ENGINEERING TRIPOS PART IA

Wednesday 8 June 2005 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer **all eight** short questions and **not more than four** long questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1 **(short)** A reservoir is going to be formed by building a dam across a valley, Fig. 1. Once completed, the water in the reservoir will be 40 m deep, 60 m wide at the bottom and 100 m wide at the surface of the reservoir. The wetted area on the dam is trapezoidal in shape.

(a) Show that the horizontal force on the dam due to the hydrostatic pressure between depth z and $z+dz$ is given by: $\rho g z(100-z)dz$ [6]

(b) Calculate the horizontal force on the dam due to the water. [4]

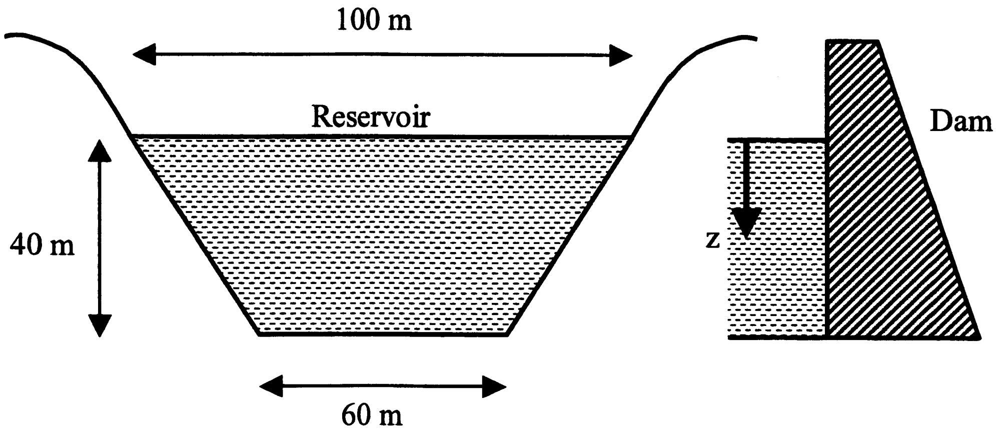

Fig. 1

2 **(short)** A thick-walled copper pipe has internal diameter 20 mm, external diameter 50 mm and thermal conductivity 380 $Wm^{-1}K^{-1}$. If the temperatures of the internal and external surfaces are 400 K and 350 K respectively, calculate the heat transfer per unit length of the pipe under steady-state conditions. [10]

3 (**short**) Methane (CH_4) is completely burnt with a stoichiometric quantity of air in a combustion chamber at atmospheric pressure.

(a) Write down the chemical equation for the combustion process. [4]

(b) Calculate the volumetric composition of the combustion products on both a wet and dry basis. [6]

The volumetric composition of air is $21\%\ O_2$ and $79\%\ N_2^*$ (atmospheric Nitrogen).

4 (**short**) A fluid with density ρ issues through a circular nozzle of diameter d to form a jet of velocity V. The jet of fluid hits a circular disc, of diameter D, at its centre and perpendicular to the surface as shown in Fig 2. The resultant force on the circular disc is F.

(a) Assuming that F, V, ρ, d and D are the complete set of relevant variables for this problem, how many independent non-dimensional groups are there? [4]

(b) Suggest a possible form for the non-dimensional group(s). [6]

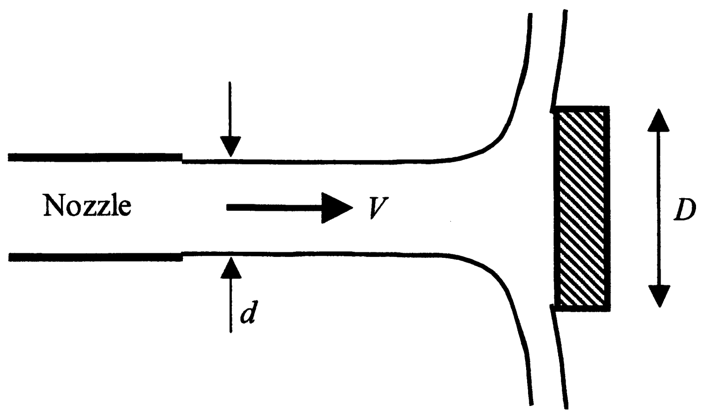

Fig. 2

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5 **(long)** A circular nozzle is designed to produce a parallel jet of water inclined at 60° to the entry direction, as shown in Fig. 3. The entry and exit diameters are 200 mm and 100 mm respectively. The water exits into the atmosphere with an exit velocity of 8 m/s. The flow through the nozzle can be assumed to be inviscid and gravity can be neglected.

