 8 even numbers, 8 odd numbers. The N° of times column above is the number of times each instruction is executed.

$$\therefore 16\times(5+5+2+4+4+3+4)+8\times6+1\times3+15\times3$$

$$= 16\times27+48+48 = 528 \text{ clock cycles}$$

$$\therefore \text{Time} = \frac{528}{8}\ \mu\text{secs} = \underline{66\ \mu\text{secs}}$$

7/ a) Data bus ⇒ A number of parallel electrical connectors (in this case 8) that are used to communicate data to the μP, memory and other peripherals

Address bus ⇒ Same as data bus but 16 connectors used to communicate locations in memory from which data is to be obtained or stored.

13 bit address bus ⇒ 2^{13} memory locations each of 8 bits

$\therefore$ ~~No~~ $2^{13} \times 2^{3} = 2^{16}$ bistables $= \underline{65536}$

1 kbyte = 1024 bytes = 2^{10} bytes

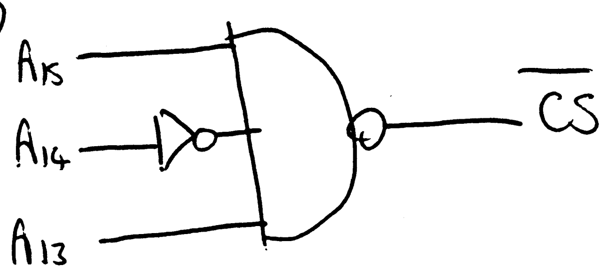
Capacity = 2^{13} bytes = $2^{3} \times 2^{10}$ = $\underline{\underline{8 \text{ kbytes}}}$

6800 has 16 address lines, so 3 lines are spare, and these are used to select which memory chip is being addressed via the $\overline{CS}$ line. Since there are $2^3 = 8$ possible combinations for these 3 lines, 8 such chips may be connected.

For address range A000 → BFFF

A_{15}	A_{14}	A_{13}	
1	0	1	0 0 0 0 ... 1 1 1 1 ...

needs to give $\overline{CS} = 0$



b) For a positive 8 bit binary number the columns mean:

$$2^7 \; 2^6 \; 2^5 \; 2^4 \; 2^3 \; 2^2 \; 2^1 \; 2^0$$

$$128 \; 64 \; 32 \; 16 \; 8 \; 4 \; 2 \; 1$$

In 2's complement, the most significant bit (MSB) means -128 instead of $+128$.

