ENGINEERING TRIPOS PART IA

Monday 7 June 2004 9 to 12

Paper 1

MECHANICAL ENGINEERING

Answer not more than ***eight*** *questions, of which not more than* ***four*** *may be taken from Section A and not more than* ***four*** *from Section B.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

*Answer not more than **four** questions from this section.*

1 To measure the mass flow rate of air entering an apparatus the air is drawn through a bellmouth, of diameter 60 mm, as shown in Fig. 1. The air can be assumed to be incompressible. A vertical column manometer is connected at A. When there is no air flow entering the apparatus the water level within the manometer is at the "zero-flow" reference level which is 100 mm below the bellmouth. The manometer reservoir has a diameter of 20 mm and the vertical column is 5 mm in diameter. Ambient conditions are 15 °C and 1 bar.

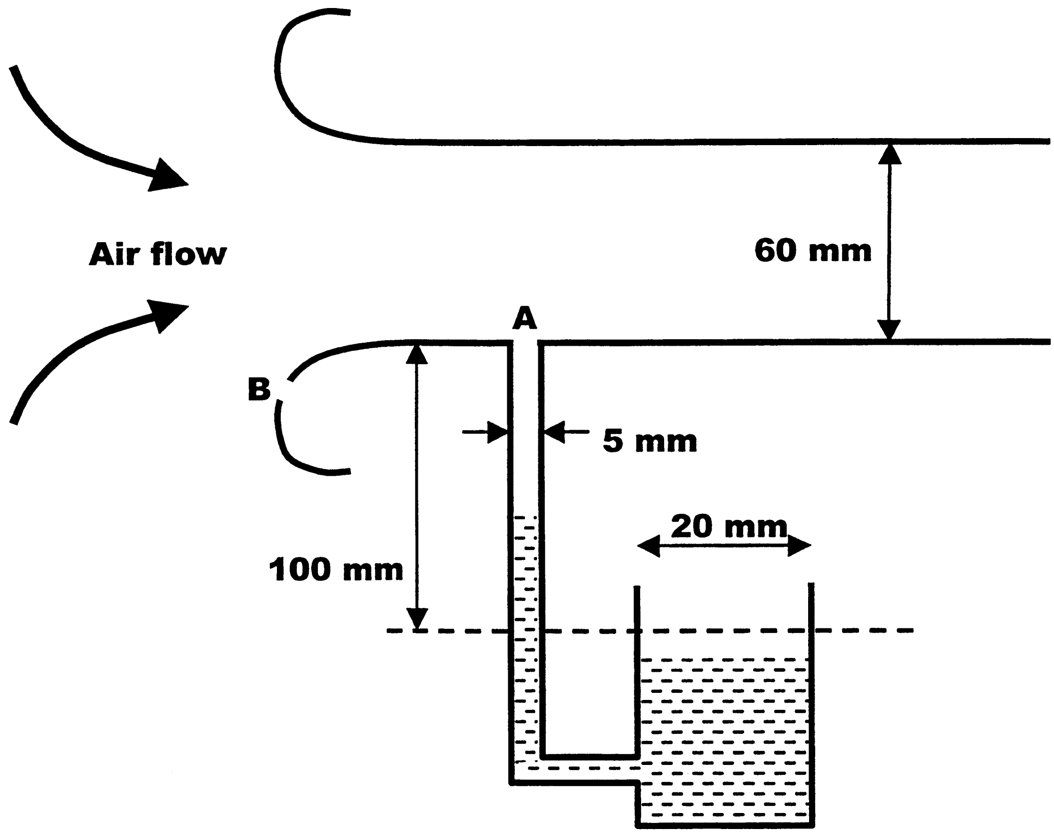

Fig. 1

(cont.

(a) Explain why it is more accurate to connect the manometer at location A rather than location B. [10%]

(b) When the gauge pressure at location A is –500 N/m^2, what is the air mass flow rate entering the apparatus? [30%]

(c) At what air mass flow rate will the water just be drawn into the apparatus? [40%]

(d) At the above maximum air mass flow rate, is it still reasonable to assume incompressible flow? Justify your answer. [20%]

2 The thrust produced by an aeroengine when stationary is measured by mounting it as shown in Fig. 2. The cross-sectional area of the exit jet is 0.5 m^2, the jet velocity is 400 m/s and the temperature is 500 K. The mass flow rate of fuel into the aeroengine may be ignored. The exhaust jet may be treated as having the same properties as air. Ambient pressure is 1 bar and the ambient temperature is 288 K.

