Engineering Tripos Part 1A

2006

Paper 1 Mechanical Engineering

Section A: Numerical Answers

1. –
2. –
3. $\rho_1 = 1.201$ kg/m^3, $\dot{m} = 2.163$ kg/s, $\dot{W} = -172.7$ kWatts
4. –
5. –
6. –

Engineering Tripos Part 1A 2006
Paper 1 Mechanical Engineering
Section B Mechanics

Answers

7 a) $\frac{ma^2}{2}$ b) $\frac{2g}{3a}$

8 a) $\omega/3$ clockwise b) $\frac{4}{3}Q$

9 a) $\frac{1}{\sqrt{2}}\mathbf{e}_t + \frac{1}{\sqrt{2}}\mathbf{e}_n$ m/s^2 ($\mathbf{e}_t$ is NE, $\mathbf{e}_n$ is SE)

b) $90\sqrt{2}$ km

c) $\frac{300}{\sqrt{2}}\mathbf{e}_r + \frac{300}{\sqrt{2}}\mathbf{e}_\theta$ m/s, $1\mathbf{e}_r + 0\mathbf{e}_\theta$ m/s^2 ($\mathbf{e}_r$ is E, $\mathbf{e}_\theta$ is N)

d) 46 m/s^2, -0.09 rad/s^2

10 b) $f = \lambda\left(1 - \exp\left(-\frac{k}{\lambda}t\right)\right)$

11 b) 208 Ns/m

12 b) $\omega_1 = \sqrt{\frac{\left(3-\sqrt{5}\right)}{2}\frac{k}{m}}$, $\omega_2 = \sqrt{\frac{\left(3+\sqrt{5}\right)}{2}\frac{k}{m}}$

ANSWERS

Part IA Paper 2 SECTION A STRUCTURES

1. tan 15°, or 0.268

2. a) $T_{max} = \frac{wL}{2}\sqrt{1+\left(\frac{L}{4D}\right)^2}$

   b) exponential limiting tension ratio in rope due to friction around peg

3. a)

Bar:	AC	AD	BD	CD	CE	CF	CG	DE	EG	FG	FH	GH	GI	HI	HJ	IJ	IK	JK
T/W:	2	0	-3	0	0	2	0	-3	-3	$-2\sqrt{2}$	2	$-\sqrt{2}$	-1	1	1	$-\sqrt{2}$	0	1

   b)

Bar:	AC	AD	BD	CD	CE	CF	CG	DE	EG	FG	FH	GH	GI	HI	HJ	IJ	IK	JK
$\frac{eE}{L\alpha\sigma_y}$	2	0	-1	0	0	2	0	-1	-1	$-\sqrt{2}$	2	$-\sqrt{2}$	-1	1	1	$-\sqrt{2}$	0	1

   c) $\delta_v = 33\ L\alpha\sigma_y/E$; $\delta_h = 8\ L\alpha\sigma_y/E$

4. S: at A, -3W/2; at B, +W/2; at C, +W/2 to -W/2; at D, -W/2 to +W/2; at E, +W/2 to 0

   M: at A, 0; at x = 3L/4, $M_{max} = -9WL/16$; at B, -WL/2; at C, 0; at D, -WL/2; at E, 0

5. 3W/2

6. (i) $p < \frac{\pi}{4}\frac{\left(b^4 - a^4\right)}{b^2h^2}(\sigma_c - \gamma h)$

   (ii) $p < \frac{\pi}{4}\frac{\left(b^4 - a^4\right)}{b^2h^2}\gamma h$

M D Bolton July 2006

PART 1A ENGINEERING TRIPOS 2006
PAPER 2 STRUCTURES AND MATERIALS

Q10)
(a)
(i) 510 MPa, 710 MPa, 30%.
(ii) n/a, 27 MPa, 6%.
(ii) 17 MPa, 23 MPa, 160%.

