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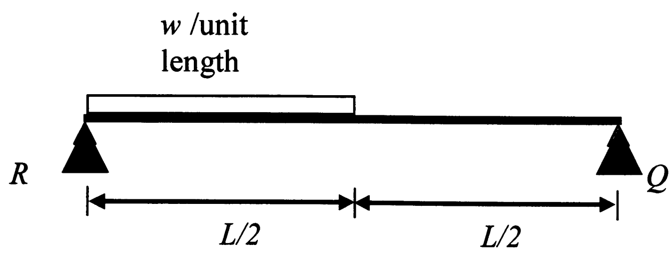
crib prepared by Andrew Palmer

1 (a) moments about left end for whole beam



$$QL = \frac{wL}{2}\frac{L}{4}$$

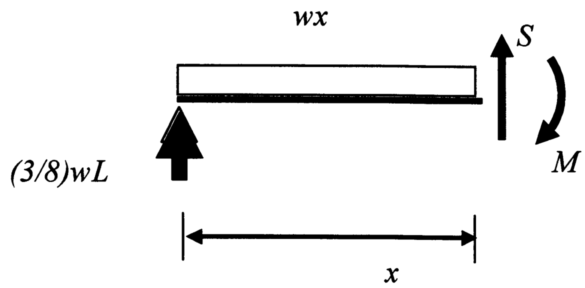
$$Q = \frac{wL}{8}$$

vertical equilibrium

$$R+Q = \frac{wL}{2}$$

$$R = \frac{3wL}{8}$$

(b) free body diagram for section length x from left end

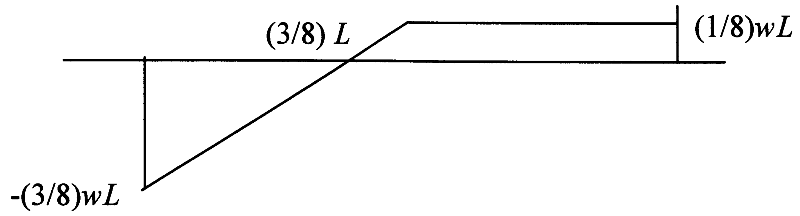


$$S = -(3/8)wL + wx \text{ in } x<L/2$$

in right half

$$S = +(1/8)wL \text{ in } x>L/2$$

shear force diagram



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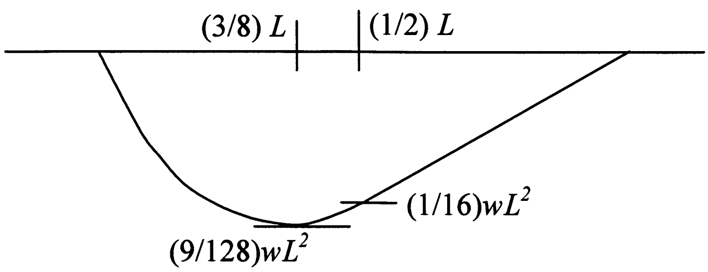
(c) from moment equilibrium of section drawn in (b)

$$\begin{aligned} 0 &= M + (3/8)wLx - (wx)((1/2)x \\ M &= -(3/8)wLx + (1/2)wx^2 \end{aligned} \quad \text{in } x<L/2$$

and in $x>L/2$ varies linearly from $wL^2/16$ when $x=L/2$ to zero at the right-hand simple support.

The bending moment is a maximum when the shear force is zero, at $x=3L/8$, and there the moment is $\left(-\frac{9}{64}+\frac{1}{2}\frac{9}{64}\right)wL^2 = \frac{9}{128}wL^2$
the bending moment diagram is



extract from examiner's report

This was plain-vanilla forces and bending moments in a beam. Almost everyone could calculate the end reactions correctly, and most could draw a correct shear force diagram, although some were confused about signs and could not relate the end reactions to the end shear forces. Bending moments produced more confusion; weaker candidates guessed the shape of the bending moment diagram, and imagined that the largest bending moment must necessarily be at the mid-point. This is the same difficulty that I remarked on last year. Once again, relatively few went back to the wholly reliable method of drawing a free body diagram for part of the beam, and I have come to suspect that weak supervisors may encourage their weak students to guess rather than going back to first principles.