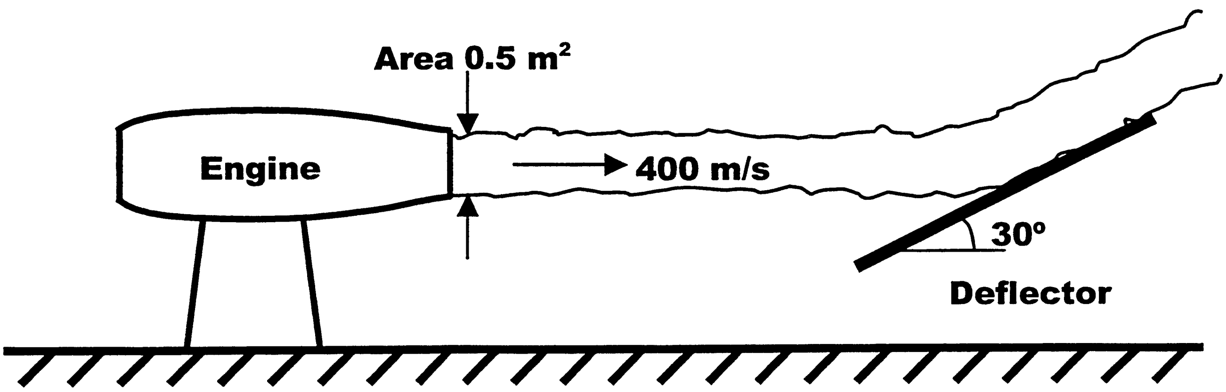

Fig. 2

(a) Calculate the mass flow rate through the aeroengine. [10%]

(b) By using a control volume analysis, calculate the required heat input into the aeroengine per unit mass flow. [20%]

(c) By using a control volume around just the aeroengine, calculate the horizontal thrust produced. [30%]

As shown in Fig. 2, the exhaust jet hits an exhaust deflector and is turned upwards through 30°. The deflector is sufficiently long so that the exhaust leaves the deflector as a parallel jet.

(d) Explain why the jet velocity leaving the deflector is still 400 m/s. [20%]

(e) Calculate the horizontal and vertical forces exerted by the jet on the exhaust deflector. [20%]

3 (a) Define a thermo-fluid system and state the First Law of Thermodynamics for such a system. Carefully describe the sign convention for heat and work interchange between the environment and the system. [10%]

(b) Starting from the First Law for a thermo-fluid system, derive expressions for the heat transfer required to raise the temperature of a mass m of a perfect gas from temperature T_1 to T_2 for the two processes:

(i) constant volume; [10%]

(ii) constant pressure. [10%]

(c) A Diesel engine is to be modelled using the Ideal Air Standard Diesel cycle with a volumetric compression ratio of 18 and a heat input of 2000 kJ per kg of air. At the start of the compression process, the air temperature is 300 K.

(i) Sketch and label this cycle on a p-V diagram. [10%]

(ii) Show that the temperature at the end of the compression process is approximately 953 K. [10%]

(iii) Calculate the cut-off ratio α and the temperature at the end of the constant pressure process. [20%]

(iv) Calculate the heat rejected between the end of the expansion process and the start of the compression process per kg of air. [20%]

(v) Calculate the efficiency of the cycle. [10%]

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4 A flat-sided water boiler, which is at a constant temperature T_h, is to be insulated to reduce heat loss to the environment which is at a temperature T_0. The insulation has thickness D and thermal conductivity λ. Edge effects and heat transfer through radiation may be ignored. The inside surface of the insulation is in good thermal contact with the boiler whilst the outside has surface heat transfer coefficient h. The heat loss per unit area is denoted by $\dot{q}$.

(a) Describe, along with appropriate equations, the mechanisms of convective and conductive heat transfer. [20%]

(b) Assuming that $\dot{q}$, h, λ, D and $(T_h - T_0)$ form the complete set of relevant physical quantities, use dimensional analysis arguments to suggest the number of non-dimensional groups for this problem. [30%]

(c) Show that hD/λ is one possible non-dimensional group and suggest the form(s) for the other group(s). [20%]

(d) By using the thermal resistance method, or otherwise, find the relationship between $\dot{q}$ and $(T_h - T_0)$ in terms of h, λ, D. [20%]

(e) Sketch the form of the relationship using the non-dimensional groups. [10%]

5 A deep-sea diving-bell is suspended by a cable in sea water of density 1030 kg/m^3 and uniform temperature 15 °C. Air can be supplied into the diving-bell via a flexible hose as shown in Fig. 3. When the diving-bell is completely immersed, but when there is no air within it, the tension in the suspension cable is 12 kN. All buoyancy effects due to the suspension cable and the flexible hose may be ignored. Atmospheric pressure is 1 bar.

