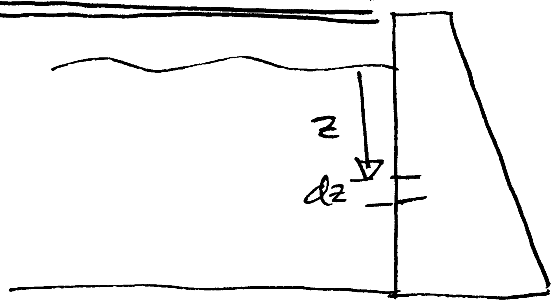
IA 2005 P1 SA

Q1



a) Hydrostatic pressure @ depth z is $\rho g z$ $\boxed{2}$

width $= (100 - z)$ $\boxed{2}$

$$\Rightarrow \quad \delta F = \rho g z (100 - z)\, dz \quad \boxed{2}$$

b)
$$F = \int_0^{40} \rho \cdot g \cdot z(100 - z)\, dz$$
$$= 1000 \times 9.81 \left[50 z^2 - \tfrac{1}{3} z^3 \right]_0^{40}$$
$$= 1000 \times 9.81 \left(50 - \frac{40}{3} \right) \times 40^2$$
$$F = 575.5 \text{ MN} \quad \boxed{4}$$

Examiner's comments:

(i) Many candidates were unable to write $P = \rho R T$.

(ii) Many unable to write width $= (100 - z)$.

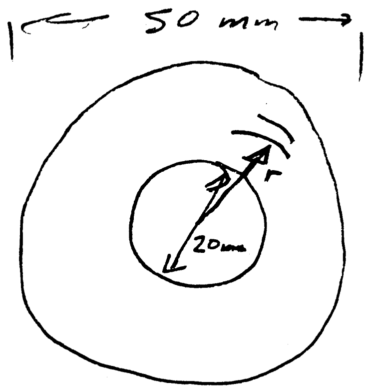
(iii) Many confused between "N" & "Pa" as units.

JPL 1/JULY/05

Q2

IA 2005 P1 SA

$$\dot{Q} = -\lambda 2\pi r L \frac{dT}{dr} \quad \text{(DATABOOK)} \quad \boxed{2}$$



$$\frac{\dot{Q}}{L} \int_{r_1}^{r_2} \frac{1}{r}\, dr = -\lambda 2\pi \int_{T_1}^{T_2} dT$$

$$\frac{\dot{Q}}{L} \ln(r_2/r_1) = -\lambda 2\pi (T_2 - T_1)$$

$$\frac{\dot{Q}}{L} = \frac{-\lambda 2\pi (T_2 - T_1)}{\ln(r_2/r_1)} \quad \boxed{4}$$

$$\frac{\dot{Q}}{L} = \frac{-380 \times 2 \times \pi (350 - 400)}{\ln(50/20)}$$

N.B. $\frac{r_2}{r_1} = \frac{D_2}{D_1}$

$$\frac{\dot{Q}}{L} = 130.3 \text{ kW/m} \quad \boxed{4}$$

Examiner's comments:

(i) Almost everyone got correct formula from Databook.

(ii) Only about 30% integrated the differential eqn!

JPL 1 JULY 05.

IA 2005 P1 SA

Q3 a) $CH_4 + x\left(\frac{79}{21}N_2^* + O_2\right) \rightarrow CO_2 + 2H_2O + x\frac{79}{21}N_2^*$

$O: \quad x \times 2 = 2 + 2 \quad \Rightarrow x = 2$ $\boxed{4}$

$$CH_4 + 2\left(\frac{79}{21}N_2^* + O_2\right) \rightarrow CO_2 + 2H_2O + 2\times\frac{79}{21}N_2^*$$

b) Wet $\Rightarrow$ total kmol $= 1 + 2 + 2\times\frac{79}{21} = 10.52$

$CO_2 = \frac{1}{10.52} = 9.5\%$

$H_2O = \frac{2}{10.52} = 19.0\%$

$N_2^* = \frac{2\times 79/21}{10.52} = 71.5\%$ $\boxed{3}$

DRY : IGNORE $H_2O \Rightarrow 8.52$ kmol total

$CO_2 = \frac{1}{8.52} = 11.7\%$

$N_2^* = \frac{2\times 79/21}{8.52} = 88.3\%$ $\boxed{3}$

Examiner's comments:

(i) Most managed the chemical equation.

