ENGINEERING TRIPOS PART IA

Monday 7 June 2004 1.30 to 4.30

Paper 2

STRUCTURES AND MATERIALS

Answer not more than ***eight*** *questions, of which not more than* ***four*** *may be taken from Section A, and not more than* ***four*** *from Section B.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

*Answer not more than **four** questions from this section.*

1 Figure 1 shows a pin-jointed plane framework attached to a rigid foundation by hinges at A and F. The dimensions are in mm.

(a) Horizontal loads of 15 kN are applied at each of D and E, as shown in the diagram. Calculate the forces in each of the bars. [30%]

(b) Each bar remains elastic and does not buckle. For each bar Young's modulus multiplied by the cross section area ('EA') is 6000 kN. Calculate the horizontal deflection of joint D induced by the loads given in part (a). [40%]

(c) The dimensions shown in the diagram are nominal dimensions. Suppose now that the lengths of the bars may be slightly different from the nominal lengths. The maximum difference between the true length and the nominal length is Δ for each bar: some bars may be up to Δ longer than nominal, and other bars up to Δ shorter than nominal. How should a designer specify Δ so that she or he can be certain that when the framework is assembled (and before it is loaded), joint D is not more than 20 mm horizontally to the right of its nominal position? [30%]

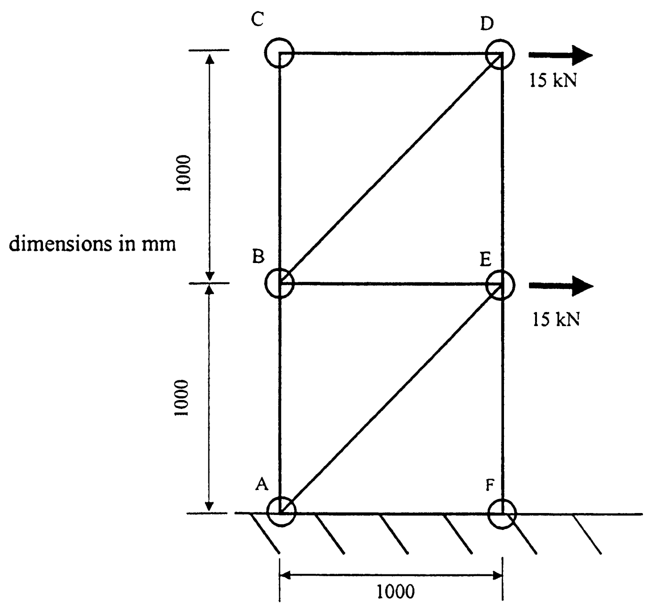

Fig.1

2 Figure 2 shows a plane semi-circular three-pinned arch with radius R, hinged to a rigid foundation at A and D and hinged at the midpoint C. The arch carries a vertical load W at point B.

(a) Find the vertical and horizontal reactions at the hinges A and D. [25%]

(b) Find the magnitude and direction of the force between the two halves of the arch at C, and draw a sketch indicating its direction clearly. [25%]

(c) Find the bending moment at B. [25%]

(d) Find the location and magnitude of the largest bending moment in the sector CD. [25%]

The diagram is redrawn to a larger scale on a separate page at the end of the paper. If you choose to solve part or all of the question graphically on the diagram, please remember to submit it with your answer.