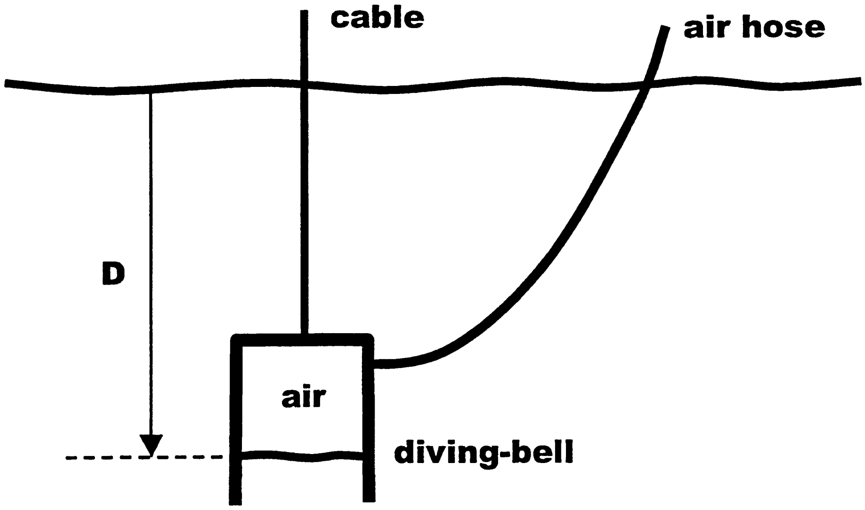

Fig. 3

(a) Describe Archimedes' principle. [10%]

After 3 kg of air, at 15 °C, has been supplied from the surface into the diving-bell the air-water interface is at a depth D = 30 m.

(b) Explain why the pressure of the air within the diving-bell may be taken as the hydrostatic pressure corresponding to the depth D of the air-water interface. [10%]

(c) By calculating the pressure and the volume of the air within the diving-bell, determine the tension force in the suspension cable. [40%]

The diving-bell is now raised towards the surface.

(d) State the conditions under which the expansion of the air within the diving-bell can be assumed to be isothermal. [10%]

(e) Calculate the depth D at which the tension force in the suspension cable first becomes zero assuming that no air escapes from the diving-bell. [30%]

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SECTION B

*Answer not more than **four** questions from this section.*

6 A particle P, of mass 250 g, is attached to one end of a light elastic string having an unstretched length of 300 mm and a stiffness of 40 N/m. The other end of the string is fixed to a point O at the centre of a horizontal frictionless table, as shown in plan in Fig. 4, so that the particle is free to orbit about the fixed point.

(a) At what speed must the particle travel so that it orbits around O in a circular path whose radius, r, is 500 mm? [20%]

(b) The particle is orbiting in this manner when a second particle Q, also of mass 250 g, is placed on the table in its path. The two particles collide, and remain locked together after the collision. What are the radial and tangential components of the velocity and acceleration of the new combined particle, immediately after the collision? [30%]

(c) Describe the subsequent motion of the combined particle qualitatively, and obtain a fourth-order expression in r which will determine the minimum and maximum values of r. Show that one solution is approximately r = 391 mm, and state whether you would expect this to be the minimum or the maximum value of r. What will be the other limiting value of r ? [50%]