(a) Explain why the gauge pressure of the water is zero at the exit of the nozzle. [3]

(b) Calculate the velocity at inlet to the nozzle and the mass flow rate of water. [6]

(c) Calculate the gauge pressure of the water at the entry to the nozzle. [6]

(d) Calculate the force on the nozzle due to the water flow. Express the answer in terms of the x and y coordinate system shown in the figure. [15]

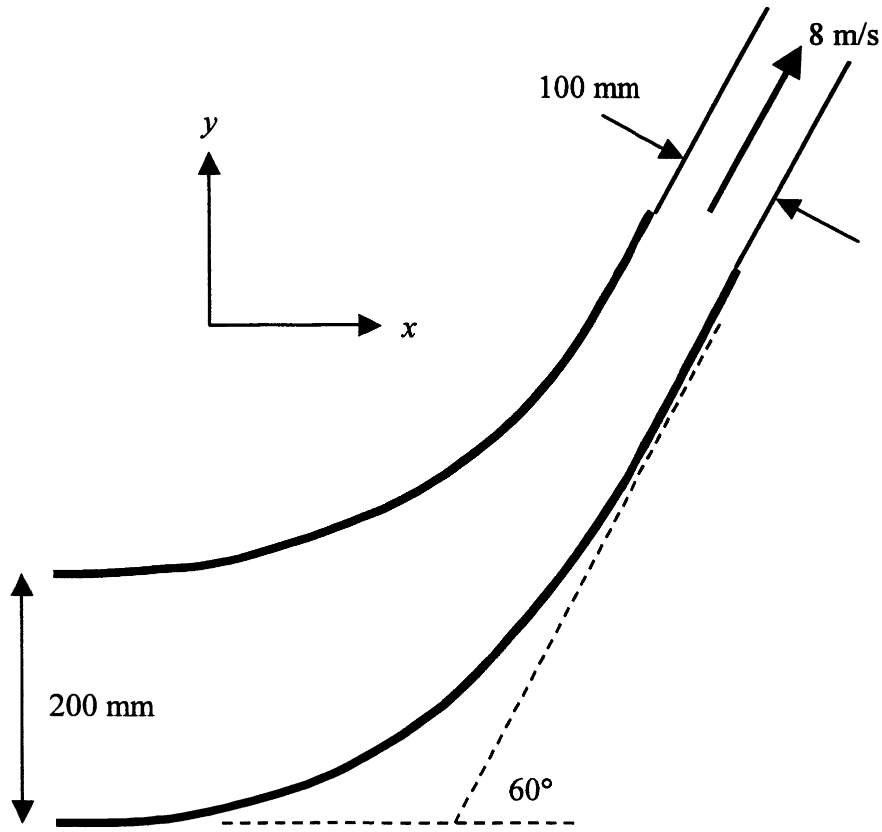

Fig. 3

6 (**long**) (a) Starting from the First Law, show that for a quasi-equilibrium (reversible) adiabatic expansion or compression process of a perfect gas (with adiabatic index γ) the pressure and temperature are related as follows:

$$p \propto T^{\frac{\gamma}{(\gamma-1)}}$$ [6]

In order to increase the efficiency of the air-standard Joule cycle, the turbine exit exhaust gases are used to pre-heat the compressor delivery air before it enters the 'combustion chamber' where the heat input occurs, Fig. 4. The air temperature at the compressor inlet is $T_1 = 300$ K, the temperature at exit from the pre-heater and entry to the 'combustion chamber' is $T_3 = 600$ K, and the turbine entry temperature is $T_4 = 1200$ K. The pressure ratio of the cycle is 5.