(ii) About 50% got "wet" and "Dry" incorrect.

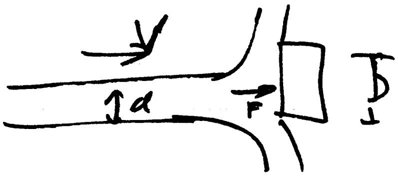
"Wet" when water included

"Dry" when water excluded

JPL 1 JULY 05

1A 2005 P1 SA

Q4



a) $F = fn(\rho, V, d, D)$

5 VARIABLES, 3 (M, L, T) DIMS

$\Rightarrow 5 - 3 = 2$ or more N-D groups $\boxed{4}$

b) $\dfrac{F}{\rho V^2 d^2} = fn\left(\dfrac{D}{d}\right)$ etc. $\boxed{6}$

Examiner's comments:

(i) Very easy question, done very well.

(ii) Many candidates gave non-dimensional groups as:

$$\frac{F}{\rho V^2 d^2} \quad \& \quad \frac{F}{\rho V^2 D^2}$$

These are still independent groups so still gained full marks.

JPC 1 JULY 05.

IA 2005 P1 SA

Q5 (a) Parallel streamlines ⇒ no transverse pressure gradient. $\boxed{2}$

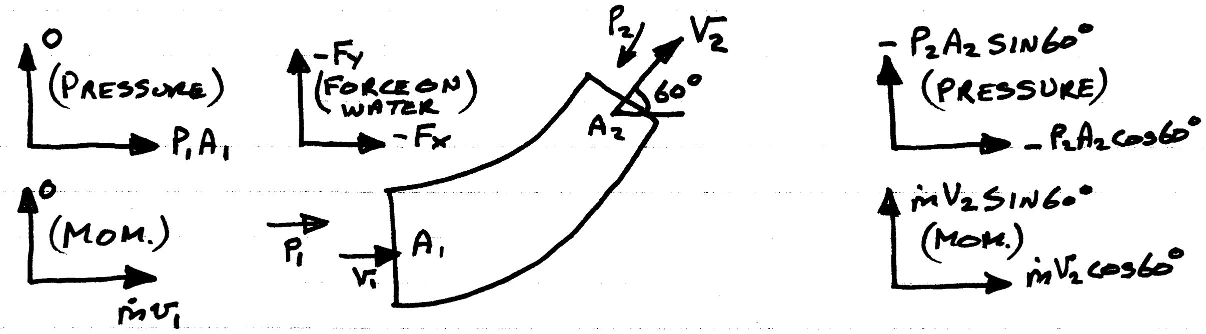
(b) $A_1 = \frac{\pi}{4}(0.2)^2 = \underline{0.031416\ m^2}$ $\qquad A_2 = \frac{\pi}{4}(0.1)^2 = \underline{0.007854\ m^2}$

$v_1 = A_2 v_2 / A_1 = (0.1/0.2)^2 \times 8 = \underline{\underline{2\ m/s}}$ $\boxed{2}$

$\dot{m} = \rho A_2 v_2 = 1000 \times 0.007854 \times 8 = \underline{\underline{62.83\ m/s}}$ $\boxed{2}$

(c) $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P_1 = 0 + \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2} \times 1000 \times (8^2 - 2^2) = \underline{\underline{30 kPa}}$ $\boxed{4}$

(d)



$\underline{R\rightarrow}$ $\quad P_1A_1 - F_x - P_2A_2\cos 60° = \dot{m}V_2\cos 60° - \dot{m}V_1$

$\underline{R\uparrow}$ $\quad 0 - F_y - P_2A_2\sin 60° = \dot{m}V_2\sin 60° - 0$

USE GAUGE PRESSURE.