3

Q11)

(b) (i) $a_{wp} = 49.74$mm 3

(ii) $a_{tp} = 22.11$mm 3

(c) (i) N= 8149 10

Q 12)

(a) (i) Materials performance index is $\left(\frac{E^{1/2}}{\rho}\right)$ 8

(ii) Materials performance index is $\left(\frac{\sigma_f}{\rho}\right)$ 8

(b) (i)
Mass from result above.

$$t_b = \frac{m_b}{4\pi r^2 \rho}$$

(ii)
Mass from result above.

$$t_f = \frac{m_f}{4\pi r^2 \rho}$$

Set $r = 1$ m and $p_b = p_f = 200$ MPa. Values for ρ, E and σ_f are to be taken from the Table of data.

D R H Jones
Michaelmas Term, 2005

Short answers for Section A of 1A Paper 3 2006

1 b) Gain = -20 Input resistance = 600 Ω

c) Gain = -19.96 Input resistance = 601.2 Ω

d) Mid-band voltage gain = -7.5 3 dB frequency = 3.98 MHz

2 b) i) I = 0.25 A ii) I = 0. 5 A

c) i) $I = 126.6\ A \angle +6.4^0$ ii) $f_0 = 73.4$ Hz $V_{cap} = 369\ V \angle -90$ Q = 3.07

3. a) $R_1 = 40\ k\Omega$ $R_2 = 2\ k\Omega$

b) Small-signal gain = -9.375

4 a) Load referred to primary = (10 + j 10) Ω

b) Load current = 117 A

c) Load power = 1369 W Transformer power loss = 479 W Efficiency = 74.1 %

5. a) Load real power = 3.62 kW Load reactive power = 1.81 kVAr

b) C = 140 μF to correct power factor to unity.

Engineering Tripos Part 1A 2006 - Paper 3 - Sections B and C numerical answers

A R L Travis

Section B

q 6 $C_4 = A_1A_0B_1B_0$,

$C_3 = A_1A_0B_1 + A_1B_1B_0$

$C_2 = A_1\overline{A}_0B_1\overline{B}_0$

$C_1 = A_1A_0B_0 + A_1\overline{B}_1B_0$

$C_0 = A_0 + \overline{B}_1\overline{B}_0$

q 7 3 bistables

q 8 V = 0, C = 1, N = 0, Z = 0, B = $3D^H$, A = 39^H, 1 microsecond

q 9 $C = \overline{X}_1X_0 + X_3\overline{X}_1 + X_3X_0 + X_2X_1\overline{X}_0$

Section C

q 10 (a) $\underline{B} = 1000 \times 4\pi \times 10^{-7} \frac{100}{2\pi r} \underline{e}_\theta$

(b) $\underline{B} = 4\pi \times 10^{-7} \frac{100}{2\pi r} \underline{e}_\theta$

(c) L = 8.1 x 10^{-4} H

q 11 200 kV, $2\pi\varepsilon_0$ x 10^9

q 12 $\underline{D} = \frac{ar^2}{4} \underline{e}_r$

Part IA 2006

Paper 4: Mathematical Methods

Answers

1. $\mathbf{R} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$; $\mathbf{u} = [0 \;\; 1 \;\; 0]^T$; $\lambda = 1$.

2. $\mathbf{A} = \frac{1}{2}\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$.

3. $x_n = \left[\frac{1}{2}(\sqrt{5}-1)\right]^n$.

4. (b) $y = e^{-3x} + (x-1)e^{-2x}$.

5. (a) (ii) $a = 12/\sqrt{26}$, (iii) $z = \frac{6}{13}(5+i)$.
   (b) $z = (-1)^n \sinh^{-1} 2 + n\pi i$.

6. $2e^{-2t}(1-e^{-t})$.

7. $(-1, 0)$ and $(-1, 2)$.

8. $x(t) = 1 - \dfrac{8}{\pi^2}\displaystyle\sum_{n=1}^{\infty} \frac{\cos(2n-1)\frac{\pi}{2}t}{(2n-1)^2}$.

9. (b) $y(t) = \frac{1}{4}e^{-t} - \frac{1}{20}e^{-3t} + \frac{1}{10}\sin t - \frac{1}{5}\cos t \quad (t \geq 0)$.
   (c) $y(0) = \dot{y}(0) = 0$.

10. (a) 105.
    (b) (ii) $a = (1+p-q)/(2-q)$, $b = (1-p)/(2-q)$, (iii) $q = 2p$.

11. $\mathcal{O}(2^n)$.

12. 4000 bytes; `g.lines[2].points[7].x`