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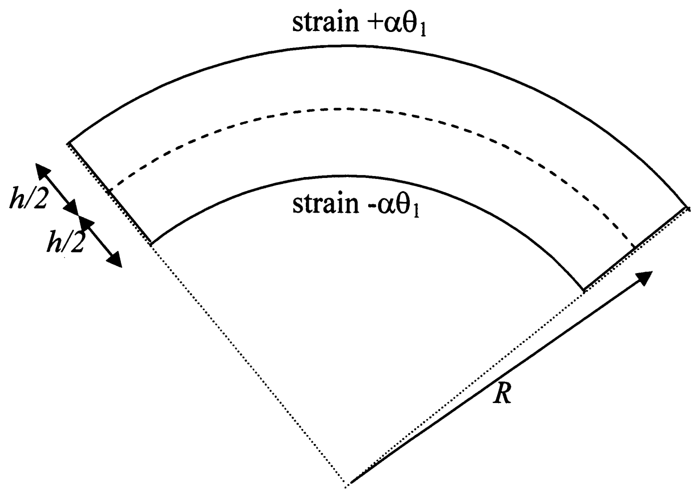
2 (a) Free expansion creates a linear variation of longitudinal strain, from $+\alpha\theta_1$ at the top to $-\alpha\theta_1$ at the bottom. That is identical with the distribution of strain in pure bending, following the Bernoulli-Euler model. The beam bends into a circular arc with radius R:



(curvature much exaggerated)

From geometry $\alpha\theta_1 = \dfrac{h/2}{R}$ and so the curvature is $\kappa = \dfrac{1}{R} = \dfrac{2\alpha\theta_1}{h}$

(b) The applied bending moment has to create an equal and opposite curvature, so that the resultant curvature is 0. The required bending moment is

$$\text{(flexural rigidity EI)(change of curvature)} \quad = E\frac{bh^3}{12}\frac{2\alpha\theta_1}{h} = \frac{1}{6}E\alpha\theta_1 bh^2$$

<u>extract from examiner's report</u>

Part (a) was easy for anyone who understood the relationship between strain and curvature in pure bending, but most candidates do not. The general standard was dreadful. Part (b) tests awareness of superposition, and was answered much better than (a). A significant number solved (b) first and worked back to (a), which is of course acceptable (and confirms the adage that in structural mechanics some of us are strain people and some are stress people).

Many answers were dimensionally incorrect in obvious ways, or were in conflict with common sense, for example by appearing to assert that if α or θ_1 is zero than the curvature is infinite.

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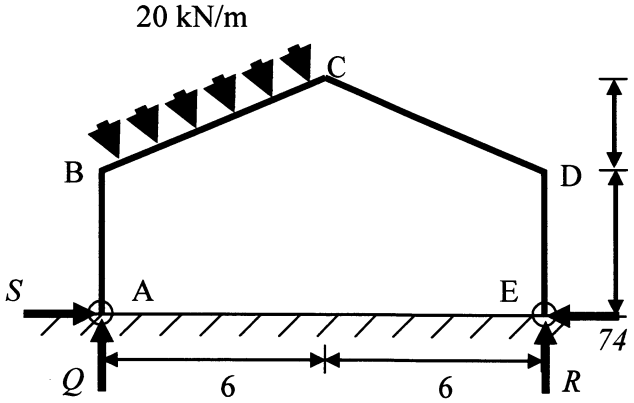
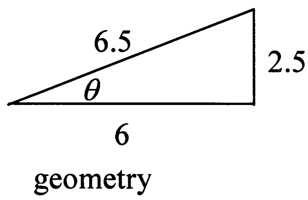


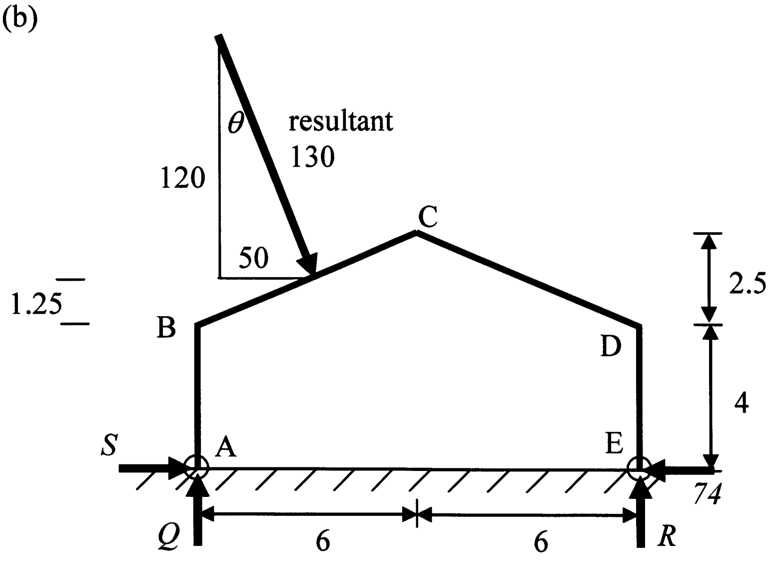


(a) load on BC is (20 kN/m)(6.5 m) = 130 kN
horizontal component is 130sinθ= 130(2.5/6.5) = 50 kN
from horizontal equilibrium