$F_x = (P_1A_1 - P_2A_2\cos 60°) + \dot{m}(V_1 - V_2\cos 60°)$

$= (30000 \times 0.031416 - 0) + 62.83(2 - 8 \times \tfrac{1}{2})$

$= \quad 942.48 \quad - \quad 125.66$

$\underline{\underline{F_x = 816.82\ N}}$ $\qquad \boxed{10}$

$F_y = (-P_2A_2\sin 60°) - \dot{m}V_2\sin 60°$

$= \quad 0 \quad - \quad 62.83 \times 8 \times \sqrt{3}/2$

$\underline{\underline{F_y = -435.30\ N}}$ $\qquad \boxed{10}$

Examiner's Comments:

(i) Very well done question (note mark re-distribution)

(ii) Common mistake was to omit $\underline{F}_{WATER} = -\underline{F}_{NOZZLE}$

(iii) Some candidates did not appreciate use of gauge pressure.

JPL 1 JULY 05.

IA 2005 P1 SA

Q6) $Q - W = \Delta U = mC_v dT$ $\qquad W = pdV = mpdv \quad Q = 0$

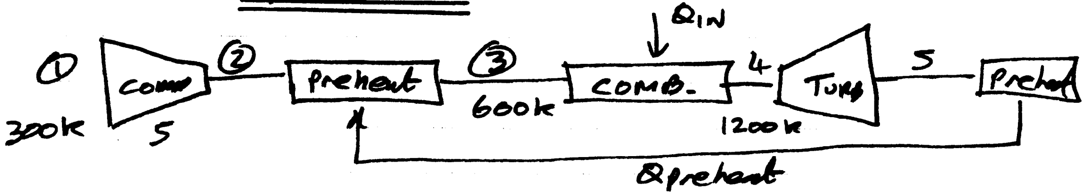
$\Rightarrow -pdv = c_v dT \qquad pv = RT \qquad \frac{dp}{p} + \frac{dv}{v} = \frac{dT}{T}$

$\Rightarrow -\frac{pdv}{pv} = \frac{c_v}{R}\frac{dT}{T}$

$\frac{dp}{p} - \frac{dT}{T} = \frac{c_v}{R}\frac{dT}{T} \Rightarrow \frac{dp}{p} = \frac{c_p}{R}\frac{dT}{T} \qquad \frac{c_p}{R} = \frac{c_p}{c_p - c_v} = \frac{\gamma}{\gamma - 1}$

$\Rightarrow p \propto T^{\frac{\gamma}{\gamma-1}}$ $\boxed{6}$

a)



$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 300(5)^{.4/1.4} = \underline{\underline{475.1\ K}}$ $\boxed{3}$

$W_c = \dot{m} c_p \Delta T \Rightarrow \frac{W_c}{\dot{m}} = -1005(475.1 - 300) = \underline{\underline{-176.0\ kJ/kg}}$ $\boxed{3}$

b) $\dot{Q}_{PH} = \dot{m} c_p \Delta T \Rightarrow \frac{\dot{Q}_{PH}}{\dot{m}} = 1005(600 - 475.1) = \underline{\underline{125.5\ kJ/kg}}$ $\boxed{4}$

c) $\dot{Q}_{IN} = \dot{m} c_p \Delta T \Rightarrow \frac{\dot{Q}_{IN}}{\dot{m}} = 1005(1200 - 600) = \underline{\underline{603\ kJ/kg}}$ $\boxed{4}$

d) $T_5 = T_4 \left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = 1200\left(\frac{1}{5}\right)^{\frac{0.4}{1.4}} = \underline{\underline{757.7\ K}}$ $\boxed{3}$

$W_T = \dot{m} c_p \Delta T \Rightarrow \frac{W_T}{\dot{m}} = 1005(1200 - 757.7) = \underline{\underline{444.5\ kJ/kg}}$ $\boxed{3}$

e) $\eta = \frac{W_{NET}}{\dot{Q}_{IN}} = \frac{444.5 - 176.0}{603.0} = \underline{\underline{44.5\ \%}}$ $\boxed{4}$

Examiner's Comments:

(i) Many candidates used C_v rather than C_p!