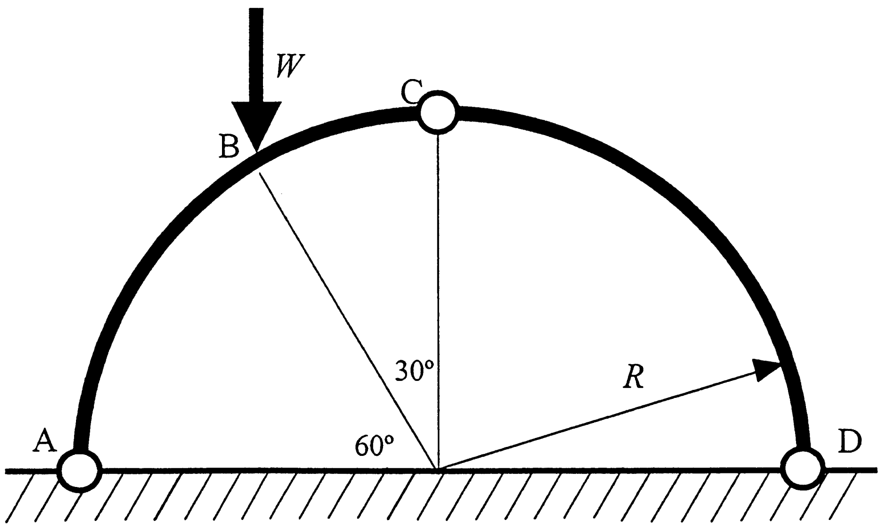

Fig. 2

3 Figure 3 is an elevation of a cargo barge hull. The hull is a rectangular box 100 m long, 20 m wide and 7 m high. The barge floats in seawater that has a weight per unit volume of 10 kN/m^3.

An offshore oil platform module weighing 50 MN is lifted onto the barge. The module can be idealised as a point load applied at the centre of the barge, point B.

(a) Calculate how far the barge sinks; [10%]

(b) draw a diagram showing the additional loads on the hull; [10%]

(c) draw the corresponding shear force diagram; [20%]

(d) draw the bending moment diagram. [20%]

In each of (a), (b), (c) and (d) idealise the hull as a rigid beam.

(e) Treating the barge as an elastic beam with flexural rigidity ('*EI*') 2×10^{12} N m^2, calculate the relative bending deflection between point B and a straight line between points A and C. Take the loads as those described above, and neglect the effect of the deflection on the loads. [20%]

(f) Discuss in qualitative terms how the flexure of the hull alters the loading, the shear force diagram and the bending moment diagram. [20%]

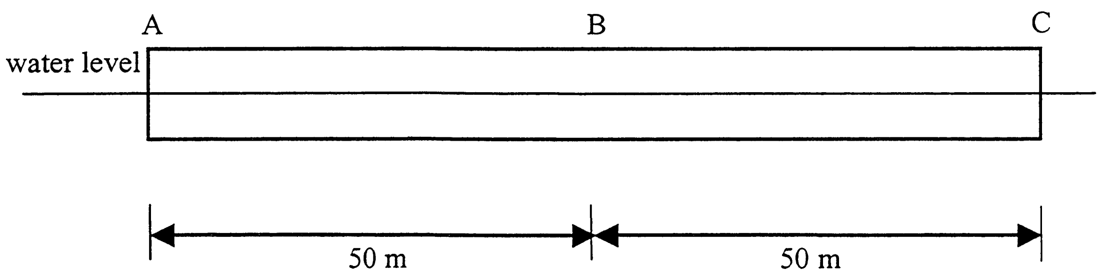

Fig. 3

4 Figure 4(a) shows a composite beam, composed of a wooden beam made of pine, 400 mm deep and 120 mm wide, with a steel plate 120 mm wide and 10 mm thick connected to one 120 mm face by screws at 100 mm centres. The Young's modulus E is 210 kN/mm^2 for steel and 14 kN/mm^2 for pine. The beam is loaded in pure bending in the vertical plane, up and down in the sketch.

(a) Explain briefly which of the following statements are true and which are false:

(i) plane sections perpendicular to the beam's longitudinal axis remain plane; [10%]

(ii) the strain is proportional to the distance from the neutral axis; [10%]

(iii) the stress is proportional to the distance from the neutral axis; [10%]

(iv) the neutral axis must be at the line where the wood and the steel meet; [10%]

(v) the largest stress must be the same in the wood and the steel; [15%]

(vi) the screws could be removed, and that would make no difference to the stresses. [15%]

(b) A rectangular beam has breadth b and depth d (Fig. 4(b)). It is made from a material that has the stress–strain relation shown in Fig. 4(c). In tension the material is elastic with Young's modulus E. In compression the material 'locks', so that there is no strain however high the compressive stress.