S+50 = 74

S = 24 kN

(b)



moments about A
12R = (120)(30) + (50)(5.25)
R = 51.875 kN
vertical equilibrim
Q+R = 120
Q = 68.125 kN

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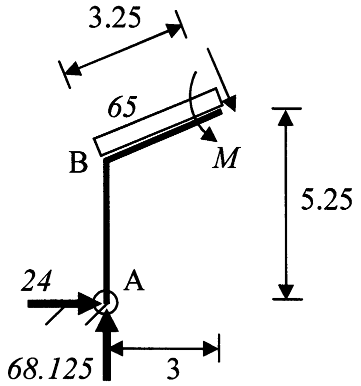
(c) free body diagram for section to left of midpoint of BC; take moments about midpoint

$0 = M + (65)(1/2)(3.25) + (24)(5.25) - (68.125)(3)$
$M = -27.25$ kN m



extract from examiner's report

Almost everyone had the basic ideas here, but there were many mistakes. (a) is about horizontal force equilibrium of the whole frame, (b) about moment and vertical force equilibrium of the whole frame, and (c) about a part of the frame to the left or right of an imaginary cut. (a) and (b) are decoupled, so that an error in one part does not make the other wrong. (c) is not decoupled, but I did not penalise candidates a second time for errors in (a) or (b).

Common mistakes were:

(i) remembering the horizontal component of the wind force in (a), but forgetting it in (b);
(ii) exchanging the horizontal component and the vertical component;
(iii) taking moments, but confusing the lever arms;
(iv) in (c), treating BC as a beam but forgetting that there are end moments at B and C.

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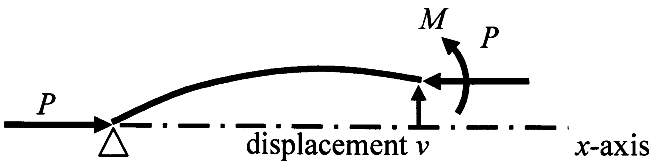
4 (a) draw the free body diagram for a section from the left-hand simple support to a distance x. Taking moments for the whole beam shows that there is no vertical reaction at that support.



Taking moments, from equilibrium $M + Pv \quad = 0$

The bending moment is the flexural rigidity EI multiplied by the <u>change</u> of curvature

$$M = EI\left(\frac{d^2v}{dx^2} - \frac{d^2v_0}{dx^2}\right)$$

and therefore

$$EI\left(\frac{d^2v}{dx^2} - \frac{d^2v_0}{dx^2}\right) \quad + Pv = 0$$

which is equation (iii).

(b) At the ends $x=0$ and $x=L$, $v=0$

In addition, the bending moment M is zero, and so $\frac{d^2v}{dx^2} = \frac{d^2v_0}{dx^2}$ at the ends, which is in fact implied by the combination of (iii) and $v=0$.

<u>extract from examiner's report</u>

This question was straightforward but of an unfamiliar kind. It was generally answered very well. Weak candidates guessed or did not read the definitions of v and v_0. There was uncertainty about the end conditions: many imposed conditions on v', or forgot that v_0'' is not necessarily zero at the ends.

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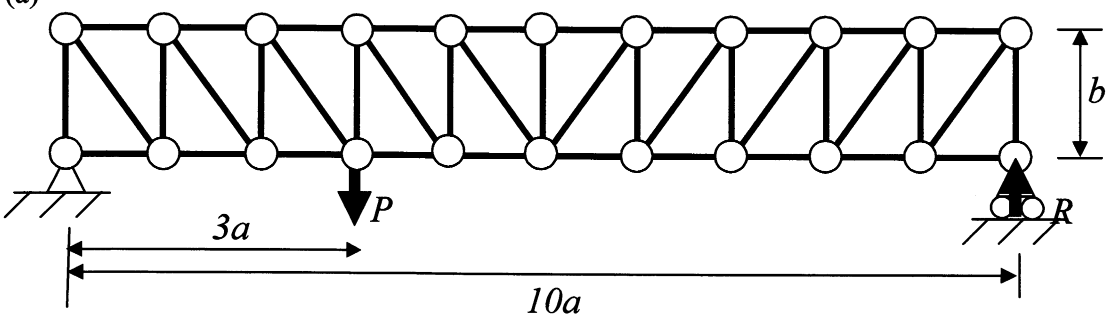
(a)