(b) Calculate the temperature at the compressor exit and the compressor work per unit mass of air. [6]

(c) Calculate the heat transferred in the pre-heater per unit mass of air. [4]

(d) Calculate the heat input into the cycle per unit mass of air. [4]

(e) Calculate the turbine exit temperature and the turbine work per unit mass of air. [6]

(f) Calculate the efficiency of the cycle. [4]

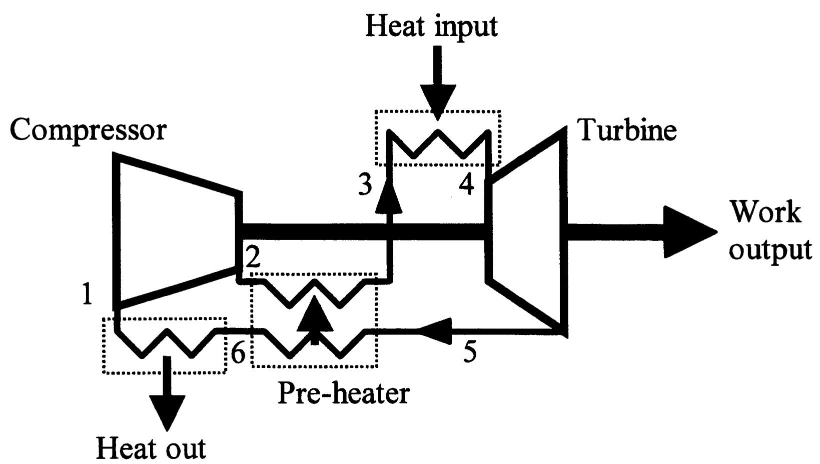

Fig. 4

7 (**long**) A frictionless piston is locked in a cylinder such that the total volume enclosed is $0.8\ m^3$. This volume is divided into two chambers by a partition. The left-hand chamber, volume $0.5\ m^3$, contains helium at 2 bar and 300 K and the right-hand chamber, volume $0.3\ m^3$, contains oxygen at 1 bar and 400 K. The initial arrangement is shown in Fig. 5.

(a) Calculate the mass contained within each chamber. [6]

(b) The partition between the chambers bursts (the piston does not move) and the contents fully mix. Show that the gas constant and the specific heat capacity at constant volume for the helium-oxygen mixture are approximately $910\ J\ kg^{-1}\ K^{-1}$ and $1535\ J\ kg^{-1}\ K^{-1}$ respectively. [8]

(c) Assuming that the contents mix and reach equilibrium sufficiently quickly for the process to be considered adiabatic, calculate the temperature and pressure of the helium-oxygen mixture. [10]

(d) The frictionless piston is now unlocked (released) so that it is free to move. The helium-oxygen mixture expands adiabatically, displacing the piston, until it reaches a pressure of 1 bar. Given that the work done by the helium-oxygen mixture during the expansion is 30 kJ, calculate the temperature and the volume of the mixture at the end of the expansion. [6]

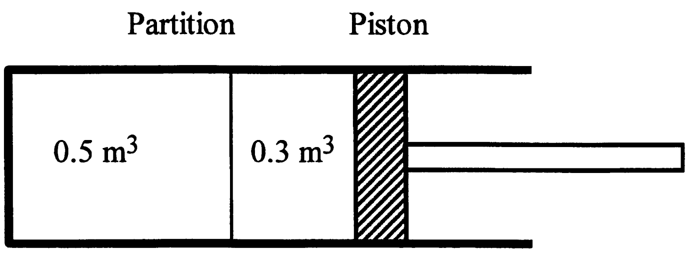

Fig. 5

SECTION B

8 (**short**) Figure 6 shows a plan view of a fishing vessel deploying its net, using a 'paravane' (a submerged board). The vessel is travelling due North at 5 m/s ; the length l of the cable (which remains taut) between the vessel and the paravane is currently 50 m, and is increasing at a steady rate of 2 m/s . The angle θ between the cable and the vessel's wake is $\pi/4$ radians (as shown), and is increasing at a steady rate of 0.1 radians per second.