$$FF = \overbrace{1}^{-128}\,\overbrace{1111111}^{+127} = -1$$

$$8C = \overbrace{1000}^{-128}\,\overbrace{1100}^{12} = -116$$

$$3B = \overbrace{0011}^{3\times16}\,\overbrace{1011}^{11} = +59$$

Most positive $01111111 = +127_{10} = 7F_{16}$

Most negative $10000000 = -128_{10} = 80_{16}$

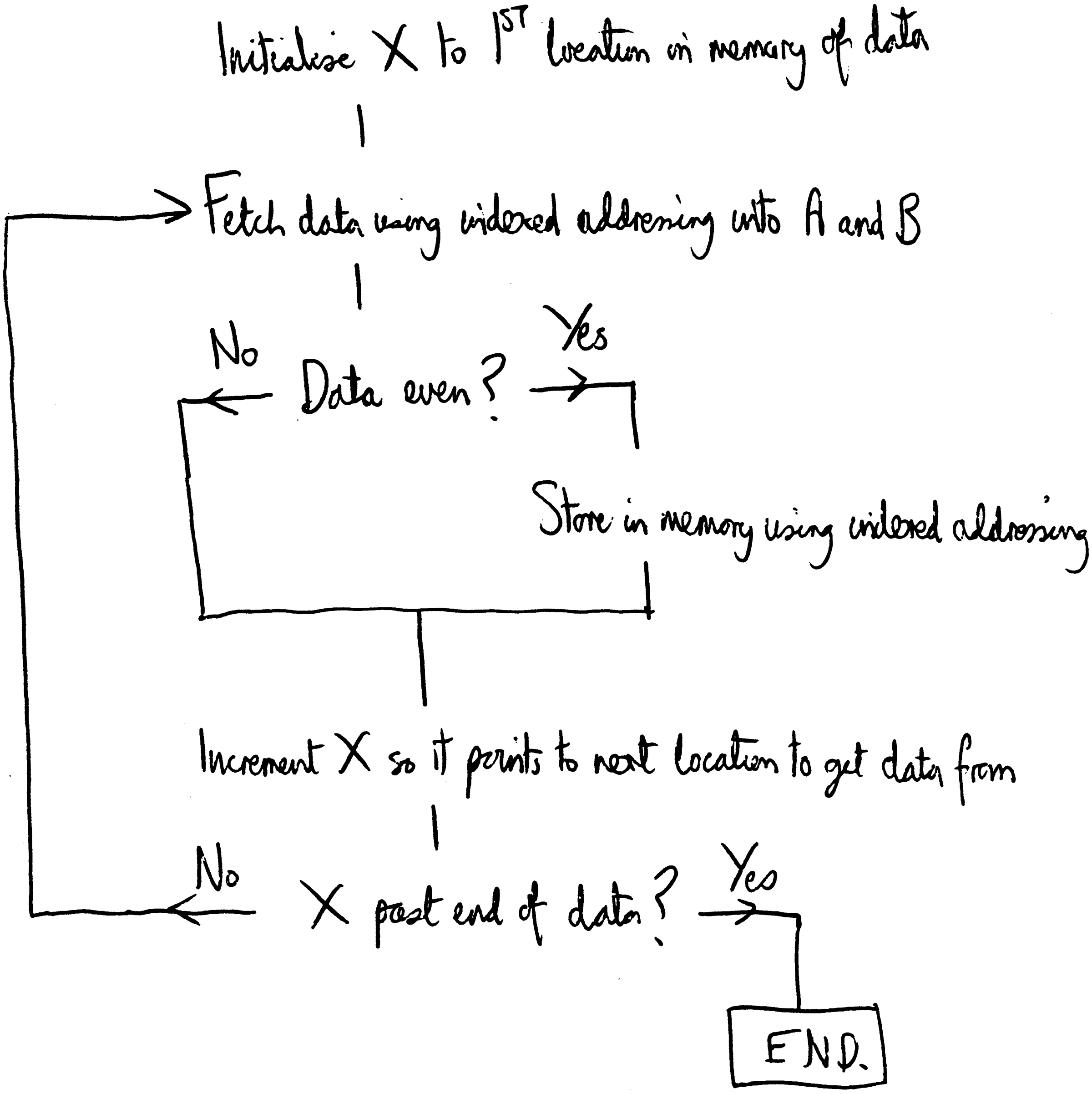
$$-53 = -128 + X \Rightarrow X = 75 = 11001011$$

$$-115 = -128 + Y \Rightarrow Y = 13 = \underline{10001101}$$

$$(1)\,01011000$$

$V = N \oplus C$ $C = 1$ since a carry from MSB occurred. $N = 0$ since MSB is itself 0 $\therefore$ $V = 0 \oplus 1 = 1$ so V flag is set. This is to be expected since $-53 - 115 = -168$ is outside the 2s complement range ($-128 \Rightarrow +127$) so there will be a 2's complement overflow. $\therefore$ V flag set.

8/ a) Accumulator B holds data to be tested. RORB is a right rotate by one, with the LSB going into the carry bit. An even number has an LSB of zero, so $C = 0$ after RORB. An odd number has an LSB of one, so $C = 1$ after RORB. BCS branches if $C = 1$, so for odd numbers the STAA 10,X instruction is missed, for even numbers it is not, so the data gets stored in memory.



PART IA, PAPER 3, SECTION C

9 (a)

Charge stored in oil-filled capacitor, $Q = \dfrac{\varepsilon_0 \varepsilon_r A}{d} V$

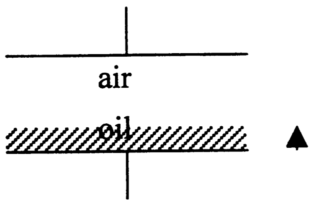
Voltage across air-filled capacitor, $V_{air} = \dfrac{d}{\varepsilon_0 A} Q = \varepsilon_r V = 2.2 \times 50 =$ **110 volts**