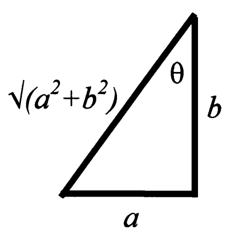


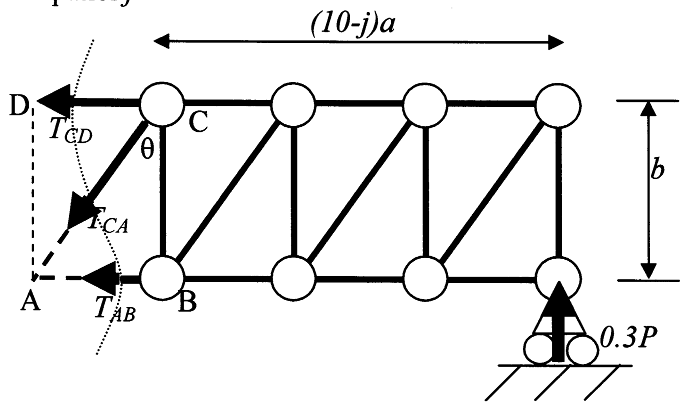
take moments about left support

$$10aR = 3aP$$

$$R = 0.3\,P$$

consider portion to the right of a section across panel *j*





take moments about A

$T_{CD}b + 0.3P(11\text{-}j)a = 0$

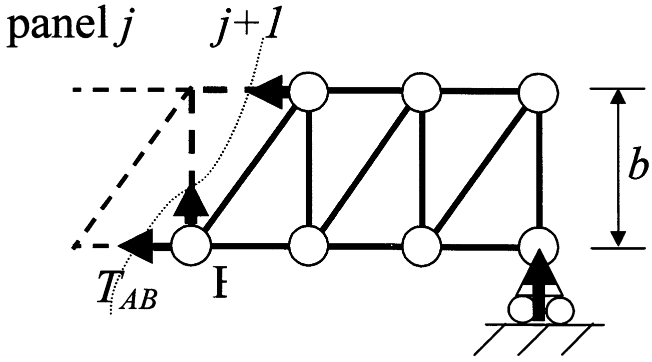
$$T_{CD} = -\frac{0.3P(11-j)a}{b}$$

(b) take moments about C

$T_{AB}b - 0.3P(10\text{-}j)a = 0$

$$T_{AB} = +\frac{0.3P(10-j)a}{b}$$

(or alternatively by drawing the section differently



and noting that horizontal equilibrium requires that the tension in the bottom horizontal bar of panel *j* is equal and opposite to the tension in the top horizontal bar of panel *j+1)*

Going back to the previous section, vertical equilibrium requires that

$$T_{CA}\cos\theta = \quad 0.3P$$

$$T_{CA} \quad = \quad 0.3P\sec\theta \quad = \quad 0.3P\frac{\sqrt{a^2+b^2}}{b}$$

and vertical equilibrium of joint B requires that

$$T_{BC} = \quad -T_{CA}\cos\theta = \quad -0.3P$$

The same result can be obtained by considering vertical equilibrium of the segment to the right of the alternative section drawn on this page.

(c) This is most easily done by virtual work. The compatible set of bar elongations and displacements has a vertical displacement *v* downward at the third joint from the left in the lower chord, zero displacements at the supports at the ends, an elongation of -*Δ* in the top bar of panel *j,* and no other elongations. The equilibrium set of bar tensions and external loads has unit vertical load at the third joint from the left in the lower chord, and a tension of

$$T_{CD} = \quad -\frac{0.3(11-j)a}{b}$$

in the top horizontal bar of panel *j,* as calculated previously. The end reactions are not needed, because the corresponding displacements in the compatible set are zero. The other bar tensions are not needed, because the corresponding elongations in the compatible set are zero. Virtual work says that

$$1u = \left(-\frac{0.3(11-j)a}{b}\right)(-\Delta) \quad = \quad \frac{0.3(11-j)a\Delta}{b}$$

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(d) In configuration (a), downwards-acting vertical loads put all the diagonal bars into tension, whereas in configuration (b) downwards-acting vertical loads put all the diagonal bars into compression. The diagonal bars are longer than the vertical or horizontal bars. Long bars in compression have to be designed to resist buckling, and therefore may have to have heavier sections or additional bracing.