The beam is subjected to pure bending in the vertical plane. Find the position of the neutral axis, and calculate the bending moment if the curvature is κ. [30%]

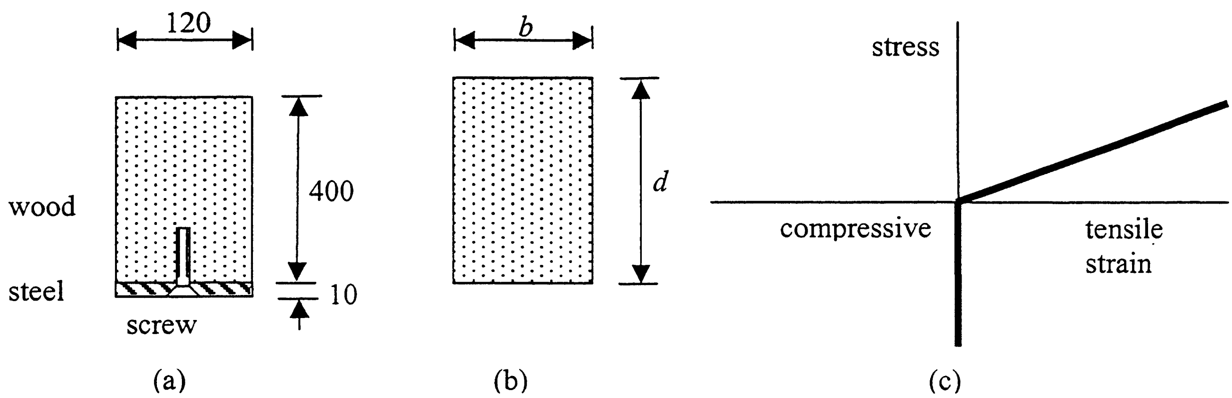

Fig. 4

(TURN OVER

5 A straight and vertical column of length L is built in at its lower end and free to move in any direction at its upper end (Fig. 5). The column is elastic with flexural rigidity EI.

(a) Derive from first principles the vertical load W at which the column will buckle. [40%]

(b) Taking L as 6 m and W as 200 kN, select from the table of universal column sections (structures data book, page 14) the lightest steel section that gives a load factor of 2 against elastic column buckling, so that the elastic column buckling load is 400 kN. [30%]

(c) Calculate the mean compressive stress under the 400 kN load if the load is applied axially. [10%]

(d) The column is inadvertently installed slightly out of vertical. Using the results from (b) and (c), comment in qualitative terms on the effect of this imperfection on the maximum vertical load the column can carry. The yield stress of the steel can be taken as 250 N/mm^2. [20%]

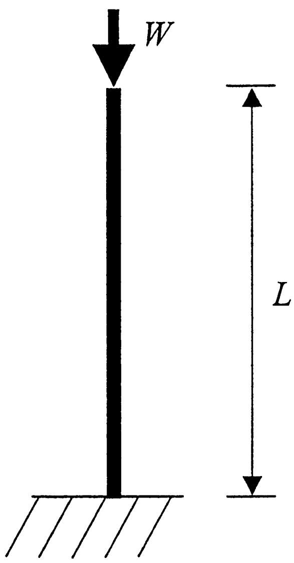

Fig. 5

SECTION B

Answer not more than ***four*** *questions from this section.*

6 A long cylindrical steel rod of radius $a = 0.1\,\text{m}$ and length l hangs vertically from the bottom of an offshore oil platform. A design requirement is that the stress in the rod resulting from self-weight and water pressure must not exceed one-tenth of the yield strength σ_y of the rod. The rod is made of steel, with a density $\rho = 7.9\,\text{Mg/m}^3$, Young's modulus $E = 200\,\text{GPa}$, Poisson's ratio $\nu = 0.3$ and $\sigma_y = 700\,\text{MPa}$. Assume that the steel rod and the platform are both at rest, that the top of the steel rod is at sea level, and that the bottom of the rod is not in contact with the sea bed. Let x represent the upward vertical distance measured from the end of the rod so that at the bottom of the platform $x = l$.