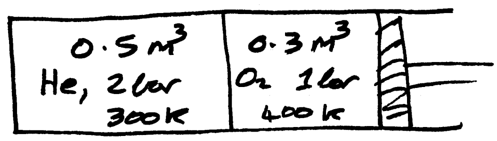
(ii) Many gave incorrect units.

JPL 1 JULY 05.

IA 2005 P1 SA

Q7

a) $PV = mRT$



	He	O_2	
$R =$	2.080	0.260	kJ/kg K
$C_v =$	3.11	0.660	kJ/kg K

$$m = \frac{PV}{RT}$$

Helium : $m_{He} = \dfrac{2\times10^5 \times 0.5}{2080 \times 300} = \underline{\underline{0.160\ kg}}$ $\boxed{3}$

Oxygen : $m_{O_2} = \dfrac{1\times10^5 \times 0.3}{260 \times 400} = \underline{\underline{0.288\ kg}}$ $\boxed{3}$

b) MIXTURE : MASS WEIGHT R & C_v

$$R_{MIX} = \frac{0.160 \times 2080 + 0.288 \times 260}{0.160 + 0.288} = \underline{\underline{910\ J/kg\,K}} \quad \boxed{4}$$

$$C_{vMIX} = \frac{0.160 \times 3110 + 0.288 \times 660}{0.160 + 0.288} = \underline{\underline{1535\ J/kg\,K}} \quad \boxed{4}$$

c) FINAL VOLUME = $\underline{0.8\ m^3}$ FINAL MASS = $0.160 + 0.288 = \underline{0.448\ kg}$ $\boxed{3}$

ENERGY CONSERVATION $Q - W = \Delta E = \Delta U$ $Q = W = 0$

$$M_{MIX} C_{vMIX} T_{MIX} = M_{He} C_{vHe} T_{He} + M_{O_2} C_{vO_2} T_{O_2}$$

$$T_{MIX} = \frac{0.160 \times 3110 \times 300 + 0.288 \times 660 \times 400}{0.448 \times 1535}$$

$$\underline{\underline{T_{MIX} = 327.6\ K}} \quad \boxed{4}$$

$$P_{MIX} = \frac{M_{MIX} R_{MIX} T_{MIX}}{V_{MIX}} = \frac{0.448 \times 910 \times 327.6}{0.8}$$

$$\underline{\underline{P_{MIX} = 1.67\ Bar}} \quad \boxed{3}$$

d) System : $Q - W = \Delta U$ $W = 30\ kJ$

$$-30\times10^3 = m C_v \Delta T = 0.448 \times 1535 \times (T_{END} - 327.6)$$

$$T_{END} = 327.6 - 30\times10^3/(0.448 \times 1535) = \underline{\underline{284.0\ K}} \quad \boxed{3}$$

$$V_{END} = \frac{M_{MIX} R_{MIX} T_{END}}{P_{END}} = \frac{0.448 \times 910 \times 284.0}{1\times10^5} = \underline{\underline{1.16\ m^3}} \quad \boxed{3}$$

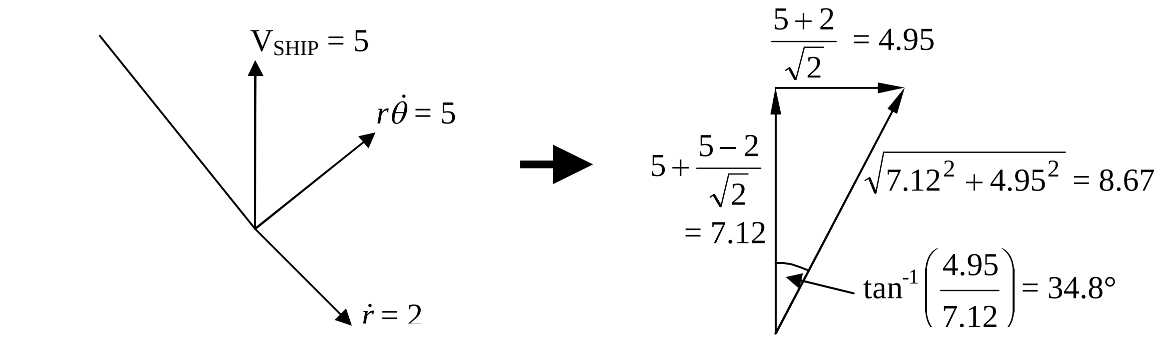
Examiners Comments:

(i) Many candidates "refused" to use energy conservation.

