ENGINEERING TRIPOS PART IA

Thursday 9 June 2005 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all eight** short questions and **not more than four** long questions*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1 **(short)** A beam of length L is simply supported at either end (Fig. 1). The left-hand half of the beam carries a uniformly-distributed load w per unit length. The other half of the beam is unloaded.

(a) Calculate the reaction forces at the supports. [3]

(b) Draw the shear force diagram. [3]

(c) Draw the bending moment diagram, identifying the location and magnitude of the largest bending moment. [4]

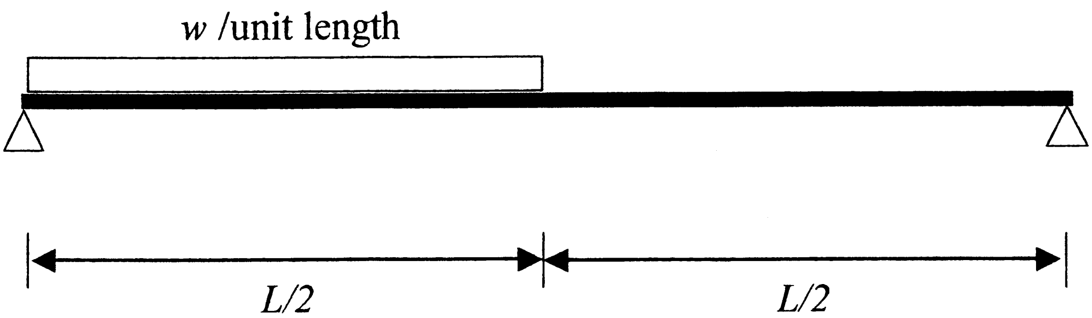

Fig. 1

2 **(short)** A solid rectangular aluminium beam has depth h and breadth b. At a temperature θ_0 the beam is straight. Its response to the loadings (a) and (b) described below is elastic, with Young's modulus E, Poisson's ratio ν and thermal expansion coefficient α.

(a) The top of the beam is heated and the bottom is cooled, so that the temperature at the top is $\theta_0+\theta_1$, the temperature at the bottom is $\theta_0-\theta_1$, and the temperature varies linearly between the top and the bottom. No other loading is applied. Show that the beam bends into a curve, and find its curvature. [5]

(b) In addition to the temperature changes in (a), the beam is completely restraightened by applying a bending moment. Find how large the moment has to be. [5]

3 **(short)** Figure. 2 shows a steel frame ABCDE that supports the roof of a building used for storing sugar beet. The frame is hinged to a foundation at A and E. The joints at B, C and D are rigid.

The frame carries a wind load of 20 kN/m, applied to BC in the direction perpendicular to BC. There are no other external loads. The frame is statically indeterminate, but a measurement shows that the horizontal reaction at E is 74 kN, directed inwards from the foundation to the frame.

(a) Calculate the horizontal reaction at A. [2]

(b) Calculate the vertical reactions at A and E. [3]

(c) Calculate the bending moment at the midpoint of BC. [5]

