Engineering Tripos Part 1A

2006

Paper 1 Mechanical Engineering

Section A: Numerical Answers

1. –
2. –
3. $\rho_1 = 1.201$ kg/m^3, $\dot{m} = 2.163$ kg/s, $\dot{W} = -172.7$ kWatts
4. –
5. –
6. –

Engineering Tripos Part 1A 2006
Paper 1 Mechanical Engineering
Section B Mechanics

Answers

7   a) $\dfrac{ma^2}{2}$ b) $\dfrac{2g}{3a}$

8   a) $\omega/3$ clockwise   b) $\dfrac{4}{3}Q$

9   a) $\dfrac{1}{\sqrt{2}}\mathbf{e}_t + \dfrac{1}{\sqrt{2}}\mathbf{e}_n$ m/s^2 ($\mathbf{e}_t$ is NE, $\mathbf{e}_n$ is SE)

     b) $90\sqrt{2}$ km

     c) $\dfrac{300}{\sqrt{2}}\mathbf{e}_r + \dfrac{300}{\sqrt{2}}\mathbf{e}_\theta$ m/s, $1\mathbf{e}_r + 0\mathbf{e}_\theta$ m/s^2 ($\mathbf{e}_r$ is E, $\mathbf{e}_\theta$ is N)

     d)   46 m/s^2, -0.09 rad/s^2

10   b) $f = \lambda\left(1 - \exp\left(-\dfrac{k}{\lambda}t\right)\right)$

11   b)   208 Ns/m

12   b) $\omega_1 = \sqrt{\dfrac{\left(3-\sqrt{5}\right)}{2}\dfrac{k}{m}}$, $\omega_2 = \sqrt{\dfrac{\left(3+\sqrt{5}\right)}{2}\dfrac{k}{m}}$

ANSWERS

Part IA Paper 2   SECTION A   STRUCTURES

1. tan 15°, or 0.268

2. a) $T_{max} = \dfrac{wL}{2}\sqrt{1+\left(\dfrac{L}{4D}\right)^2}$

    b) exponential limiting tension ratio in rope due to friction around peg

3. a) Bar: AC AD BD CD CE CF CG DE EG FG FH GH GI HI HJ IJ IK JK
        T/W: 2   0 -3   0   0   2   0 -3 -3 -2√2 2 -√2 -1   1   1 -√2   0   1

    b) Bar: AC AD BD CD CE CF CG DE EG FG FH GH GI HI HJ IJ IK JK
        $\dfrac{eE}{L\alpha\sigma_y}$ 2   0 -1   0   0   2   0 -1 -1 -√2 2 -√2 -1   1   1 -√2   0   1

    c) $\delta_v = 33\, L\alpha\sigma_y/E$; $\delta_h = 8\, L\alpha\sigma_y/E$

4. S: at A, -3W/2; at B, +W/2; at C, +W/2 to -W/2; at D, -W/2 to +W/2; at E, +W/2 to 0

    M: at A, 0; at x = 3L/4, M_{max} = -9WL/16; at B, -WL/2; at C, 0; at D, -WL/2; at E, 0

5. 3W/2

6. (i) $p < \dfrac{\pi}{4}\dfrac{\left(b^4 - a^4\right)}{b^2 h^2}(\sigma_c - \gamma h)$

    (ii) $p < \dfrac{\pi}{4}\dfrac{\left(b^4 - a^4\right)}{b^2 h^2}\gamma h$

M D Bolton   July 2006

Q10)

(a)

(i) 510 MPa, 710 MPa, 30%.

(ii) n/a, 27 MPa, 6%.

(ii) 17 MPa, 23 MPa, 160%.

3

Q11)

(b) (i) a_{wp} = 49.74mm

3

(ii) a_{tp} = 22.11mm

3

(c) (i) N= 8149

10

Q 12)

(a) (i) Materials performance index is $\left(\dfrac{E^{1/2}}{\rho} \right)$

8

(ii) Materials performance index is $\left(\dfrac{\sigma_f}{\rho} \right)$

8

(b) (i)

Mass from result above.

$$t_b = \frac{m_b}{4\pi r^2 \rho}$$

(ii)

Mass from result above.

$$t_f = \frac{m_f}{4\pi r^2 \rho}$$

Set r = 1 m and $p_b = p_f$ = 200 MPa. Values for ρ, E and σ_f are to be taken from the Table of data.

