Part IA 2005, Paper 1, Section A Answers.

Q1: 575.5 MN

Q2: 130.3 kW/m

Q3: (a) $CH_4 + 2(O_2+79/21\ N_2^*) \rightarrow CO_2 + 2H_2O + 2\times79/21\ N_2^*$

(b) WET: CO_2=9.5% H_2O =19.0% N_2^*=71.5%

DRY: CO_2=11.7% N_2^*=88.3%

Q4: (a) Two or more non-dimensional groups.

(b) $F/(\rho V^2 d^2)$ and D/d

Q5: (a) Parallel streamlines

(b) 2 m/s 62.8 kg/s

(c) 30 kPa

(d) F_x= 816.8 N F_y= -435.3 N

Q6: (b) 475.1 K -176.0 kJ/kg

(c) 125.5 kJ/kg

(d) 603.0 kJ/kg

(e) 757.7 K 444.5 kJ/kg

(f) 44.5%

Q7: (a) He: 0.160 kg O_2: 0.288 kg

(c) 0.448 kg 327.7 K 1.67 bar

(d) 284.2 K 1.16 m^3

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Answers

8 (a) 8.67 m/s at a bearing of 34.8° (b) 0.640 m/s^2 at a bearing of 353.7°

9 (a) 1.5 m/s^2 (b) 3 m/s^2 at $t = 10$s (c) 20.8 m/s

10 (b) $\dfrac{11Mr^2}{9}$ (c) K.E. increases; from work done pulling arms in

11 (a) $A = \dfrac{1}{k}$ and $T_1 = \dfrac{\lambda}{k}$ (b) $B = \dfrac{3}{k}$, $T_2 = \dfrac{\lambda}{k}$ and $T_3 = \dfrac{2\lambda}{3k}$

12 (a) 433 mm/s upwards, 0.433 rad/s anticlockwise and 0.060 rad/s anticlockwise
(b) 15.4 N (c) 14.6 N

13 (b) 76.5 V at $t = 0.32$ ms (c) 65 V, −128° (d) 1.2 Ω

A.L. Johnson

Engineering Tripos Part 1A 2005 paper 2 section A
Numerical answers prepared by Andrew Palmer

1 (a) at left end $3wL8$, at right end $wL/8$
(b) $S = -(3/8)wL + wx$ in $x<L/2$, where x is distance from left end; $S = +(1/8)wL$ in $x>L/2$
(c) $M = -(3/8)wLx + (1/2)wx^2$ in $x<L/2$, and in $x>L/2$ varies linearly from $wL^2/16$ when $x=L/2$ to zero at the right-hand simple support. The maximum bending moment is at $x=3L/8$, and is $(9/128)wL^2$

2 (a) $2\alpha\theta_1/h$
(b) $(1/6)E\alpha\theta_1 bh^2$

3 (a) 24 kN
(b) 51.875 kN, 68.125 kN
(c) – 27.25 kN m

4 (a) equation (iii)
(b) at both ends, $v=0$ and $d^2v/dx^2 = d^2v_0/dx^2$

5 (b) lower bar $+\dfrac{0.3P(10-j)a}{b}$; diagonal $+0.3P\dfrac{\sqrt{a^2+b^2}}{b}$; vertical $-0.3P$
(c) $\dfrac{0.3(11-j)a\Delta}{b}$

6 (a) 309 kN m
(c) at A 225 MPa, at B 42 MPa
(d) 215 MPa

Engineering Tripos Part 1A 2005 paper 2 Section B
A.M.Campbell

Numerical Answers

10c. 28 Microns
12b. Wall thickness 7.8 mm.
12c Crack size 0.58 mm. Proof pressure 14.6MPa
12d Crack extension 0.048 mm.

2005 Part IA Engineering, Paper 3 (Electrical and Information Engineering), Section A (Linear Circuits)

Numerical Answers **Dr T J Flack**

1. 0.175 A

2. -

3. Total complex impedance is $(4 + j1)\ \Omega$.
Magnitude of the current is 58.2 A, phase wrt voltage source is 14 degrees lagging.