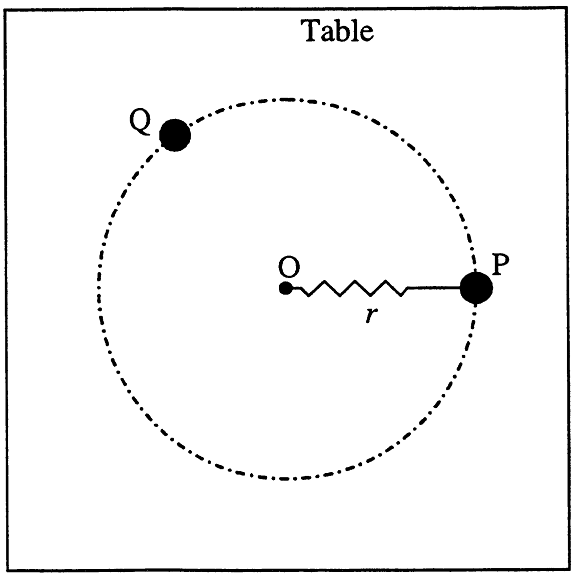

Fig. 4

7 A system for slowing down a drag-racing car is shown in plan in Fig. 5. As the rear of the vehicle passes the finishing line, a trailing hook picks up the ends of two heavy chains, each of length L and mass m_c, laid out in a straight line on either side of the car's path. Friction between the chain and the ground can be ignored throughout this question.

(a) The car has mass M, and crosses the finishing line with velocity V_0. Given that friction can be ignored, state:

(i) whether kinetic energy and/or momentum are conserved during the process;

(ii) what the final velocity of the car will be, when the chains have become fully straight;

(iii) how far the car will travel past the line before the chains become straight;

(iv) the total mass m that is in motion when the vehicle has travelled a distance x past the finishing line, as shown (you can assume that the loops in the chain behind the car have a small radius). [30%]

(b) Obtain an expression for the velocity v of the vehicle as a function of x. Verify that your expression gives the correct velocity of the vehicle when the chains are fully straightened. [35%]

(c) Using the identity:

$$\frac{dv}{dt} = v\frac{dv}{dx}$$

or otherwise, express the deceleration of the vehicle as a function of x. What is the greatest deceleration, and when does it occur? [35%]

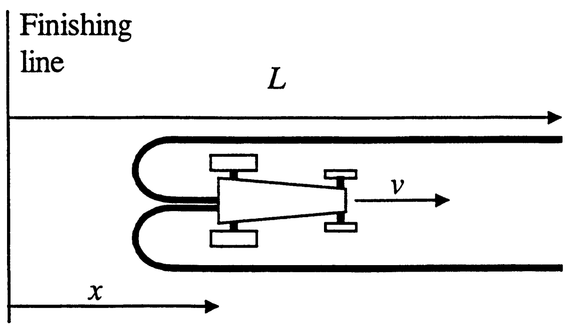

Fig. 5

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8 Figure 6 shows a scale drawing of a mechanism used in a paper-folding machine, in elevation. Crank OA is 100 mm long and rotates at 3 rad/s; and it is vertical at the instant shown. It is attached to a straight rod AD, which is 600 mm long and passes through a slider pinned at point B. A second rigid rod, CFG, is hinged to rod AD at C, and passes through a second slider pinned at E. Both sliders can rotate freely, and B is fixed at the same height as point O.

(a) Show the positions of the instantaneous centres of the rods AD and CFG on the attached sheet, which is drawn to 1/5 scale and should be handed in as part of your answer. [30%]

(b) By means of a velocity diagram or otherwise determine the velocities (magnitude and direction) of points D and G. If you choose to draw a velocity diagram, you should draw it on the attached sheet using the given origin, and a scale of 1 mm to represent 5 mm/s. [35%]

(c) Assuming the mechanism has negligible mass, find the driving torque which must be applied to the crank to overcome:

(i) an upwards load of 10 N at point D;

(ii) a frictional resistance of 2 Nm in the hinge at C. [35%]