<u>extract from examiner's report</u>

Many candidates did not attempt to answer this question, which is easy but perhaps looked daunting.

(a) was answered well, almost invariably by the method of sections as suggested. A few attempted a solution by virtual work, but hardly any were successful. Not one of those drew a displacement diagram. Disappointingly, a number of candidates wrote equations that did not correspond to the required solution, and then magically produced the final result: I marked such dishonesty down heavily.

(b) was answered less well than (a), which confirms that questions like 4(a) are more searching than questions like 5(a). I didn't have a sense that candidates had a feeling for how structures work: for instance, some found that the force in the vertical bars is zero, suggesting that those bars could be removed (for this loading), and failed to point out that without the vertical bars the structure becomes a mechanism.

(c) was answered well by candidates who realised that it was a simple application of virtual work, badly by everyone else.

(d) was answered reasonably well.

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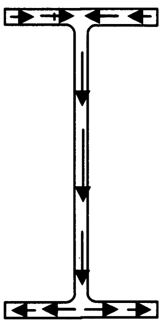
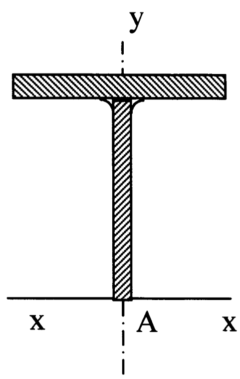
6(a) elastic section modulus $Z =$ 687 cm^3 = 687×10^{-6} m^3
yield stress Y = 450 MN/m^2 = 4.5×10^5 kN/m^2
moment at yield = ZY= (687×10^{-6} m^3)(4.5×10^5 kN/m^2) = 309 kN m

(b)



(c) point A

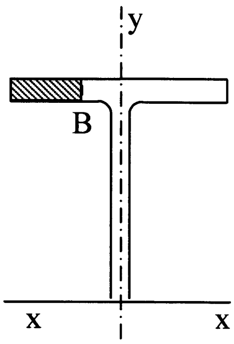


I = 12070 cm^4 = 1.207×10^8 mm^4

$\int ydA =$ (171.1)(9.7)(175.7-4.85) + (7)(166)(83) = 283554.6 + 96446 = 380000.6 mm^3
flange half-web

$$\text{shear stress} = \frac{S\int ydA}{I(web\ thickness)} = \frac{(5\times10^5\ N)(380000\ mm^3)}{(1.207\times10^8\ mm^4)(7\ mm)} = 225\ N/mm^2$$

point B



$\int ydA =$ (60)(9.7)(175.7-4.85)

shear stress =

$$\frac{S\int ydA}{I(\mathit{flange\ thickness})} = \frac{(5\times10^5\,N)((60)(9.7)(175.7-4.85)mm^3)}{(1.207\times10^8\,mm^4)(9.7\ mm)} = 42\ N/mm^2$$

(d)

$$\frac{\mathit{shear\ force}}{\mathit{web\ area}} = \frac{(5\times10^5\,N)}{(7\ mm\ )(332\ mm)} = 215\ N/mm^2$$

which agrees well with the 'exact' calculation, because the shear stresses that act vertically are almost entirely within the web, as can be seen from (b), and therefore the shear stresses in the web carry the shear force. The shear stresses in the flanges act horizontally.

<u>extract from examiner's report</u>

(a) was answered well, though some came up with absurd answers such as 309 N m (where the number is right but the units out by a factor of 1000), and failed to remark on that. The units used in the data book do not exactly help, but an engineer ought to be able to cope with that. I wonder if the students are gaining enough knowledge of magnitudes.

(b) was answered badly: many candidates can calculate the shear stresses in (c), but have no clue about how and in which directions they act;

(c) and the first part of (d) were answered quite well. There were many blunders in (c), among them:

(i) saying that since point A is on the neutral axis, the shear stress there must be zero;
(ii) recalculating *I* in the shear flow formula as the value for the cut-off portion of the section, instead of the whole;

(iii) calculating $A\tilde{y}$ for point A by saying that the cut-off area is half the total area, and the corresponding distance for the neutral axis is a quarter of the section depth.