(a) How fast is the paravane travelling through the water, and in what direction? [5]

(b) What is the direction and magnitude of the paravane's acceleration? [5]

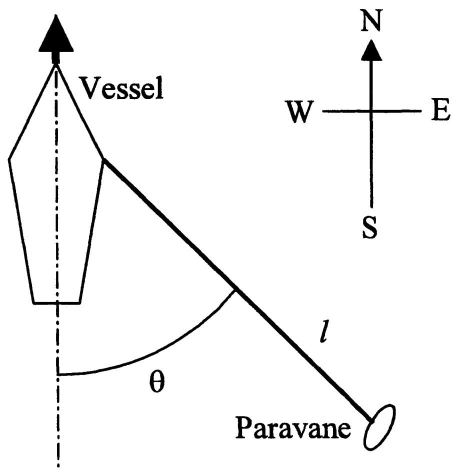

Fig. 6

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9 (**short**) A model 'rocket sledge' runs without friction on a straight horizontal railway track. The empty sledge has mass 2 kg, and it is filled with 2 kg of water under pressure. The sledge is at rest when a valve is opened and the water is ejected backwards at a constant rate of 0.2 kg/s with a *relative* velocity of 30 m/s, until the sledge is empty.

(a) What is the initial acceleration of the sledge? [2]

(b) What is the greatest acceleration of the sledge, and when does it occur? [3]

(c) What is the final speed of the sledge? [5]

10 (**short**) (a) If a body is spinning about a fixed axis, state briefly the circumstances under which the moment of momentum of the body about this axis is conserved. [2]

(b) A figure skater spinning with her arms outstretched can be modelled in plan view as a uniform cylinder of radius r and mass M, plus two thin uniform rods each of length $2r$ and mass $M/12$, as shown in Fig. 7. What is the moment of inertia I of the skater about the axis of her body when her arms are outstretched? [3]

(c) The kinetic energy E of a body spinning about a fixed axis is given by:

$$E = \frac{I\omega^2}{2}$$

Does the skater's kinetic energy change when she pulls her arms in? If so, where has the energy come from, or gone to? [5]

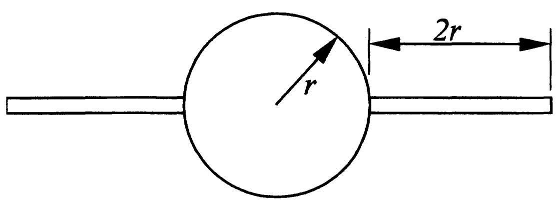

Fig. 7

11 (**short**) Figure 8 shows the mechanical model for an elastomer, which is being loaded by a force f that varies with time t.

(a) Show that the displacement y can be expressed in the form:

$$y + T_1 \dot{y} = Af$$

and find an expression for A, and the time constant T_1. [4]

(b) Show that the displacement z can be expressed in the form:

$$z + T_2 \dot{z} = B\left(f + T_3 \dot{f}\right)$$

and find expressions for B, T_2 and T_3. [6]

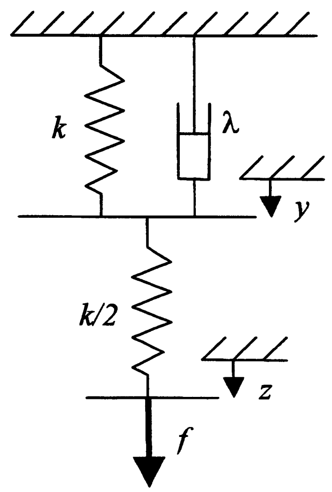

Fig. 8

12 (**long**) Figure 9 shows a side elevation of the mechanism for an 'up and over' garage door. The door is supported by fixed horizontal rails at point A, and by two spring-assisted support arms BD (one on either side of the door), hinged to the door at B and to the frame at C. At the instant shown, the support arms are horizontal and an automatic opener is applying a horizontal force F to point A, which is moving to the left at a speed of 250 mm/s.