D R H Jones
Michaelmas Term, 2005

Short answers for Section A of 1A Paper 3 2006

1 b) Gain = -20   Input resistance = 600 Ω

c) Gain = -19.96 Input resistance = 601.2 Ω

d) Mid-band voltage gain = -7.5   3 dB frequency = 3.98 MHz

2 b) i) I = 0.25 A   ii) I = 0. 5 A

c) i) I = 126.6 A $\angle$ +6.4^0 ii) f_0 = 73.4 Hz V_{cap} = 369 V $\angle$ -90   Q = 3.07

3. a) R_1 = 40 kΩ R_2 = 2 kΩ

b) Small-signal gain = -9.375

4 a) Load referred to primary = (10 + j 10) Ω

b) Load current = 117 A

c) Load power = 1369 W   Transformer power loss = 479 W   Efficiency = 74.1 %

5. a) Load real power = 3.62 kW   Load reactive power = 1.81 kVAr

b) C = 140 μF to correct power factor to unity.

Engineering Tripos Part 1A 2006 - Paper 3 - Sections B and C numerical answers

A R L Travis

Section B

q 6 $C_4 = A_1 A_0 B_1 B_0$,

         $C_3 = A_1 A_0 B_1 + A_1 B_1 B_0$

         $C_2 = A_1 \overline{A_0} B_1 \overline{B_0}$

         $C_1 = A_1 A_0 B_0 + A_1 \overline{B_1} B_0$

         $C_0 = A_0 + \overline{B_1} \overline{B_0}$

q 7   3 bistables

q 8   V = 0, C = 1, N = 0, Z = 0, B = $3D^H$, A = 39^H, 1 microsecond

q 9 $C = \overline{X_1} X_0 + X_3 \overline{X_1} + X_3 X_0 + X_2 X_1 \overline{X_0}$

Section C

q 10   (a) $\underline{B} = 1000 \times 4\pi \times 10^{-7} \dfrac{100}{2\pi r} \underline{e}_\theta$

      (b) $\underline{B} = \qquad 4\pi \times 10^{-7} \dfrac{100}{2\pi r} \underline{e}_\theta$

      (c)   L = 8.1 x 10^{-4} H

q 11   200 kV, $2\pi\varepsilon_0$ x 10^9

q 12 $\underline{D} = \dfrac{ar^2}{4} \underline{e}_r$

Part IA 2006

Paper 4: Mathematical Methods

Answers

1. $\mathbf{R} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$; $\mathbf{u} = [0 \ 1 \ 0]^T$; $\lambda = 1$.

2. $\mathbf{A} = \dfrac{1}{2} \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$.

3. $x_n = \left[\frac{1}{2}(\sqrt{5} - 1) \right]^n$.

4. (b) $y = e^{-3x} + (x - 1)e^{-2x}$.

5. (a) (ii) $a = 12/\sqrt{26}$, (iii) $z = \frac{6}{13}(5 + i)$.
   (b) $z = (-1)^n \sinh^{-1} 2 + n\pi i$.

6. $2e^{-2t}(1 - e^{-t})$.

7. $(-1, 0)$ and $(-1, 2)$.

8. $x(t) = 1 - \dfrac{8}{\pi^2} \displaystyle\sum_{n=1}^{\infty} \dfrac{\cos(2n - 1)\frac{\pi}{2}t}{(2n - 1)^2}$.

9. (b) $y(t) = \frac{1}{4}e^{-t} - \frac{1}{20}e^{-3t} + \frac{1}{10}\sin t - \frac{1}{5}\cos t \quad (t \geq 0)$.
   (c) $y(0) = \dot{y}(0) = 0$.

10. (a) 105.
    (b) (ii) $a = (1 + p - q)/(2 - q)$, $b = (1 - p)/(2 - q)$, (iii) $q = 2p$.

11. $\mathcal{O}(2^n)$.

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