A few interpreted point B as the point where the flange and the web meet, rather than the point on the flange 60 mm in from the edge. I was generous to correct answers to that different problem.

The second part of (d) was answered badly.

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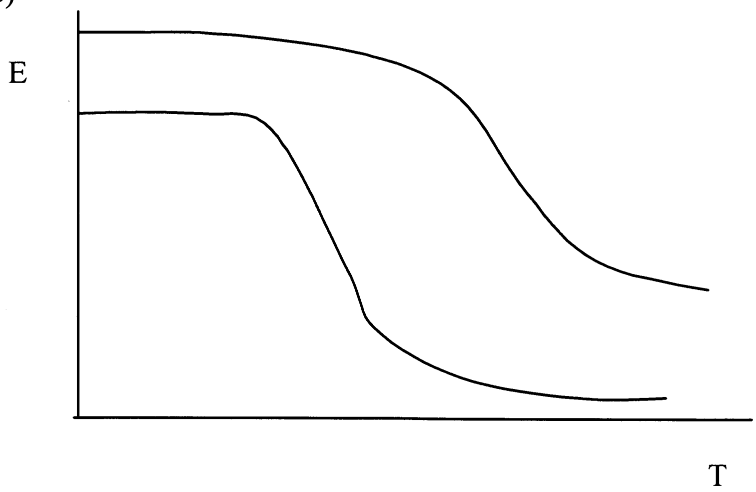
Crib prepared by A.M.Campbell

7)
a) The important features are:-:-
i) Number of Cross links
ii) Size and type of side chains
iii) Degree of crystallinity.

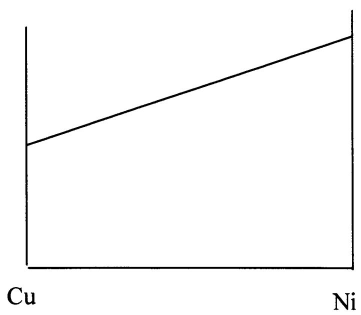
All increase modulus and yield stress, so reduce fracture toughness.

b)



Comments: Many candidates did not read the question but put down a whole lot of material on melting points and decomposition which was not asked for and got no marks.

8)
a) The modulus depends on the mean strength of atomic bonds so varies roughly linearly with composition. (The exact shape will depend on whether the mean of the A-A and B-B bonds is greater or smaller than the A-B bond).



Modulus

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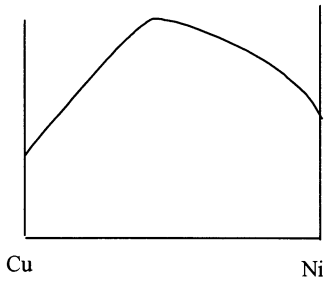
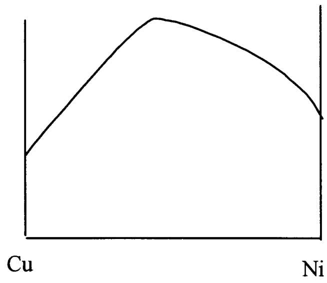
Crib prepared by A.M.Campbell

b) The yield stress increases with defect number so is a minimum at the two pure elements. Dislocations move freely in pure metals but are pinned by foreign atoms, hence there is a maximum between the pure metals



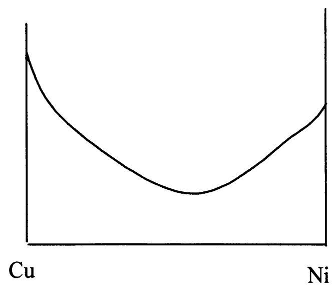
Yield Stress

c) Electrons are scattered by impurities so the resistivity s a maximum between the pure metals.



Electrical resistivity

d) The fracture strength decreases as yield strength increases because the plastic zone size decreases. (Since a number of other actors affect the yield stress this correlation only works if all other factors remain the same).



Fracture Toughness

Comments: Although all the material needed for this question was in the notes somewhere, it required some analysis and few candidates got many marks. This is only a starting point, supervisors may find this a useful question for more detailed discussion.