(b) In effect we have two capacitors in series, each of half the original thickness and one filled with air (C_{air}), one filled with oil (C_{oil}). The combined capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_{air}} + \frac{1}{C_{oil}}$$
$$= \frac{d/2}{\varepsilon_0 A} + \frac{d/2}{\varepsilon_0 \varepsilon_r A}$$

The ratio of this capacitance to $\varepsilon_0 A/d$ is $2/(1+1/\varepsilon_r):1 = 2/(1+1/2.2):1 =$ **1.375:1**

(c)



$$\frac{1}{C} = \frac{h}{\varepsilon_0 \varepsilon_r A} + \frac{d-h}{\varepsilon_0 A}$$

$$\text{Stored energy, } E = \frac{Q^2}{2C} + \frac{\rho g A h^2}{2}$$

$$\frac{dE}{dh} = \frac{Q^2}{2\varepsilon_0 A}\left(\frac{1}{\varepsilon_r} - 1\right) + \rho g A h = 0$$

$$h = \frac{Q^2}{2\varepsilon_0 \rho g A^2}\left(1 - \frac{1}{\varepsilon_r}\right)$$

Now the charge stored on the capacitor, $Q = CV = (\varepsilon_0 \varepsilon_r A/d)V$, so

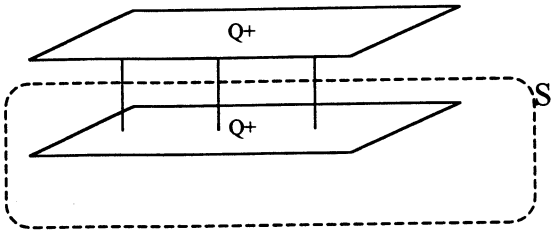
$$h = \frac{(V\varepsilon_0 \varepsilon_r A/d)^2}{2\varepsilon_0 \rho g A^2}\left(1 - \frac{1}{\varepsilon_r}\right)$$

$$h = \frac{\varepsilon_0 \varepsilon_r (V/d)^2}{2\rho g}(\varepsilon_r - 1)$$

$$= \frac{8.854 \times 10^{-12} \times 2.2(50/10^{-3})^2}{2 \times 1500 \times 98}(2.2 - 1) = \mathbf{1.99\ \mu m}$$

Examiner's comments: an unpopular question, and part (c) is difficult with only one candidate completing it successfully. Success depended on noting that it is charge Q which is conserved when the capacitor is isolated. I was pleased to see that most candidates realised the implication that D is conserved, and uniform across the oil/air boundary.

10 (a)



Gauss: $\int_S \mathbf{D.dS} = Q$

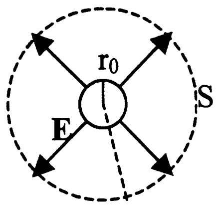
D is only non-zero when surface S encloses one plate and not the other. If area A is large relative to separation d, then by symmetry **D** must be perpendicular to the plates and uniform so that $DA=Q$.

Now $\mathbf{D} = \varepsilon\mathbf{E}$ so $E = \dfrac{Q}{\varepsilon A}$ and **E** is perpendicular to the plates of the capacitor.

Also $V = \int \mathbf{E.dl}$ so integrating parallel to E field, we get $V = \dfrac{Qd}{\varepsilon A}$

Now by definition, $Q = CV$ so the capacitance of the plates is given by $C = \dfrac{\varepsilon A}{d}$

(b)



By symmetry, **D** must radiate from the centre of the sphere and if we enclose the sphere with a spherical surface S of radius r then D will be constant at all points on that surface. **D** will be parallel to elements **dS** of the surface at all points, so we get from Gauss that:

$$D.4\pi r^2 = Q$$

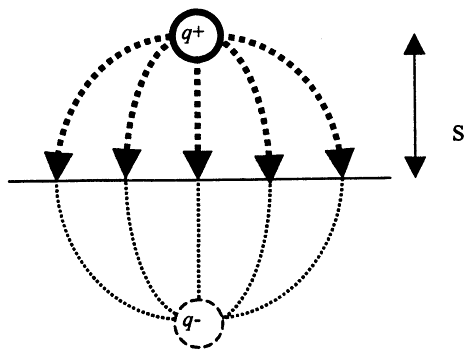
Now $\mathbf{D} = \varepsilon\mathbf{E}$ and $V = \int \mathbf{E.dl}$. If we integrate radially from the surface of the metal sphere to a point infinitely far from the sphere, then $\int \mathbf{E.dl} = \int_{r_0}^{\infty} E dr$ so:

$$V = \int_{r_0}^{\infty} \frac{Q}{\varepsilon 4\pi r^2} dr$$

$$= \frac{Q}{\varepsilon 4\pi r_0}$$

so the capacitance of the sphere is <u>$\varepsilon 4\pi r_0$.</u>

(c)



We use the method of images, where we note that the fields between a charged object and a flat metal plate are equal to the sum of the fields between the same object and infinity, and between the duplicate of the object with opposite charge reflected in the plane of the plate, also to infinity.