JPL 1 JULY 05.

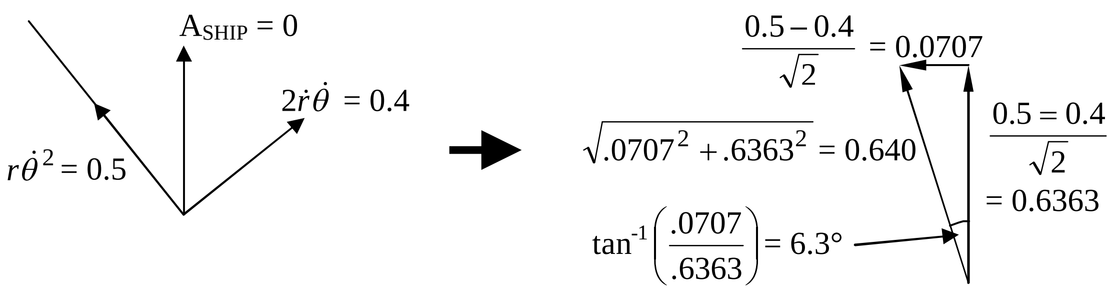
Solutions

8 (a) velocities:



So total velocity is <u>8.67 m/s</u> at a bearing of <u>34.8°</u>

(b) accelerations:



So total acceleration is <u>0.640 m/s^2</u> at a bearing of <u>353.7°</u> (or 6.3° West of North)

9 Let accelerating force be P, mass of sledge be m, and acceleration of sledge be a
Then $P = -\dot{m}V_{relative} = 0.2 \times 30 =$ <u>6 N</u>

(a) $P = ma$ and $m = 4$ kg at $t = 0$, so $a = \frac{6}{4} =$ <u>1.5 m/s^2</u>

(b) Maximum acceleration occurs when mass is at a minimum, i.e. just before the water runs out. This happens at $t = 10$ s, when $m = 2$ kg.
Then, $a = \frac{6}{2} =$ <u>3 m/s^2</u>

(c) We can write $$m\frac{dV}{dt} = -30\frac{dm}{dt}$$
whence $$\int_0^{V_{final}} dV = -30\int_{m=4}^{m=2}\frac{dm}{m}$$
giving $V_{final} = 30 \log 2 =$ <u>20.8 m/s</u>

(Several candidates assumed that the acceleration would change linearly from 1.5 m/s^2 to 3 m/s^2 over the period of acceleration. This gives a final velocity for the sledge of 22.5 m/s – which is a reasonable estimate, but not exactly correct.)

10 (a) Moment of momentum is conserved if there is no net moment about the axis: typically if all external forces pass though the axis, or are parallel to it.

(b) $$I = \frac{Mr^2}{2} + 2\times\frac{M}{12}\left(\frac{4r^2}{12} + (2r)^2\right) = Mr^2\left(\frac{1}{2}+\frac{1}{18}+\frac{2}{3}\right) = \frac{11Mr^2}{9}$$

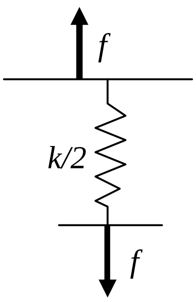
I_G for cylinder I_G for rod Parallel axis term for rod

(c) No net moment about the axis of rotation, so $\underline{I\omega \text{ remains constant}}$. Since I will decrease when the skater pulls her arms in, ω must increase: this means that ½($I\omega \times \omega$) will <u>increase</u>. The additional K.E. comes from the work she does pulling her arms in towards her body, against the centrifugal force that is pulling them outwards.

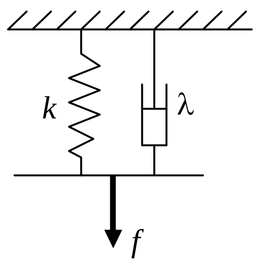
(Many candidates answered part (a) correctly, but then failed to apply the same logic to part (c). Most of these candidates assumed that the kinetic energy would remain constant, as no work appeared to be involved; some thought that ω would remain constant.)