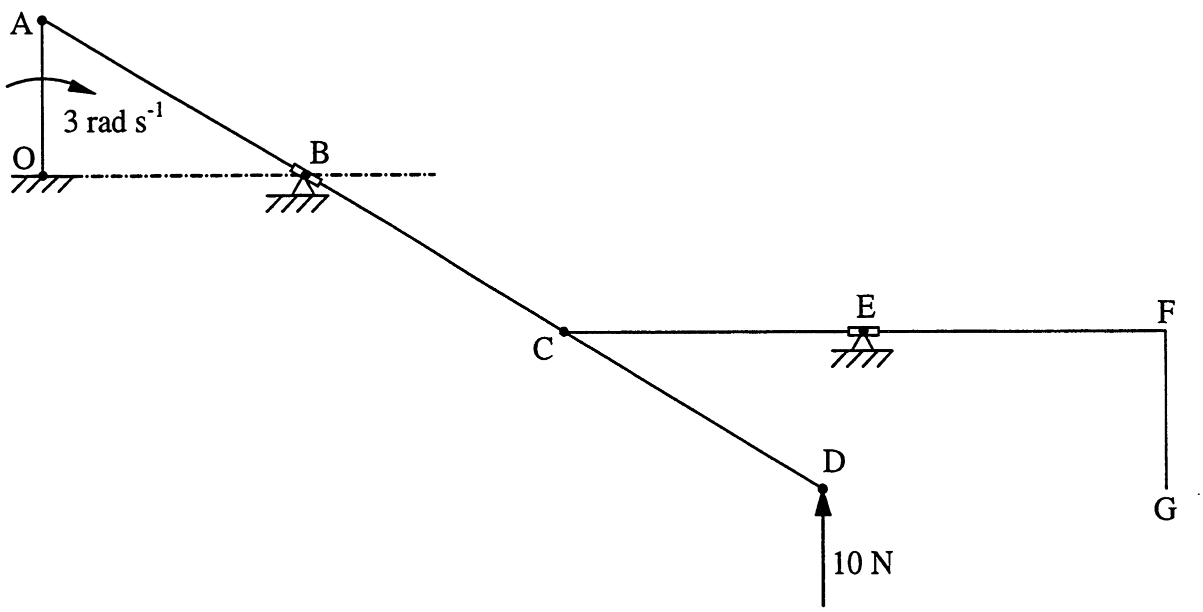

Fig.6

9 A mass m is attached to a rigid support by means of a spring of stiffness k and a damper of rate constant λ, as shown in Fig. 7. A further spring of stiffness $2k$ is also attached to the mass.

(a) The system is at rest when a steady force of magnitude F is suddenly applied at point A. What will be:

(i) the initial acceleration of the mass;

(ii) the final displacement of the mass; and

(iii) the final displacement of point A? [20%]

(b) Write down the equation of motion of the mass when a time-dependent *displacement* of z is applied at point A. Show that this equation can be written in the form:

$$\frac{\ddot{y}}{\omega_n^2} + 2\zeta\frac{\dot{y}}{\omega_n} + y = x$$

and find expressions for ω_n, ζ and x in terms of m, λ, k and z. [30%]

(c) A sinusoidal displacement is applied to the system at point A, having a frequency of 6 Hz, and the amplitude of the mass's motion is observed to be ± 5 mm. If m = 1 kg, λ = 12 Ns/m and k = 300 N/m, what will be the amplitude of the displacement at point A? [25%]

(d) By means of a phasor diagram or otherwise, find the amplitude of the sinusoidal force at A which is required to maintain the motion described above. [25%]

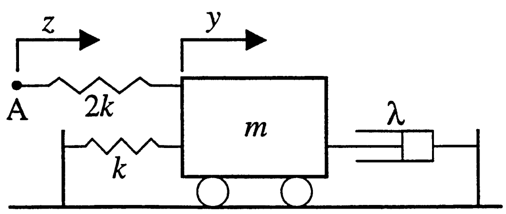

Fig. 7

10 Two rotors having polar moments of inertia $2J$ and J are rigidly connected to a shaft which is in turn fixed to a rigid support, as shown in Fig. 8. The shaft has torsional stiffness $2k$ between the support and the first rotor, and k between the two rotors. A torque T, which can vary with time, is applied to the outboard rotor.

(a) Write down the equations of angular motion for the two rotors in the form:

$$[M]\underline{\ddot{\theta}} + [K]\underline{\theta} = \underline{Q}$$

and find the elements of the mass matrix and the stiffness matrix. [30%]

(b) Find the frequencies of the two natural modes of the system, and (without further calculation) sketch the corresponding modal shapes. [40%]

(c) A torque of the form $T \cos(\omega t)$ is now applied to the outboard rotor. At what values of ω would there be no movement in one or other of the rotors? [30%]

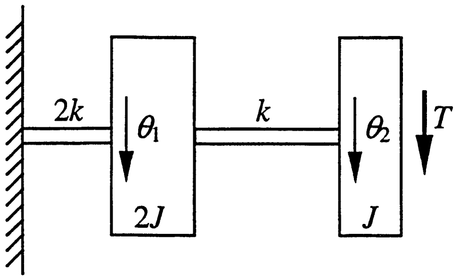

Fig. 8

END OF PAPER

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Sheet to be handed in with your answer to Question 8.