We cannot allocate a fixed charge Q to the cylinder (because it has infinite length) so allocate a charge q per unit length. Put a surface comprising a cylinder of radius r and length δl round one metal cylinder. By symmetry, the **D** field from a charged cylinder to infinity must radiate perpendicularly to the axis of the cylinder, so it cuts the surface at right angles at all points:

$$\int_S \mathbf{D.dS} = Q$$

$$D.2\pi r\delta l = q\delta l$$

$$E = \frac{q}{2\pi\varepsilon r}$$

The electric field at a distance x along a line joining one cylinder to the other is the sum of the electric fields of each cylinder alone:

$$E = \frac{q}{2\pi\varepsilon(s-x)} - \frac{-q}{2\pi\varepsilon(s+x)}$$

The voltage from the plane of symmetry to the surface of one cylinder is the integral of electric field:

$$V = \int_0^{s-a} E dx = \frac{q}{2\pi\varepsilon}\left[\ln\left(\frac{(s+x)}{(s-x)}\right)\right]_0^{s-a} = \frac{q}{2\pi\varepsilon}\ln\left(\frac{2s-a}{a}\right)$$

c, the capacitance per unit length, is q/V:

$$c = \frac{2\pi\varepsilon}{\ln\left(\frac{2s-a}{a}\right)}$$

Examiner's comments: Almost everybody did this question. Success depends on working from first principles rather than memory. A surprising number of candidates could do parts (b) and (c) but not part (a).

11 (a)

Maxwell-Ampere:

$$\oint \mathbf{H.dl} = \int_S (\mathbf{J} + \dot{\mathbf{D}}) \mathbf{dS}$$

Now the rate of change of D is zero, and $\int_S \mathbf{J.dS} = NI$. Also, the cross-section of the bar and C-shaped bar is uniform, so **H** will be constant and parallel to **dl** round the dotted line.

Hence:

$$HL = NI$$

B= $\mu_0\mu_r$**H** so:

$$B = \frac{\mu_0 \mu_r NI}{L}$$

$$= \frac{4\pi \times 10^{-7} \times 1000 \times 100 \times 5}{0.075}$$

$$= \underline{\mathbf{8.38\ T}}$$

...big, but this is a lot of current in a small device.

(b) We can expect that the H-field in air (H_a)will differ from that in the iron (H_i). So:

$$H_a L_a + H_i L_i = NI$$

so

$$\frac{B_a}{\mu_0} L_a + \frac{B_i}{\mu_0 \mu_r} L_i = NI$$

Now $\int_S \mathbf{B.dS} = 0$ so B is continuous across a boundary, so $B_a = B_i$

$$B = \mu_0 \frac{NI}{L_a + L_i / \mu_r}$$

$$= 4\pi \times 10^{-7} \frac{100 \times 5}{0.002 + 0.075/1000}$$

$$= \underline{\mathbf{0.303\ T}}$$

(c) The energy per unit volume stored by a magnetic field = $\int BdH = B^2/2\mu$ if B and μ are constant. If the bar moves a small distance δx, then it sweeps out a volume $2A\delta x$ in air where A is the cross-sectional area of the curved bar. The energy used to move the bar equals the applied force, F, times the distance δx, so:

$$F\delta x = \frac{B^2}{2\mu} 2A\delta x$$

so

$$F = \frac{B^2}{\mu_0} A = \frac{8.38^2}{4\pi \times 10^{-7}} 0.01^2$$

$$= \underline{\mathbf{5.59\ kN}}$$

Examiner's comments: Answers to this question were good, and it was good to see that many candidates were aware that this is a big magnetic field and force. Remember that there are two poles to the magnet, so their cross-sectional area is $2A$.