(a) Show that the axial stress in the rod is given by:

$$\sigma_x(x) = \rho g x - \rho_f g l$$

where $\rho_f = 1020\,\text{kg/m}^3$ is the average density of sea water and $g = 9.8\,\text{m/s}^2$. [20%]

(b) Show that the maximum length l_{max} of the rod for which the design requirement is satisfied is equal to 1038 m. [20%]

(c) With reference to the stress-free state of the rod, show that the total elongation of the rod having the maximum length l_{max} determined in part (b) above is equal to 0.171 m. [35%]

(d) If the rod of length l_{max} falls from the platform and lies in a horizontal position on the seabed that is 2000 m deep, calculate the shrinkage relative to its stress-free state and the corresponding volume change of the rod, also relative to its stress-free state. [25%]

7 (a) Discuss briefly the effect of the density of dislocations on the yield stress of a material. [20%]

(b) Show that $\sigma_y \approx 2\tau_y$ in a single crystal and explain why $\sigma_y \approx 3\tau_y$ in a polycrystalline material, where σ_y is the tensile yield stress and τ_y is the shear yield stress. [25%]

(c) How would the gradient of a *true* stress-*true* strain curve vary before, at and after the onset of necking ? Under what conditions can a neck in a tensile specimen be stable? [25%]

(d) Using the model of a few sliding blocks underneath a flat indenter, derive the approximate relationship $H \approx 3\sigma_y$ for the indentation hardness H of a polycrystalline material. Discuss the approximations used in the above model. [30%]

8 The probability of survival P_s of a ceramic material of volume V is given by:

$$P_s(V) = \exp\left[-L_F \left(\frac{V}{V_0} \right) \left(\frac{\sigma_{\max}}{\sigma_0} \right)^m \right]$$

where σ_0 is the characteristic strength of an element of volume V_0, $\sigma_{\max}$ is the maximum stress in the material under a given loading, and m is the Weibull modulus. The loading factor L_F measures how effectively the material body is stressed relative to uniaxial tension, with $L_F = 1$ for uniaxial tension, $L_F = 1/\left[2(m+1)^2\right]$ for 3-point bending and $L_F = (2+m)/\left[4(m+1)^2\right]$ for 4-point bending. For the same probability of survival, let σ_t, σ_{b1} and σ_{b2} denote separately the uniform tensile strength, the modulus of rupture associated with 3-point bending, and the modulus of rupture associated with 4-point bending.

(a) Sketch the bending moment diagram for the ceramic beam under both 3-point bending and 4-point bending. For the same total force applied, discuss which loading situation is more critical. [35%]

(b) Using the Weibull model and the stress field developed in the ceramic beam under 3-point bending, show that $\sigma_{b1} / \sigma_t = [2(m+1)^2]^{1/m}$. [45%]

(c) Assuming $m = 2$, determine the ratios σ_{b1} / σ_t and σ_{b2} / σ_t. Explain why σ_{b1} / σ_t and σ_{b2} / σ_t are typically larger than unity. [20%]

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9 (a) A multi-layer laminate consists of many alternating layers of thickness t_1 and thickness t_2, as shown in Fig. 6. Layer 1 has a Young's modulus E_1 and layer 2 has a Young's modulus E_2. Derive from first principles expressions for the longitudinal modulus $E_{//}$ and the transverse modulus $E_{\perp}$ of the laminate. [30%]

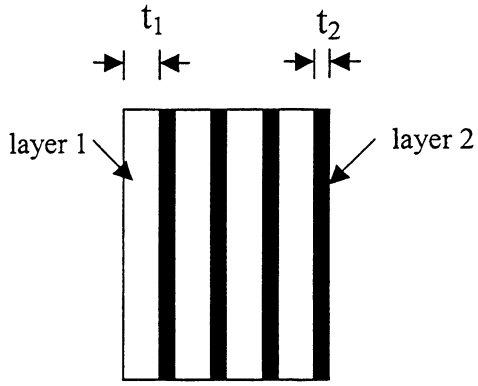

Fig. 6

(b) A simply supported column of *fixed* length l and rectangular cross-section with side lengths a and b is subjected to a *fixed* axial compressive force P. The column is made from a material with density ρ, Young's modulus E and yield strength σ_y.