Engineering Tripos Part 1A 2005 paper 2 Section B
Crib prepared by A.M.Campbell

9) The electrode potential is the energy in electron volts required to remove an electron from a metal, or the energy gained when added to a non-metal. The higher the electrode potential the greater the tendency to corrode. If two metals are place in an electrolyte the voltage of the resulting cell is the difference in electrode potentials, and the one with the lower potential will corrode if the plates are connected while the other will be protected. This is galvanic protection
We would expect the rate of corrosion would put Al highest then iron then copper. This is true, except that Al forms a protective oxide layer which prevents further corrosion. Copper also forms a protective layer (patina) depending on the atmosphere round it.

i) Copper roof, copper nails. uniform material, low electrode potential.

ii) Iron roof, iron nails, uniform material, higher electrode potential more rapid corrosion.

iii,iv) Copper roof with iron nails or iron roof with copper nails will lead to rapid corrosion of the iron due to galvanic action. In both cases it will fail at the nails, with little difference in lifetime. (Any sensible answer here was acceptable).

Comments: Nearly all knew about galvanic protection, although nobody pointed out that you need not only a higher electrode potential but also a reasonably protective oxide layer to prevent the sacrificial electrode corroding too quickly (there would be no point in using sodium). A significant minority thought that Aluminium corroded more quickly than iron which shows a distinct lack of observation. About half got the point about not using different materials in the roof. A number were a bit literal in the interpretation of 'failure of the roof' assuming that the roof had not failed even if all the nails had fallen out. I am not sure a customer would agree.

10)
a) A dislocation is an extra row of atoms in a crystal which allows atomic layers to slip over each other when the dislocation moves. The ease with this happens determines the yield stress of the material.

b) Dislocations can be pinned by foreign atoms, precipitates, other dislocations and grain boundaries.

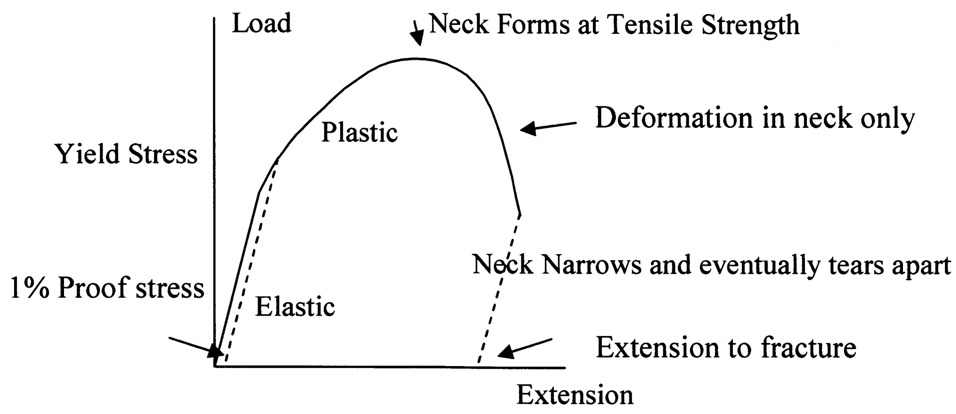
c) For spheres of radius r spaced a apart the volume fraction is $4/3\ \pi r^3/a^3$.
Hence $a=7.5\text{x}(4\pi/0.24)^{1/3}=28$ microns.

Comments: Most got (a) and (b) right. In (c) some got confused with the crystal structure, some used 2D rather than a 3D structure. There were many numerical mistakes and the candidate who came out with a spacing greater than the diameter of the galaxy might have pointed out that this was a bit on the large side.

Engineering Tripos Part 1A 2005 paper 2 Section B
Crib prepared by A.M.Campbell

11)
a) The 'nominal stress' is the load divided by the starting area, the 'true stress' is the load divided by the instantaneous value of the cross sectional area.
The 'nominal strain' is the extension divided by the original length, the 'true strain' is the sum of the increments of strain (extension divided by instantaneous length) up to the point of interest.

b) 'Necking' occurs when the increase in stress due to the thinning of the specimen is greater than the increase in yield stress due to work hardening.
If the length increases by δl, then since volume is constant $\delta(Al)=0$, so
$\delta A=-A\delta l/l=A\delta\varepsilon$.
The increase in yield strength is $\delta\sigma_y=\delta\varepsilon d\sigma_y/d\varepsilon=\delta\varepsilon\, d\sigma/d\varepsilon$ where $d\sigma/d\varepsilon$ is the gradient of the true stress -strain curve.
If the load is F the increase in stress is $\delta\sigma=\delta(F/A)=-F\,\delta A/A^2=\sigma\,\delta\varepsilon$.
Hence necking starts when $d\sigma/d\varepsilon=\sigma$.
c)



Load- Extension Curve

d) In the elastic region bonds are stretched by a small amount. At the yield stress dislocations begin to move and plastic deformation occurs. The strain generates more dislocations which interfere with each other causing work hardening. When the work hardening rate is low enough a neck forms and all subsequent deformation takes place here. This continues until after a large amount of deformation in the neck voids open up and coalesce so that ductile fracture occurs.

e) Since copper is fcc it does not become brittle at low temperatures. The only changes are small increases in the modulus and yield stress (about 15% in each).