11 (a) Free body diagrams for the two parts of the model:

bottom:



top:



Note that, since there is no mass, the downwards force on the top part <u>must be f</u>.

Equating forces for the top part gives: $ky + \lambda\dot{y} = f$ whence $\underline{y + \frac{\lambda}{k}\dot{y} = \frac{f}{k}}$ (1)

This gives $\underline{A = \frac{1}{k}}$ and $\underline{T_1 = \frac{\lambda}{k}}$

(b) Considering the tension in the bottom spring gives:

$\frac{k}{2}(z-y) = f$ whence $y = z - \frac{2f}{k}$ and $\dot{y} = \dot{z} - \frac{2\dot{f}}{k}$

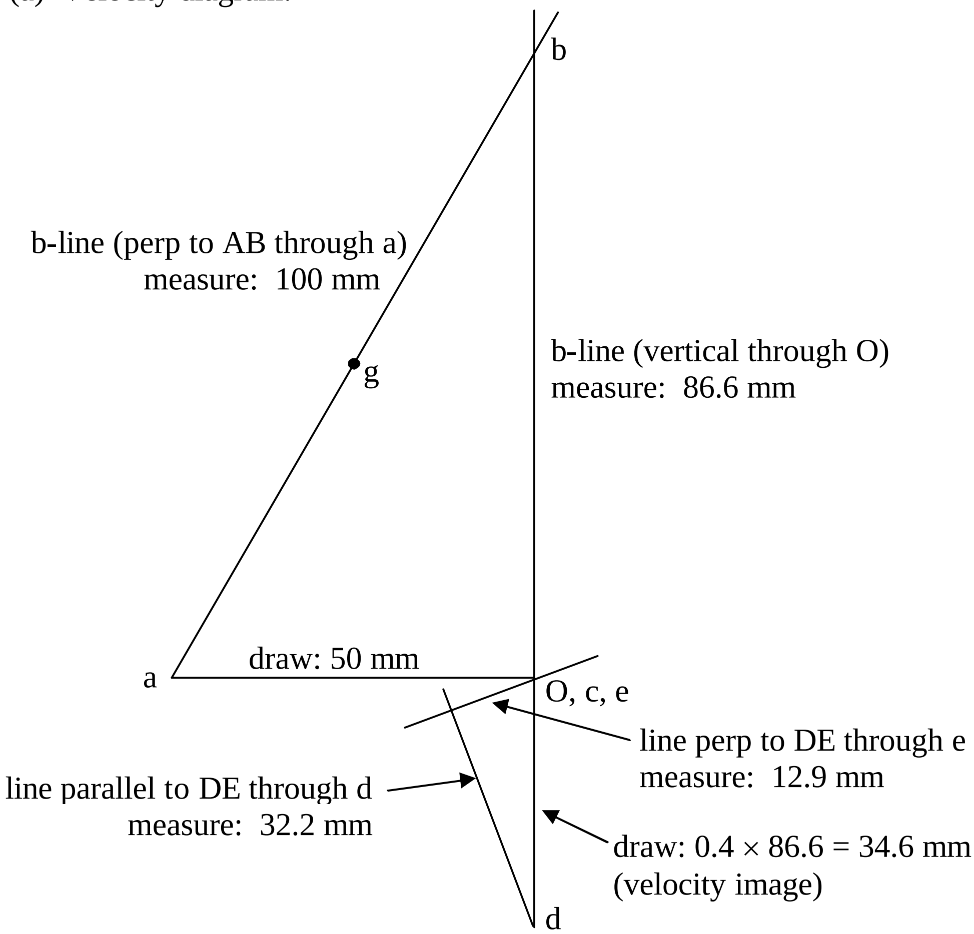
Substituting these into equation (1) gives $z - \frac{2f}{k} + \frac{\lambda}{k}\left(\dot{z} - \frac{2\dot{f}}{k}\right) = \frac{f}{k}$

whence $z + \frac{\lambda}{k}\dot{z} = \frac{3f}{k} + \frac{2\lambda\dot{f}}{k^2} = \frac{3}{k}\left(f + \frac{2\lambda}{3k}\dot{f}\right)$

giving $\underline{B = \frac{3}{k}}$, $\underline{T_2 = \frac{\lambda}{k}}$ and $\underline{T_3 = \frac{2\lambda}{3k}}$

(Note that the three time constants must all have the same dimensions!)