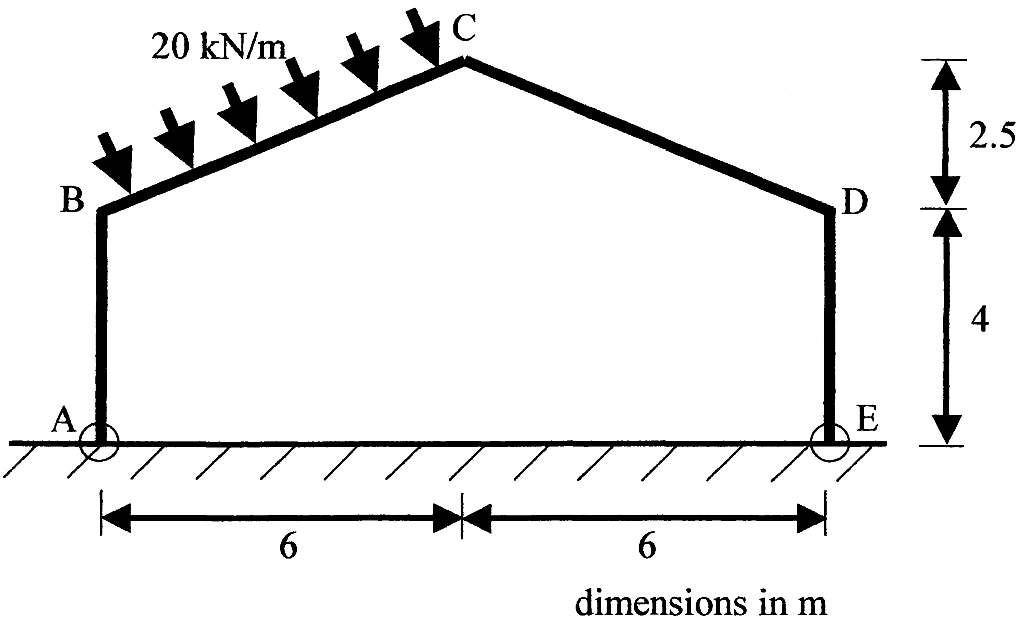

Fig. 2

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4 **(short)** A uniform beam with uniform flexural rigidity is horizontal and simply-supported at both ends (Fig. 3). In its initial unloaded state the beam is **not** perfectly straight, and has an initial displacement $v_0(x)$ from the x-axis; $v_0(x)$ is small compared to the length L. The beam carries an axial compressive force P whose line of action is along the x-axis.

(a) Under load the deflection v, measured upward from the x-axis, is governed by **one** of the following four equations:

$$\text{(i)} \quad EI\frac{d^2v}{dx^2} + Pv \quad = 0$$

$$\text{(ii)} \quad EI\frac{d^2v}{dx^2} + P(v - v_0) \quad = 0$$

$$\text{(iii)} \quad EI\frac{d^2}{dx^2}(v - v_0) + Pv \quad = 0$$

$$\text{(iv)} \quad EI\frac{d^2}{dx^2}(v - v_0) + P(v - v_0) \quad = 0$$

where

EI	is flexural rigidity
v	is deflection measured upwards from the x-axis
x	is distance along the beam
P	is longitudinal force (compressive positive).

Derive the correct equation. Any equations in the data book can be used without proof. [7]

(b) State the boundary conditions that apply to the deflection v, and its derivatives, at each end. [3]

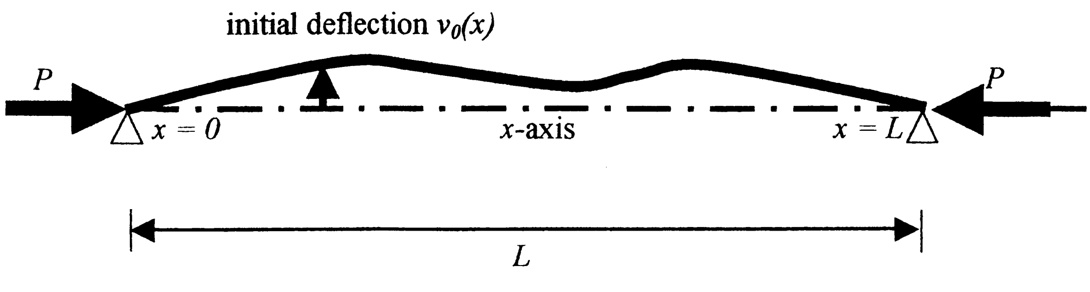

Fig. 3

5 **(long)** A simply supported bridge truss has the layout shown in Fig. 4(a). It has 10 panels, each a broad and b high, so that the span is $10a$. The truss carries a vertical load P at the joint in the lower chord at a distance $3a$ from the left-hand support.

(a) Panel j is one of the panels in the right-hand half of the structure, so that $6 \leq j \leq 10$. Using the method of sections or otherwise, show that the tension in the upper horizontal bar of panel j is

$$-\frac{0.3P(11-j)a}{b}$$ [7]

(b) Derive similar formulae for the other three bars in panel j, counting the vertical bar between panel j and panel $j+1$ as belonging to panel j. [15]

(c) The upper bar in panel j is inadvertently made shorter than intended, so that its length is a-Δ rather than a; Δ is small compared to a. How much does this error change the position of the joint at the bottom right-hand corner of panel 3? [4]

(d) Railway bridges and gantries that support signs on motorways often have the above layout. Explain why it is much more commonly used than the superficially similar layout in Figure 4(b). [4]