4. b) $R_1 = 1\ k\Omega$, $R_2 = 2\ k\Omega$

c) Gain = - 8.33, output resistance = 1.67 kΩ

d) Capacitor $C_0 = 2.39\ \mu F$

e) Range of available voltages at the drain is 4 V up to 20 V, so a V_{DS} of 8 V gives a drain voltage exactly midway between these values, allowing maximum voltage swings.

5 a) Current = 118 A, feeder power loss = 6.91 kW, source voltage = 1.15 kV

b) Capacitor = 197 μF, feeder current = 100 A, feeder power loss = 5 kW

d) Factory voltage = 943 V

2005 Part IA Engineering Paper 3 - Sections B and C

Numerical Answers **Dr A R L Travis**

Section B

6 9, 9

7 E7

8 N/A

9 (a) V_2= 7.1 volts

10 (b) 2 bistables

(c) $J_a = Q_b$, $K_a = \overline{B}.Q_b$, $J_a=Q_a+B$, $K_b=1$

Section C

11 10 μT

12 contract

13 (a) 0

(b) 1 Coulomb

(c) 1/24ε Joules

Part IA 2005

Paper 4: Mathematical Methods

Answers

1. $x - \frac{7}{6}x^3 + O(x^5)$.

2. $y = e^{-x}(A\cos 3x + B\sin 3x) + 0.1e^{-2x}$.

3. $z = e^{\pm i(\pi/6 + n\pi)}$.

4. (a) $12x + 21y - 34z = -56$.
   (b) 1.84.
   (c) $\mathbf{R} = (\mathbf{I} - 2\mathbf{n}\mathbf{n}^T)$; $\mathbf{R} = \frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ 2 & 2 & 1 \\ -2 & 1 & 2 \end{bmatrix}$; $\det \mathbf{R} = -1$.

5. (a) $x_n = \frac{1}{2}(5^n - 3^n)$.
   (b) (i) $\mathbf{A} = \begin{bmatrix} 0.6 & 0 & 0 \\ 0.1 & 0.9 & 0.2 \\ 0.3 & 0.1 & 0.8 \end{bmatrix}$, (ii) $\lambda_1 = 0.6, \mathbf{u}_1 = [1\ 1\ -2]^T$; $\lambda_2 = 1.0, \mathbf{u}_2 = [0\ 2\ 1]^T$;
   $\lambda_3 = 0.7, \mathbf{u}_3 = [0\ 1\ -1]^T$; matrix is not symmetric,
   (iii) $\mathbf{p}_0 = N\mathbf{u}_1 + (N/3)\mathbf{u}_2 - (5N/3)\mathbf{u}_3$; $\mathbf{p}_n \to (N/3)\mathbf{u}_2$ as $n \to \infty$.

6. $x = 2e^{-t} - e^{-2t}$.

7. $[0\ -1]^T$; 1.

8. 0.953.

9. (b) $\begin{cases} 0 & t < 0\,, \\ 1 - e^{-t} & 0 \le t \le T\,, \\ e^{-t}(2e^T - 1) - 1 & t > T\,. \end{cases}$; becomes negative step response as $T \to 0$.
   (c) $\begin{cases} 0 & t < 0\,, \\ 1 - te^{-t} - e^{-t} & 0 \le t \le T\,, \\ 1 - Te^{-t} - e^{-T} & t > T\,. \end{cases}$

10. (a) $f(x)$ has period 1; $g(x)$ has period 1; $h(x)$ has period 2.
    (b) $f(x)$ converges the slowest, $h(x)$ converges the fastest.
    (c) $a_n = \begin{cases} \frac{1}{3} & \text{for } n = 0, \\ -\frac{1}{n^2\pi^2} & \text{for } n \ge 1. \end{cases}$

11. The factorial of 5 is 120; The factorial of 1 is 120; $\mathcal{O}(n)$.

12. −25165824; −25165824.