12 (a) Velocity diagram:



So $V_B = 86.6 \times 5 =$ <u>433 mm/s upwards</u>, $\omega_{\text{ARMS}} = \dfrac{433}{1000} =$ <u>0.433 rad/s anticlockwise</u>

and $\omega_{\text{SPRINGS}} = \dfrac{12.9 \times 5}{\sqrt{1000^2 + 400^2}} =$ <u>0.060 rad/s anticlockwise</u>

(b) From the diagram, the C of G of the door has a velocity whose vertical component is $43.3 \times 5 = 217$ mm/s (point g on the diagram), and the springs are contracting at a rate of $32.2 \times 5 = 161$ mm/s. We can therefore write a work equation:

$250\,F + 161 \times 650 = 217 \times 500$ whence <u>$F = 15.4$ N</u> (all velocities in mm/s)

(c) The angular velocity of the door is $\dfrac{100 \times 5}{2000} = 0.25$ rad/s anticlockwise, so the hinges at B are opening at a rate of (0.433 – 0.25) rad/s. The extra force needed is then given by:

$$\frac{250}{1000}\Delta F = 20(0.433 - 0.25) \quad \text{whence} \quad \underline{\Delta F = 14.6\ \text{N}}$$

(Many candidates wrote $250\,F = 217 \times 500 + 161 \times 650$ in part (b), implying that the springs are opposing, rather than assisting, the automatic opener!)

13 (a) Let the current round the circuit be i (in a clockwise direction). Summing the voltages round the loop gives:

$$e - Ri - L\frac{di}{dt} = v \quad (1)$$

But we can also write $\frac{i}{C} = \frac{dv}{dt}$ whence $i = C\dot{v}$ and $\frac{di}{dt} = C\ddot{v}$

Eliminating *i* and its derivatives from equation (1) gives

$\underline{LC\ddot{v} + RC\dot{v} + v = e}$ as required.

(b) Comparing the equation above with p. 6 of the Data Book, $v = y$ and $LC = \frac{1}{\omega_n^2}$

so $\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 10^{-5}}} = 10^4$

Also, $RC = \frac{2\zeta}{\omega_n}$ so $\zeta = \frac{\omega_n RC}{2} = \frac{10^4 \times 4 \times 10^{-5}}{2} = 0.2$

From the graph, $v_{max} \approx 1.53\, e$ = $\underline{76.5 \text{ V}}$ when $w_n t \approx 3.2$, i.e. $\underline{t \approx 0.32 \text{ ms}}$

(c) This is "case (a)", on p. 8 of the Data Book, with $v = y$ and $LC = \frac{1}{\omega_n^2}$

The frequency is $2 \times 2\pi \times 10^3$ so $\frac{\omega}{\omega_n} = 1.257$ and $\zeta = 0.2$ still, so

$v \approx 1.3\, e$ = $\underline{65 \text{ V}}$ and $\underline{\phi \approx \text{-}128°}$ from the graphs.

(d) The "locus of maxima" crosses the $|Y/X| = 2$ line at $\frac{\omega}{\omega_n} \approx 0.93$, so we can use the "maximum response" formula and write $0.93 = \sqrt{1 - 2\zeta^2}$

whence $2\zeta^2 = 1 - 0.93^2$ or $\zeta \approx 0.26$

(this is much easier than solving $2 = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$!)

To achieve this higher damping ratio, we need a new total resistance *R*' such that

$R'C = \frac{2 \times 0.26}{\omega_n}$, giving $R' = \frac{2 \times 0.26}{\omega_n C} = \frac{0.52}{10^{-1}} = 5.2$

So an extra $\underline{1.2\ \Omega}$ of resistance is required.