(i) The column is designed against plastic buckling. Assuming that $a \geq b$ and that b is *fixed*, determine the minimum weight W_{min} of the column and the corresponding side length a as functions of l, b and P. (Hint: The force P_B at which the column buckles is given by $1/P_B = 1/P_E + 1/P_Y$, where $P_E = \pi^2 EI/l^2$, with I the second moment of area and $P_Y = \sigma_y ab$.) [35%]

(ii) Assuming l/b = 10 and E/σ_Y = 1000, use the result of (i) to plot $W_{\min}/(\rho g l^3)$ and a/l as functions of $P/(\sigma_y l^2)$. Determine a performance index for selecting a material that will minimise the weight of the column for a fixed column length l and a given compressive force P. [35%]

10 (a)

(i) Describe briefly what are meant by hydrogen embrittlement and stress corrosion cracking. [20%]

(ii) Consider the wet oxidation of aluminium and iron. Which metal has a more negative electrochemical potential than oxygen, and which metal has a higher rate of reaction in water? Hence determine which metal needs galvanising for protection against aqueous corrosion, and describe how this method works. [20%]

(b) An indentation load P generates an edge crack of length a in an alumina strip, as shown in Fig. 7. The stress intensity factor of the edge crack is given by $K = 0.016(E/H)^{1/2} P/a^{3/2}$, where E and H are the Young's modulus and hardness of the ceramic, respectively. For a fixed indentation load of $P = 100\,\text{N}$, the crack will grow until K becomes smaller than the fracture toughness $K_{IC} = 4\,\text{MPa}\sqrt{\text{m}}$ of the ceramic. Determine the length of the crack at which it arrests. For alumina, $E = 300\,\text{GPa}$ and $H = 3\,\text{GPa}$. [10%]

(c) After the indentation, and after the load P has been removed, a uniform tensile stress σ is applied perpendicular to the indentation crack of length a in (b). The stress intensity factor of the crack now becomes $K = 0.016(E/H)^{1/2} P/a^{3/2} + 1.12\sigma\sqrt{\pi a}$, with the first term representing the contribution of the residual stresses generated due to inelasticity underneath the contact impression.

(i) Sketch K as a function of crack size a for selected values of σ. Superimpose on this plot the line represented by $K = K_{IC}$, and hence comment on the behaviour of crack growth as σ is gradually increased. [30%]

(ii) Determine the smallest σ at which fast fracture occurs in the indented strip and the corresponding crack length. [20%]

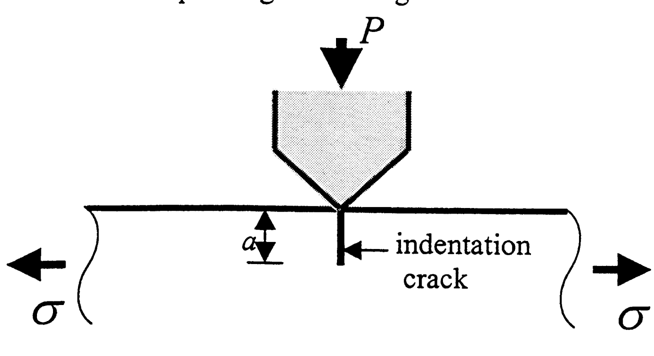

Fig. 7

END OF PAPER

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Sheet to be handed in with solution for Question 2 if question is solved graphically