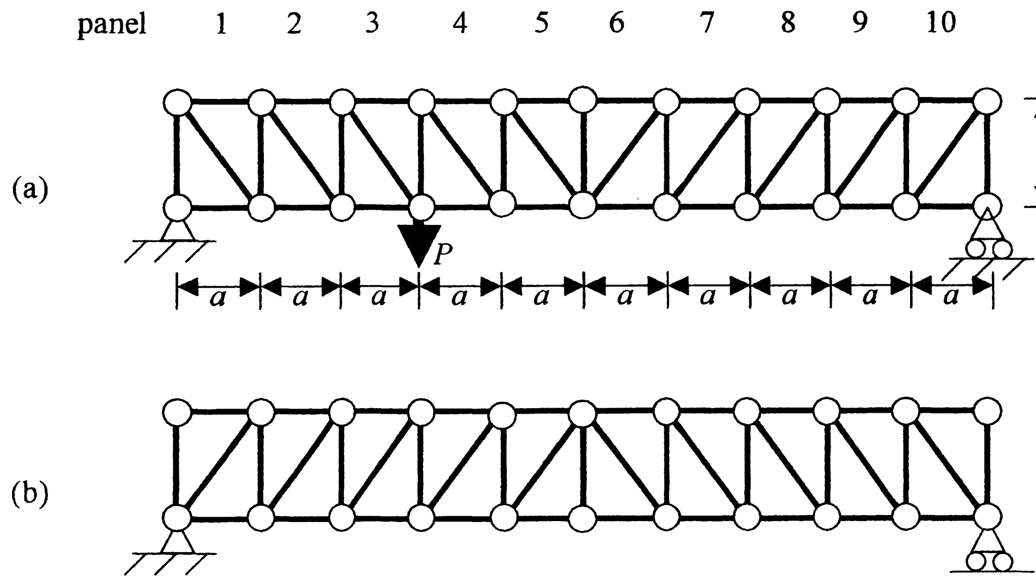

Fig. 4

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6 **(long)** Figure 5 below shows a 356×171×45 Universal Beam section (structures data book, page 13) made from steel with a yield stress of 450 MPa.

(a) Determine the maximum bending moment that can be applied to the beam before it reaches yield. The moment is directed about the *x-x* axis in the Figure, so that it loads the beam in the 'strong' direction. [8]

(b) The beam carries a 500 kN shear force whose line of action is along the *y-y* axis. Draw a sketch indicating the direction of the shear stresses on a cross-section of the beam. [4]

(c) Determine the shear stresses at points A and B on the diagram, under the loading in (b) above. [10]

(d) A useful approximation is that the shear stress in the web is equal to the shear force divided by the cross-sectional area of the web. Test this approximation against your result in (c), and explain briefly why the area taken in the approximation is the web cross-sectional area rather than the total cross-sectional area of the steel. [8]

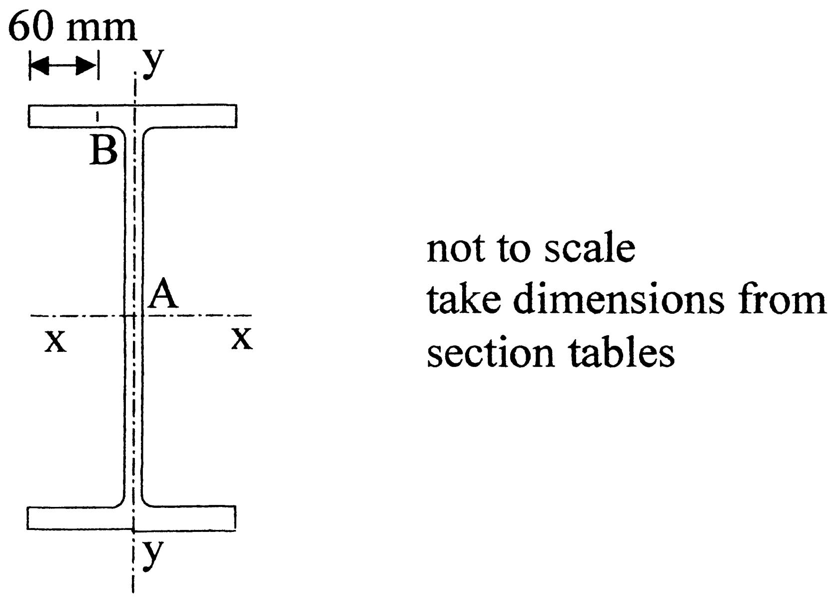

Fig. 5

SECTION B

7 (**short**) (a) Summarise the microstructural features of the structure of a polymer which determine its mechanical properties. State briefly the effect of each. [6]

(b) Sketch how the elastic modulus varies with temperature for the different classes of polymer. [4]

8 (**short**) Nickel forms a solid solution in copper at all concentrations. Table 1 shows the mechanical properties of pure copper and nickel. Sketch the variation with nickel concentration of:

(a) Young's modulus; [2]

(b) the yield stress; [2]

(c) the electrical resistivity; [3]

(d) the fracture toughness. [3]

(Only a qualitative picture is required).