(a) Draw a velocity diagram for the door mechanism, and find:
  (i) the velocity of point B;
  (ii) the angular velocity of the support arms;
  (iii) the angular velocity of the springs. [15]

You are advised to let 1 mm represent 5 mm/s, and to place the origin of the diagram at the centre of your answer paper.

(b) The door can be considered as a uniform lamina of weight 500 N, and the other moving parts have negligible weight. If inertia can be ignored and the combined force from both springs at D is 650 N, how big must F be to cause the door to open? [10]

(c) How much bigger must F be, if there is also a total frictional resistance of 20 Nm in the hinges at B? [5]

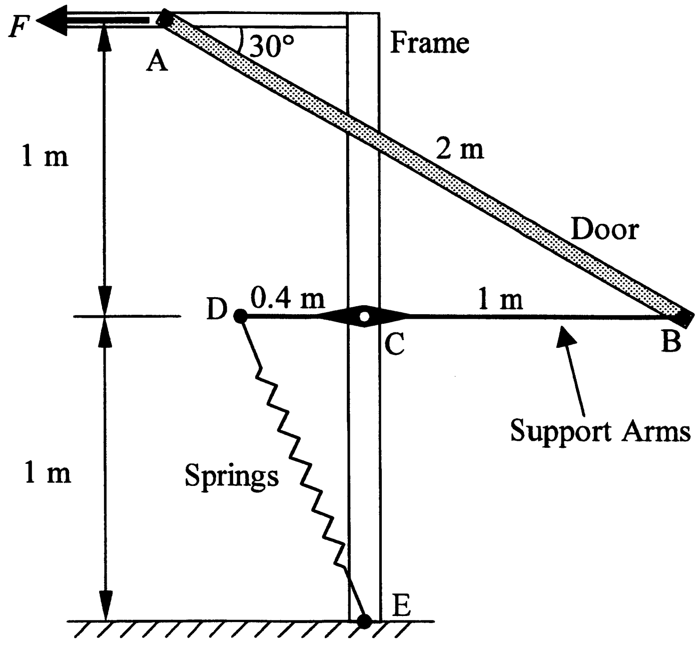

Fig. 9

13 (**long**) A length of coaxial signal cable can be modelled by the circuit shown in Fig. 10.

(a) Show that the relationship between the input voltage e and the output voltage v is given by the differential equation:

$$LC\ddot{v} + RC\dot{v} + v = e$$ [8]

(b) The voltage generator is set to deliver a steady voltage of $e = 50\text{ V}$ and R, L and C have values of $4\ \Omega$, 1 mH, and $10\ \mu\text{F}$ respectively. Initially the switch S is open, and there is no current flowing in the circuit. At time $t = 0$, the switch is closed. Estimate the greatest value that v will reach subsequently, and the time at which it occurs. [8]

(c) The voltage generator is now set to deliver an alternating voltage of amplitude 50 V at a frequency of 2 kHz. The switch S is closed and all transients are allowed to died away. Estimate the amplitude of the voltage v, and its phase in relation to e. [7]

(d) How much additional resistance would have to be added to the circuit to ensure that there was no frequency at which the amplitude of v would be more than twice that of e? [7]

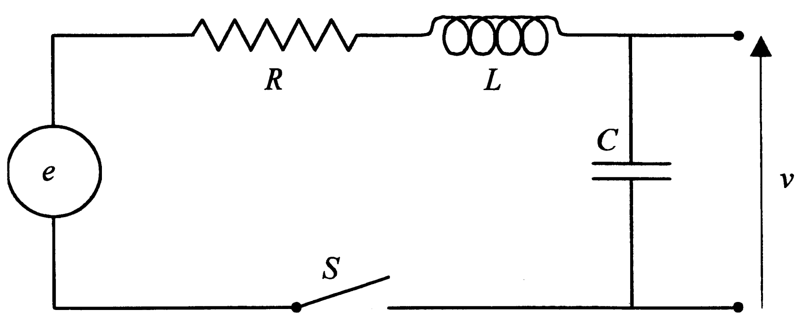

Fig. 10

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