Comments: A significant minority had not taken in either the lectures or the lab on which this is based and talked vaguely about 'bonds breaking' (no marks). However many knew that necking was a plastic instability and that fcc materials do not become brittle at low temperatures. Some candidates gave the curve for steel although they had all seen both. However since only half the class actually did the copper sample, credit was given for this.

12)

(a) The stress intensity factor is $\sigma\sqrt{\pi a}$ and is a measure of the stress at the root of a crack in a given material. The fracture toughness is the value of K needed to propagate the crack. The hoop stress is double the longitudinal stress, so is the critical stress.

(b)The hoop stress in a cylinder of diameter d with a wall thickness t is $Pd/2t$.
For a crack of half length a
$K=Pd\sqrt{(\pi a)}/2t$.
If the crack just penetrates the wall $a=t$ and to avoid fast fracture we must have $K<K_{IC}$.
Hence $t>(Pd\sqrt{\pi}/(2K_{IC})=(4\times5/(2\times200))^2\times\pi=7.8$ mm.

In fatigue $da/dN=A(Pd/2t)^4\pi^2a^2$ so
$1/a_o-1/(7.8\times10^{-3})=NA(Pd/2t)^4\pi^2=2000\times3\times10^{-14}\times(4\times5/(2\times7.8\times10^{-3}))^4\pi^2$.
Maximum Initial crack size a_o =0.58 mm.
Pressure to propagate this crack = $2tK_{IC}/(d\sqrt{(\pi a_o)})$
$=2\times7.8\times10^{-3}\times200/(5\times\sqrt{(\pi\times0.58\times10^{-3})})=14.6$ MPa
This is the minimum proof pressure P_p.
This is one cycle at the proof stress so $da=A(P_pd/(2t))^4\pi^2a_o^2$
$=3\times10^{-14}\times(14.6\times5/(2\times7.8\times10^{-3})^4\times\pi^2\times(0.58\times10^{-3})^2=0.048$ mm.
This is an 8% addition to the crack, which is not large and will be covered if we increase the proof stress by 10%, to allow a safety margin.

Comments: This is similar to the question on the examples sheet and most candidates did reasonably well. Inevitably arithmetic mistakes were rife and with the high powers involved there was a wide spread of numbers. One found a crack extension of 10^{-55} m which I think is a record low. One candidate pointed out that the maximum stress in the material was 4.7GPa which is likely to be higher than the yield stress. This is something that would have to be checked in any real calculation, but fortunately this examiner does not have to make a living out of designing pressure vessels.

13)

$w=\rho gA$ and $T=\sigma A$ sothe stress $\sigma=\rho gL^2/8h$

	Al	Cu	Steel
σ	102	336	295
$\sigma/0.8\times\sigma_y$	0.64	1.05	0.587

Hence Al or Steel but not Cu can be used.

If the material cost is C_M /kg the cost/ metre is $C_M\rho A$

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Crib prepared by A.M.Campbell

The resistance/metre is $R=\rho_E/A$ Dissipation= $I^2\rho_E/A$.
Cost=$TC_EI^2\rho_E/A$ where T is the time (20 years)
Total cost=$C_M\rho A+TC_EI^2\rho_E/A$.
This is of the form cost=aA+b/A which is a minimum when $A^2=b/a$.
With A this value both terms in the cost are $\sqrt{(ab)}$, i.e the costs are equal.
In this case ab= $C_M\rho TC_EI^2\rho_E$.
A suitable material merit index is $C_M\rho\rho_E$ which is 4.6×10^{-5} for Al and 1.3×10^{-3} for steel.
Hence choose Aluminium.

Comments: Like all design questions this is much more difficult than it looks, until you see the answer. Most candidates forgot to multiply the mass by g. More damaging, many did not multiply the stress by the area to get the tension. This meant that they needed to get a figure for the area which they did by the bizarre method of taking the line voltage (which is in fact a totally irrelevant parameter) dividing by the current to get the resistance and then using the resistivity to get the area. It would be better in these circumstances to guess an area and carry on. There were however several correct answers.