	Modulus	**Yield Strength**	**Fracture Toughness**	**Resistivity**
Cu	112 GPa	30 MPa	270 MPa $m^{1/2}$	1.5×10^{-8} Ω m
Ni	190 GPa	70MPa	80 MPa $m^{1/2}$	6.2×10^{-8} Ω m

Table 1

9 (**short**) (a) What is meant by the term 'electrode potential' and how is it related to 'galvanic protection'? [3]

(b) To what extent can it be used to explain the corrosion rates in water of aluminium, iron and copper? [3]

(c) Put the following types of roof in the order you would expect them to last (longest lasting first) giving reasons for your answer: [4]

(i) iron Roof with iron nails;

(ii) copper roof with copper nails;

(iii) copper roof with iron nails;

(iv) iron roof with copper nails.

10 (**short**) (a) Describe what is meant by a dislocation and say how the presence of dislocations affects the yield stress of a material. [3]

(b) Describe four microscopic mechanisms by which the yield stress of a material can be increased. [4]

(c) An aluminium alloy contains 8% by volume of precipitates which are spheres 15μm in diameter. Assuming they are arranged in a simple cubic lattice determine the distance between them. [3]

11 (**long**) (a) Explain the difference between the 'nominal' and 'true' values of stress and strain in a tensile test. [4]

(b) Explain what is meant by 'necking' and derive an expression for the point on the 'true stress–true strain' curve at which it occurs, stating any assumptions made. [6]

(c) Sketch the load-extension curve for a copper rod at room temperature marking significant points on the curve. [10]

(d) Describe the atomic processes occurring at each stage in the deformation. [6]

(e) How does the curve change if the experiment is carried out in liquid nitrogen? [4]

12 (**long**) (a) Define the terms 'Stress Intensity Factor' (K) and 'Fracture Toughness' (K_{IC}), and explain what they mean. [4]

(b) A steel cylindrical pressure vessel of 5m diameter is to hold a pressure of 4 MPa. What is the minimum wall thickness required if the vessel is to fail by leaking rather than fast fracture when a crack penetrates the wall? Assume $K=\sigma\sqrt{(\pi a)}$ where σ is the applied stress and a is the half length of the crack. The fracture toughness of the steel is 200 MPa $m^{1/2}$. [6]

(c) In fatigue conditions the rate of crack growth da/dN is given by $da/dN=A(\Delta K)^4$ where $A=3\times10^{-14}$ MPa^{-4} m^{-1}. ΔK is the amplitude of the variation in K and N the number of cycles. If the wall thickness is the thickness found in (b), find the maximum initial crack size acceptable if the vess el is to survive 2000 cycles of pressurisation to the design value. Hence calculate the pressure required in a proof test to ensure that such a crack does not exist. [12]

(d) The customer points out that the single cycle of the proof test will increase the length of all cracks so that a previously undersize crack could become unsafe due to fatigue loading. Calculate the extension of a crack of the size found above during the proof test, and state whether this is a significant effect. [8]

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13 **(long)** (a) A power line is to be designed to carry 5 kA at 11 kV using pylons 460 m apart. The dip d, in a wire of weight w per unit length strung between pylons L apart at a tension T is given by $d=L^2w/(8T)$. The maximum tension allowed is 0.8 of the yield stress. Determine which of the materials in Table 2 can be used to avoid tensile failure if the maximum dip allowed is 6 m. [8]

(b) We need to select the material which gives the lowest cost over a twenty year period. The total cost is the sum of the material cost and the cost of the power dissipated in Joule heating. The cost of electricity C_E is 8.3×10^{-9} £/Joule. The cost of the materials is in Table II. Derive an expression for the total cost per metre of cable in terms of the cross sectional area A (which is a free parameter), the material and electrical costs and the material parameters. Show that the minimum cost occurs when the two contributions to the cost are equal. Hence derive a merit index for the material and decide on the best of the materials in Table 2. [18]

(c)Suggest how the design might be improved [4]

Table 2

Material	**Electrical Resistivity ρ_c**	**Yield Stress σ_Y**	**Density ρ**	**Price C**
Units	Ωm	MPa	kg/m^3	£/kg
Al	$1.7\text{x}10^{-8}$	102	2700	1.0
Cu	$1.5\text{x}10^{-8}$	336	8900	1.5
Steel	$55\text{x}10^{-8}$	295	7800	0.3

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